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THE QUEST FOR STATUS AND R&D-BASED GROWTH

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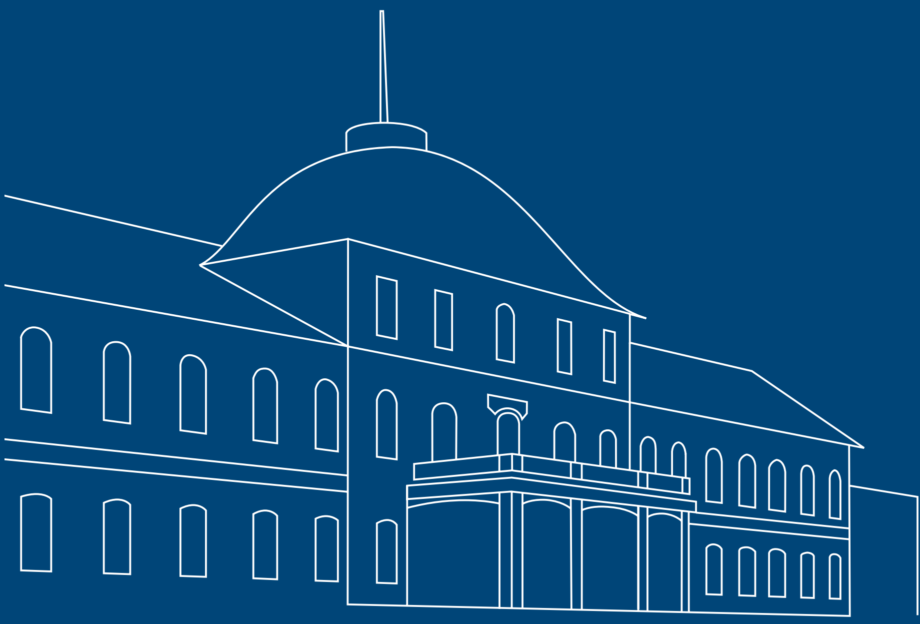

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The Quest for Status and R&D-Based Growth

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Abstract

We analyze the impact of status preferences on technological progress and long-run economic growth within an R&D-based framework. For this purpose, we extend the standard relative wealth approach by allowing the various assets held by households to differ with respect to their status relevance. Relative wealth preferences imply that the effective rate of return on saving in the form of a particular asset is the sum of its market rate of return and its status-related extra return. We show that the status relevance of shares issued by entrants to finance the purchase of new technologies is of crucial importance for long-run growth: First, an increase in the intensity of the quest for status raises the steady-state economic growth rate only if the status-related extra return of these shares is strictly positive. Second, for any given degree of status consciousness, the long-run economic growth rate depends positively on the relative status relevance of shares issued by entrants. Third, while the decentralized long-run economic growth rate is less than its socially optimal counterpart in the standard model, wealth externalities reduce this distortion.

JEL classification: D31, D62, O10, O30.
Keywords: Status concerns, relative wealth, technological progress, long-run economic growth, social optimality.

Wealth is like sea-water; the more we drink, the thirstier we become.
(Arthur Schopenhauer)

1 Introduction

The idea that individuals derive utility not only from absolute consumption, leisure, or wealth but also from their social status is by now well established. It has long been recognized that individuals compare themselves with each other and that they derive extra felicity from outperforming their peers. For example, Adam Smith wrote in *The Wealth of Nations* that "With the greater part of rich people, the chief enjoyment of riches consists in the parade of riches" and John Stuart Mill stated in his *Essay on Social Freedom* that "Men do not desire merely to be rich, but to be richer than other men". This all too human trait is also backed by empirical evidence: Clark and Oswald (1998) and Luttmer (2005) find, by analyzing 5,000 British workers and 8,000 US households, respectively, that self-reported happiness and life satisfaction are lower if, *ceteris paribus*, neighbors an/or colleagues are better off. Their results are statistically and economically significant and robust against various re-specifications of the regressions. Luttmer (2005) concludes that the most promising explanation for his result is the presence of a psychological externality that leads individuals to derive utility from their own status in relation to the status of others. For further empirical support of status concerns see McBride (2001) and Boyce et al. (2010).

In theoretical macroeconomic models, the implications of the quest for status on private consumption, saving, work effort, and the optimal design of distortionary taxation/subsidization have been analyzed extensively.[1] With respect to the analysis of long-run economic growth, however, status-related models did not yet lead to substantially new insights. The reason is that status preferences have no impact on the long-run balanced growth path in standard neoclassical growth models, regardless of whether status is determined by *relative consumption* or by *relative wealth*. While AK type of growth models imply that status preferences of the relative wealth type have an effect on long-run economic performance, i) many implications of the AK model are refuted by the available empirical evidence (cf. Aghion and Howitt, 2009, pp. 56–60) implying that it is not a suitable framework for the analysis of long-run economic growth, ii) there is only one asset (physical capital) in the AK framework, which rules out growth effects due to the possibility that individuals attach different status weights to different forms of assets such as physical capital and shares, iii) AK models do not leave an explanatory role for technological progress, which has been identified as the main driver of long-run economic growth (Acemoglu, 2009, pp. 402–403). Nowadays, multi-sector R&D-based growth models with two types of assets, physical capital and shares, are used to analyze the driving forces of

[1] For the relative consumption specification or more general specifications of consumption externalities see, for example, Abel (1990, 2005), Galí (1994), Harbough (1996), Carroll et al. (1997), Rauscher (1997), Grossmann (1998), Fisher and Hof (2000), Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005), Turnovsky and Monteiro (2007), Fisher and Heijdra (2009), and Strulik (2015). For the relative wealth specification see, for example, Corneo and Jeanne (1997, 2001a,b), Futagami and Shibata (1998), Fisher and Hof (2005, 2008), Van Long and Shimomura (2004), and Fisher (2010). For frameworks that allow for both specifications see Tournemaine and Tsoukis (2008), Ghosh and Wendner (2014), Ghosh and Wendner (2015), and Wendner (2015).

technological change and long-run economic growth. We therefore aim to extend this literature to adequately capture the impact of the quest for status on long-run economic performance.

The contribution of our paper is twofold: First, we close an important gap in the literature by introducing relative wealth preferences into the generic R&D-based growth model of the Romer (1990) type and by analyzing the implications of status concerns for technological progress and growth. To the best of our knowledge this has not been attempted before. Using a semi-endogenous growth model of the Jones (1995) type instead of the Romer (1990) type would not change the basic mechanisms and channels that we identify because all of our results would be present during the transition toward the long-run balanced growth path.[2] Second, we extend the standard relative wealth approach by allowing for the possibility that the assets held by households differ with respect to their status relevance. This extension is inspired by psychological research on whether various categories of items differ with respect to their degree of positionality.[3] As we will see, the differential status effect of traditional physical capital versus those of shares used by entrants to purchase new technologies is of crucial importance for long-run economic growth and has the potential to explain the superior growth patterns of countries in which entrants/startups have better access to new funds.

The introduction of generalized relative wealth preferences into the Romer (1990) model implies that the effective rate of return on saving in the form of a particular asset is the sum of its standard market rate of return and its status-related extra return. In both the Euler equation for consumption and in the no-arbitrage condition, the rental rate of physical capital and the market rate of return of shares issued by entrants are replaced by the corresponding effective rates of return. Hence, the status-seeking motive leads to a rise in the common steady-state effective rate of return of all assets as long as the positive effects of the status-related extra returns are not perfectly offset by the decrease in the market rates of return associated with a higher saving rate. The resulting stronger incentive to save causes the demand for both physical capital and shares issued by entrants to grow at a higher rate. Since the purchase of new technologies by entrants is financed through equity, this raises the demand for R&D. The faster-growing demand for new technologies is in turn satisfied by an increase in employment in the research sector, which leads to an acceleration of technological progress and to faster economic growth.

The main implications of the introduction of generalized relative wealth preferences can be summarized as follows: First, an increase in the intensity of the quest for status raises the steady-state economic growth rate as long as the possession of shares issued by entrants matters for social status. If, however, solely the relative holdings of physical capital are status-relevant, then the status-augmented Romer (1990) model yields the same long-run balanced growth rate as the standard Romer (1990) model. Second, for any given degree of status consciousness both the fraction of wealth held in the form of shares and the long-run economic growth rate depend positively on the relative status weight of shares issued by entrants. Third, while in the standard Romer (1990) model the decentralized long-run economic growth rate is less than its

[2]See Trimborn et al. (2008), and Prettner and Trimborn (2016) for the numerical analysis of the transitional dynamics of semi-endogenous growth models.

[3]See for example Solnick and Hemenway (1998, 2005); Solnick et al. (2007), Johansson-Stenman and Martinsson (2006), and Hillesheim and Mechtel (2013).

socially optimal counterpart, the wealth externalities are able to counterbalance this distortion provided that the effect of shares on status is large enough.

The paper is structured as follows. In Section 2 we introduce the basic assumptions with respect to status preferences and derive the status-augmented versions of i) the Euler equation for consumption, ii) the no-arbitrage condition with respect to the rates of return of physical capital and shares issued by entrants, and iii) the transversality conditions of the representative household's optimization problem. In addition, we present the three sectors of the production side of the economy and derive the system of differential equations that governs the dynamic evolution of the economy. Section 3 contains the main results with respect to the impact of the quest for status on long-run growth and with respect to the importance of the status-relevance of shares in this context. In Section 4 we discuss the results and conclude.

2 The model

2.1 Basic assumptions

Consider a modern knowledge-based economy with three sectors in the vein of Romer (1990)[4]: final goods production, intermediate goods production, and R&D. These sectors employ two production factors, physical capital and labor. Homogeneous labor is employed in the final goods sector and in the R&D sector (for simplicity we refer to labor employed in final goods production as "workers" and to labor employed in R&D as "scientists"). The final goods sector produces a single homogeneous commodity that is used either as consumption good or as physical capital. The varieties produced by the intermediate goods sector are used as inputs in the production of the final good. The R&D sector develops patents for intermediate goods, which are sold to the new firms that enter intermediate goods production. To put it differently, an entrant into the intermediate goods sector has to purchase a new intermediate-specific patent from the R&D sector as a fixed up-front investment to be able to start the production process. These up-front investments are financed by issuing shares that are bought by the households in the economy, which, in turn, receive the associated dividend income and may experience valuation gains.[5]

There exists a continuum of homogeneous households of mass one. The flow budget constraint of the representative household has the following form:

$$\dot{K} + p_Z \dot{Z} = rK + DZ + wL - C, \tag{1}$$

where K denotes physical capital employed by incumbent firms, r is the rental rate of physical capital, Z is the number of shares issued by entrants up to time t, p_Z denotes the price of these shares, D refers to the dividend payments per share, L is exogenously given supply of labor, w is the real wage rate, C refers to consumption, and for any variable x the derivative with

[4]For the sake of simplicity, we follow the literature on horizontal innovations. Similar effects would, however, also be present in case of vertical innovations. For R&D-based growth models in general see Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Segerström (1998), Peretto (1998), Young (1998), Howitt (1999), Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008), and Strulik et al. (2013). For extensive surveys see Gancia and Zilibotti (2005) and Aghion and Howitt (2005).

[5]See, for example, Lehmann-Hasemeyer and Streeb (2016) for the importance of the stock market for innovative firms.

respect to time t is denoted by $\dot{x} \equiv dx/dt$.[6]

Individuals earn labor income and asset income. The former is given by wL, while the latter consists of capital income rK and dividend payments DZ. For simplicity and without loss of generality, we ignore the depreciation of physical capital. To summarize, the right-hand side of the flow budget constraint refers to total saving, while the left-hand side shows that total saving is either used for investments in physical capital or for purchasing the shares that are newly issued by entrants into the intermediate goods sector to finance the patents of the new technologies needed to start production.

In contrast to the standard framework, we employ status preferences to account for the fact that one's own felicity also depends on the comparison with others. More specifically, we assume that instantaneous utility u of the representative consumer depends not only on her consumption C, but also on her status S, i.e., the utility function has the form $u = u(C, S)$, where we assume the following:

$$\frac{\partial u}{\partial C} > 0, \quad \frac{\partial^2 u}{\partial C^2} < 0, \qquad \frac{\partial u}{\partial S} > 0, \quad \frac{\partial^2 u}{\partial S^2} < 0, \qquad \frac{\partial^2 u}{\partial C^2}\frac{\partial^2 u}{\partial S^2} - \left(\frac{\partial^2 u}{\partial C \partial S}\right)^2 \geq 0, \tag{2}$$

$$\frac{\partial\left(\frac{\partial u}{\partial S} \Big/ \frac{\partial u}{\partial C}\right)}{\partial C} > 0, \qquad \frac{\partial\left(\frac{\partial u}{\partial S} \Big/ \frac{\partial u}{\partial C}\right)}{\partial S} < 0, \tag{3}$$

$$\lim_{C \to 0} \frac{\partial u(C, S)}{\partial C} = \infty, \qquad \lim_{C \to \infty} \frac{\partial u(C, S)}{\partial C} = 0. \tag{4}$$

Assumption (2) signifies that the representative consumer derives positive but diminishing marginal utility from both consumption and status. Moreover, the utility function u is jointly concave in C and S. According to (3), the marginal rate of substitution of status for consumption $(\partial u/\partial S) / (\partial u/\partial C)$ depends positively on C and negatively on S. These properties are normality conditions with respect to status and consumption. Finally, (4) introduces standard Inada conditions with respect to the marginal utility of consumption.

With respect to status S, two alternative specifications are employed in the literature. In the relative consumption approach, status S is determined by a comparison of own consumption with average consumption of a reference group. In models with homogeneous agents, average consumption of the total household sector serves as the benchmark. In the relative wealth approach the determination of status rests on a comparison of own wealth with average wealth. We focus our attention on the latter approach, because, as we already explained, this allows us to analyze the differential status effects of physical capital versus shares issued by entrants, which is of crucial importance for long-run growth.

A crucial and distinctive feature of our model is that the components of wealth are allowed to differ with respect to their effect on social status. More specifically, we assume that

$$S = S\left(\Omega, \bar{\Omega}\right), \qquad \frac{\partial S}{\partial \Omega} > 0, \quad \frac{\partial^2 S}{\partial \Omega^2} \leq 0, \quad \frac{\partial S}{\partial \bar{\Omega}} < 0, \tag{5}$$

[6]Note that the derivative of total wealth $K + p_Z Z$ with respect to time t is obtained by adding the valuation gains of shares $\dot{p}_Z Z$ on both sides of the flow budget constraint (1) such that $\dot{K} + p_Z \dot{Z} + \dot{p}_Z Z = rK + DZ + \dot{p}_Z Z + wL - C$.

where

$$\Omega \equiv \phi_K K + \phi_Z p_Z Z \qquad \text{and} \qquad \bar{\Omega} \equiv \phi_K \bar{K} + \phi_Z p_Z \bar{Z} \tag{6}$$

are the status-relevant measures of own wealth and average economy-wide wealth, respectively. The parameter $\phi_Z \geq 0$ is the status weight of shares issued by entrants to purchase new technologies from the R&D sector. The parameter $\phi_K \geq 0$ refers to the status weight of physical capital employed by incumbent intermediate firms. For simplicity, it is often assumed in the literature that households own the physical capital. Non-entrepreneurial households rent it out to firms, while entrepreneurial households might also employ it in their own firms. A common hypothesis of the psychological status literature is that the observability/visibility of an item exerts a significant positive effect on its degree of positionality. Hence, if households literally own physical capital, then the observability/visibility aspect implies that this possession entails a stronger status effect than the ownership of shares. Think in this context of owning the Empire State Building or other commercial properties and renting out their office space to firms. This interpretation would give rise to the parameter specification $\phi_K > \phi_Z$. The alternative assumption is that physical capital is in the possession of firms. Households supply the funds that are required for its purchase either directly (via financial markets) or indirectly (via banks). In case that firms finance investment through bank loans or corporate bonds, implying that rK has to be interpreted as interest income, it is plausible that the ownership of shares exerts a higher status effect than financial claims held in the form of loans or bonds. A rationale for the superiority of shares is that savings accounts or corporate bonds are often considered as conservative, uninspiring, or even dull forms of wealth. This interpretation would imply the parameter specification $\phi_K < \phi_Z$. Even if incumbent firms finance the purchase of physical capital through equity, these shares need not be as status relevant as those issued by startups. One may, in this context, think of innovation-friendly societies in which the segment of shares that is associated with technological progress is more status relevant or more fashionable than other shares so that $\phi_K < \phi_Z$. In such societies it could even be a social norm to act in a way that fosters progress (a suitable example might be the US). However, the opposite could be true in conservative societies in which people are afraid of the potential negative effects and dangers of innovation (examples for this might include some European countries) and as a result prefer claims against incumbent firms implying that $\phi_K > \phi_Z$. The standard relative wealth approach in which all assets are treated as equally status relevant is obtained by setting $\phi_K = \phi_Z = 1$ so that Ω is identical to the standard definition of wealth as given by $K + p_Z Z$. This specification would, for instance, fit to situations in which households are able to assess the total magnitude of wealth held by other households, but don't have any detailed information about the compositions of their portfolios.

In our comprehensive theoretical analysis we allow for all of the relative status cases mentioned above. It will become clear that the long-run economic growth rate depends crucially upon the relative status relevance of shares issued by entrants. Our generalization therefore yields additional insights that cannot be obtained in the standard relative wealth approach in which all assets are of equal importance with respect to status or in an AK growth model in which only one asset exists.

In (5) we assume that status $S = S\left(\Omega, \bar{\Omega}\right)$ increases in own wealth Ω, with marginal status being non-increasing, and decreases in average wealth $\bar{\Omega}$. The latter implies negative wealth externalities. In the status literature it is common practice to restrict attention to symmetric equilibria in which identical agents make identical choices such that $\Omega = \bar{\Omega}$ holds along an equilibrium path. With respect to symmetric situations, we follow Fisher and Hof (2005) and assume that the following condition holds:

$$S(\Omega, \Omega) = \chi = \text{constant}, \qquad \text{for } \Omega > 0. \tag{7}$$

Assumption (7) ensures that our approach corresponds to a pure *relative* wealth specification because the flow of utility is independent of the *level* of wealth Ω along any symmetric equilibrium path, i.e., $u[C, S(\Omega, \Omega)] = u(C, \chi)$.[7] It is easily verified that two standard specifications of the status literature, the difference specification

$$S\left(\Omega, \bar{\Omega}\right) = \varphi\left(\Omega - \bar{\Omega}\right), \qquad \varphi' > 0, \varphi'' \leq 0 \tag{8}$$

and the ratio specification

$$S\left(\Omega, \bar{\Omega}\right) = \varphi\left(\Omega / \bar{\Omega}\right), \qquad \varphi' > 0, \varphi'' \leq 0 \tag{9}$$

satisfy Assumption (5) and Assumption (7).

By optimally choosing the time paths of C, $\dot{K}$, and $\dot{Z}$, the representative household maximizes overall utility as given by

$$\int_0^\infty e^{-\rho t} u\left[C, S\left(\Omega, \bar{\Omega}\right)\right] dt,$$

where $\rho > 0$ denotes the time-preference rate, subject to the flow budget constraint (1), the definitions of Ω and $\bar{\Omega}$ as given by (6), and the initial conditions $K(0) = K_0$ and $Z(0) = Z_0$. A crucial feature of this optimization problem is that the representative household takes the time paths of w, r, p_Z, D, and $\bar{\Omega}$ as given. This is due to the fact that in a continuum of households each single household has mass zero and its choices do not affect aggregate variables.

A detailed analysis of this optimization problem is provided in Appendix A. Here we only mention the three aspects with respect to which the symmetric equilibrium of the status-augmented model differs from the equilibrium of the standard Romer (1990) model.

The first modification is that the no-arbitrage condition between saving in terms of physical capital and saving in terms of shares that holds in the standard Romer (1990) model

$$r = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} \tag{10}$$

has to be replaced by

$$r + \varepsilon^K\left(C, \Omega, \chi, \phi_K\right) = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} + \varepsilon^Z\left(C, \Omega, \chi, \phi_Z\right) \tag{11}$$

[7] Note that absolute wealth Ω would play a role along symmetric equilibrium paths if the instantaneous utility function $u(C, S)$ was replaced by $u(C, \Omega, S)$.

where

$$\varepsilon^K(C,\Omega,\chi,\phi_K) \equiv MRS(C,\Omega,\chi)\times\phi_K, \tag{12}$$
$$\varepsilon^Z(C,\Omega,\chi,\phi_Z) \equiv MRS(C,\Omega,\chi)\times\phi_Z, \tag{13}$$
$$MRS(C,\Omega,\chi) \equiv \frac{\partial u(C,\chi)}{\partial S}\frac{\partial S(\Omega,\Omega)}{\partial\Omega}\left[\frac{\partial u(C,\chi)}{\partial C}\right]^{-1}. \tag{14}$$

On the left-hand side of (11), $r+\varepsilon^K$ is the *effective* rate of return of wealth accumulation in the form of physical capital, where r is the market rental rate of physical capital, while $\varepsilon^K(C,\Omega,\chi,\phi_K)$ as defined in (12) is the *status-related extra return* of physical capital. On the right-hand side of (11), $(D+\dot{p}_Z)/p_Z+\varepsilon^Z$ is the *effective* rate of return of wealth accumulation in the form of shares, where the market return $(D+\dot{p}_Z)/p_Z$ results from dividend payments and valuation gains, while $\varepsilon^Z(C,\Omega,\chi,\phi_Z)$ as defined in (13) is the *status-related extra return* of shares. The term $MRS(C,\Omega,\chi)$ defined in (14) is the marginal rate of substitution of status-relevant own wealth Ω for consumption C as perceived by the representative agent in a symmetric state in which $\Omega=\bar{\Omega}$ holds. Taking into account that ϕ_K is the weight of physical capital in the status-relevant measure of wealth, $\Omega=\phi_K K+\phi_Z p_Z Z$, it follows that $\varepsilon^K = MRS\times\phi_K$ is the symmetric MRS of own physical capital K for consumption C. Analogously, $\varepsilon^Z = MRS\times\phi_Z$ is the symmetric MRS of own shares Z for consumption C. The economic interpretation of ε^K given above can also be verified as follows: From the perspective of the representative household, an increase in own physical capital K by a marginal unit causes the status-relevant measure of wealth Ω to increase by ϕ_K units. Since i) the household takes average wealth as given, and ii) $\Omega=\bar{\Omega}$ holds at the outset of our thought experiment, this increase in Ω by ϕ_K units causes status S to rise by $[\partial S(\Omega,\Omega)/\partial\Omega]\,\phi_K$ units and felicity u to increase by $[\partial u(C,\chi)/\partial S]\,[\partial S(\Omega,\Omega)/\partial\Omega]\,\phi_K$ units. Dividing the latter expression by the marginal utility of own consumption, $\partial u(C,\chi)/\partial C$, we obtain the amount of consumption C that the status-conscious household is willing to give up in exchange for an increase in K by a marginal unit. Analogous considerations can be used for the interpretation of ε^Z.

It can be shown that the partial derivatives of $MRS(C,\Omega,\chi)$ exhibit the following properties: i) The normality assumption with respect to status given by the first inequality in (3) implies that $\partial MRS/\partial C>0$; ii) under the ratio specification of the status function (9) we obtain $\partial MRS/\partial\Omega<0$. The difference specification (8) implies that $\partial MRS/\partial\Omega=0$ holds. Already at this point it is obvious that this property of the difference specification rules out the existence of a balanced growth path, because permanent growth would lead to ever-increasing status-related extra returns of physical capital and shares, ε^K and ε^Z.

The second modification as compared to Romer (1990) refers to the Euler equation for consumption. In the standard framework, where $u=u(C)$, it holds that

$$\frac{\dot{C}}{C}=\sigma(C)(r-\rho)\,,\qquad \text{with}\qquad \sigma(C)\equiv-\frac{u'(C)}{Cu''(C)}.$$

In our case this has to be replaced by

$$\frac{\dot{C}}{C} = \sigma^{S}(C,\chi)\left[r + \varepsilon^{K}(C,\Omega,\chi,\phi_{K}) - \rho\right], \tag{15}$$

where, according to Assumption (7), $\chi = \text{constant} = S(\Omega,\Omega)$, for $\Omega > 0$. The term $\sigma^{S}(C,\chi)$ in which the superscript "S" refers to "status" is the *effective* elasticity of intertemporal substitution under relative wealth preferences in a symmetric equilibrium as given by

$$\sigma^{S}(C,\chi) \equiv -\frac{\partial u(C,\chi)}{\partial C}\left[C\frac{\partial^{2} u(C,\chi)}{\partial C^{2}}\right]^{-1}. \tag{16}$$

For a given value of the rental rate of capital r, the modified Euler equation implies the following: the higher the status-related component of the effective rate of return, ε^{K}, the higher is the growth rate of consumption $\dot{C}/C$. In other words, the willingness to substitute future consumption for present consumption increases, implying that individuals save more. In a general macroeconomic equilibrium as analyzed below, r is determined endogenously. It is therefore possible that the positive impact of $\varepsilon^{K} > 0$ on the effective rate of return $r + \varepsilon^{K}$ is partially or even completely offset by a fall in the market rate of return r.

The third modification concerns the transversality conditions. In the standard model they are given by

$$\lim_{t\to\infty}\left\{\exp\left[-\int_{0}^{t} r(v)dv\right]K\right\} = 0, \qquad \lim_{t\to\infty}\left\{\exp\left[-\int_{0}^{t}\left[r(v)\right]dv\right]p_{Z}Z\right\} = 0, \tag{17}$$

such that the present values of wealth held in the form of physical capital and shares must converge to zero as time goes to infinity. In the model with relative wealth preferences, the market rate of return r is replaced by the effective rate of return $r + \varepsilon^{K}$ such that

$$\begin{aligned} &\lim_{t\to\infty}\left\{\exp\left[-\textstyle\int_{0}^{t}\left[r(v) + \varepsilon^{K}(v)\right]dv\right]K\right\} = 0, \\ &\lim_{t\to\infty}\left\{\exp\left[-\textstyle\int_{0}^{t}\left[r(v) + \varepsilon^{K}(v)\right]dv\right]p_{Z}Z\right\} = 0, \end{aligned} \tag{18}$$

where $\varepsilon^{K}(v) = \varepsilon^{K}\left[C(v),\Omega(v),\chi,\phi_{K}\right]$.

2.2 Production side

The production side of the economy follows the standard R&D-based growth literature so that our description will be short and focused on the main parts that we need in the subsequent analysis.

The final goods sector consists of a continuum of perfectly competitive firms of mass one, each of which produces the same single good by employing the same technology. The production function of the representative firm is given by

$$Y = L_{Y}^{1-\alpha}\int_{0}^{A} x_{i}^{\alpha}\, di, \tag{19}$$

where Y is output, L_Y denotes labor input, and x_i is the amount of the intermediate good of type $i \in [0, A]$ used in final goods production. In this context, A refers to the technological frontier, i.e., the spectrum of patents for specific varieties i that has already been discovered by R&D in the past. For simplicity, the elasticities of output with respect to the various types of intermediate goods are identical and given by $\alpha \in (0, 1)$. Since, by assumption, the mass of firms equals one, output and labor input of the representative firm coincide with GDP and aggregate employment in the final goods sector, respectively. The perfectly competitive representative firm takes both the real wage w_Y in the final goods sector and the real prices of intermediate goods p_i as given and maximizes profits by choosing the inputs L_Y and x_i. The corresponding first-order conditions (FOCs) are

$$w_Y = (1-\alpha)\, L_Y^{-\alpha} \int_0^A x_i^\alpha \, di = (1-\alpha) \int_0^A \left(\frac{x_i}{L_Y}\right)^\alpha di, \tag{20}$$

$$p_i = \alpha L_Y^{1-\alpha} x_i^{-(1-\alpha)} = \alpha \left(\frac{x_i}{L_Y}\right)^{-(1-\alpha)}. \tag{21}$$

These conditions require that each input is utilized up to the point at which its marginal product equals its real price (i.e., its price in terms of the final good). From (20) and (21) it follows that – in a general equilibrium – the remuneration of workers equals $(1-\alpha)$ percent of real revenue Y, while α percent are used to pay for the intermediate goods.

Entrants into the intermediate goods sector have to purchase the patent of a new technology from the R&D sector as up-front investment before they can produce the corresponding patent-specific differentiated intermediate good. The incumbent firms $i \in [0, A]$ in the intermediate goods sector employ a single variable production factor, physical capital, which it either rents from private households or finances through bonds or loans. The production function is assumed to be linear and, without loss of generality, the productivity of physical capital is normalized to one such that $x_i = k_i$. Taking into account this linear production function and the first-order condition for the optimal input of x_i in the final goods sector [see (21)], operating profits of intermediate goods producers can be written as

$$\pi_i = p_i x_i - r x_i = \alpha L_Y^{1-\alpha} x_i^\alpha - r x_i.$$

Profit-maximization implies that prices are set according to the rule

$$p_i = \frac{1}{\alpha} r, \tag{22}$$

where the rental rate r represents marginal cost and $(1/\alpha) > 1$ is the gross markup we are familiar with from Dixit and Stiglitz (1977). Hence, firms have a certain degree of price setting power and operating profits will be positive. This price setting policy implies that production x_i and input of physical capital k_i in the intermediate goods sector depend negatively on the rental rate r and positively (in a linear way) on employment in the final goods sector L_Y:

$$x_i = k_i = \left(\frac{\alpha^2}{r}\right)^{1/(1-\alpha)} L_Y. \tag{23}$$

The same is true for operating profits:

$$\pi_i = (1-\alpha)\,\alpha^{(1+\alpha)/(1-\alpha)} r^{-\alpha/(1-\alpha)} L_Y. \tag{24}$$

Since all incumbent firms $i \in [0, A]$ make identical choices, we can drop the index i in the subsequent analysis and use the notation p, x, k, and π instead.

The R&D sector employs scientists L_A to discover new technologies in the form of blueprints A according to the production function

$$\dot{A} = \lambda A L_A, \tag{25}$$

where λ refers to the productivity of scientists. There is perfect competition in the research sector such that R&D firms take both the real price of blueprints p_A and the real wage of scientists w_A as given. Since the production function (25) is linear in L_A, the profit of the representative R&D firm $(p_A \lambda A - w_A)\,L_A$ is linear in L_A, too. Hence, the existence of profit-maximizing production plans with $L_A > 0$ requires that scientists are paid their marginal product, i.e.,

$$p_A \lambda A = w_A. \tag{26}$$

2.3 Market clearing and equilibrium dynamics

We close the model by introducing the market clearing conditions for all markets. Afterwards we derive a system of differential equations that governs the dynamic evolution of the economy in a symmetric macroeconomic equilibrium. In such an equilibrium households maximize utility, firms maximize profits, and all market clearing conditions are satisfied. The word "symmetric" means that households – being identical in every respect – and firms – facing identical cost and demand functions – make identical choices.

Equilibrium in the labor market requires that the wage rates earned in the final goods sector and in the R&D sector are equal because labor is homogenous. In addition, the sum of labor inputs in these two sectors must equal the exogenously given labor supply of households:

$$w_Y = w_A = w \qquad \text{and} \qquad L_Y + L_A = L. \tag{27}$$

Equilibrium in the rental market for physical capital requires that the supply of capital (K) is equal to the aggregate capital input of firms in the intermediate goods sector (Ak). Using (23), this condition can be written as

$$K = Ak = A\left(\frac{\alpha^2}{r}\right)^{1/(1-\alpha)} L_Y. \tag{28}$$

Equilibrium in the stock market requires that all previously and newly issued shares are held by households. The normalization of the number of shares that are issued by a single intermediate firm to 1 yields $Z = A$ and $\dot{Z} = \dot{A}$. Since all firms in the intermediate goods sector earn identical profits, their shares will have the same price in equilibrium. From the no-arbitrage condition under status preferences (11) it follows that the common price of shares at time t is given by

$$p_Z(t) = \int_t^{\infty} \exp\left\{-\int_t^{\tau} [r(v) + \Gamma(v)]\, dv\right\} D(\tau)\, d\tau,$$

where

$$\Gamma \equiv \varepsilon^K(C, \Omega, \chi, \phi_K) - \varepsilon^Z(C, \Omega, \chi, \phi_Z) = MRS(C, \Omega, \chi)(\phi_K - \phi_Z). \tag{29}$$

Future dividend payments are discounted by $r + \Gamma \equiv r + \varepsilon^K - \varepsilon^Z$, i.e., the sum of the market rental rate r and the difference between the status-related extra returns of wealth accumulation in the form of physical capital and shares, $\varepsilon^K - \varepsilon^Z$. If $\phi_K = \phi_Z$, the extra returns ε^K and ε^Z are equal, such that the formula for the calculation of the fundamental value of a stock simplifies to the standard expression in Romer (1990).

New entrants into the intermediate goods sector have to buy a new technology in the form of a patent at price p_A the purchase of which is financed by issuing a new share. Due to free entry, competition between new entrants will cause p_A to reach the highest possible level. Since the price of a share attains its maximum if the operating profit is fully distributed in the form of dividends, we have that $D = \pi$ holds in equilibrium and the share price is given by

$$p_A(t) = p_Z(t) = \int_t^{\infty} \exp\left\{-\int_t^{\tau} [r(v) + \Gamma(v)]\, dv\right\} \pi(\tau)\, d\tau, \tag{30}$$

where Γ is defined by (29). Differentiating (30) with respect to time t, we obtain the following differential equation for the evolution of the price of patents:

$$\frac{\dot{p}_A}{p_A} = r + MRS(C, \Omega, \chi)(\phi_K - \phi_Z) - \frac{\pi}{p_A}. \tag{31}$$

Substituting various results derived above into the flow budget constraint of the representative household (1), we show in the Supplement that the market for final goods is also in equilibrium, i.e.,

$$Y = C + \dot{K}, \tag{32}$$

such that output of final goods is either consumed or used for investment in physical capital.

Putting all the information together, we show in the Supplement that the dynamic evolution of the four variables A, K, L_A, and C is governed by the following system of differential

equations:

$$\frac{\dot{A}}{A} = \lambda L_A, \tag{33}$$

$$\frac{\dot{K}}{K} = \left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)} - \frac{C}{K}, \tag{34}$$

$$\frac{\dot{C}}{C} = \sigma^S(C,\chi)\left\{\alpha^2\left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)} + \varepsilon^K(C,\Omega,\chi,\phi_K) - \rho\right\}, \tag{35}$$

$$\begin{aligned}\dot{L}_A = (L-L_A)\Bigg\{&-(1-\alpha)\left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A \\ &-\lambda(L-L_A) + \frac{\varepsilon^K(C,\Omega,\chi,\phi_K) - \varepsilon^Z(C,\Omega,\chi,\phi_K)}{\alpha}\Bigg\},\end{aligned} \tag{36}$$

where

$$\Omega = \phi_K K + \phi_Z \frac{(1-\alpha)A}{\lambda}\left[\frac{K}{A(L-L_A)}\right]^{\alpha} \tag{37}$$

holds in (35) and (36). Inspection of the system (33)–(37) reveals that, as in the standard framework of Romer (1990), we need to impose additional structure on the preferences to ensure the existence of a balanced growth path (BGP). The BGP is defined as a stationary equilibrium in which the variables A, K, C, and Ω grow at the same constant rate

$$g^* = \left(\dot{A}/A\right)^* = \left(\dot{K}/K\right)^* = \left(\dot{C}/C\right)^* = \left(\dot{\Omega}/\Omega\right)^* > 0,$$

while the variables L_A, $K/[A(L-L_A)]$, C/K, and

$$\frac{C}{\Omega} = \frac{\frac{C}{K}}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda(L-L_A)}\left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)}} \tag{38}$$

remain unchanged at their steady-state levels L_A^*, $\{K/[A(L-L_A)]\}^*$, $(C/K)^*$, and $(C/\Omega)^*$. A crucial element of the derivation of the differential equations (34)–(36) is the fact that the variables r, p, π, w/A, p_A, and Y/K can be expressed as functions of $K/[A(L-L_A)]$ and $(L-L_A)$. Hence, along the BGP we are looking for, these variables are constant, while aggregate output, per capita output, and wages grow at rate g^*.

In the rest of the paper we restrict our attention to specifications of the instantaneous utility function $u(C,S)$ and the status function $S\left(\Omega,\bar{\Omega}\right)$ such that i) the symmetric effective elasticity of intertemporal substitution under relative wealth preferences does not depend on C, i.e.,

$$\frac{\partial \sigma^S(C,\chi)}{\partial C} = 0, \tag{39}$$

and ii) the symmetric marginal rate of substitution of status-relevant own wealth Ω for consumption C, $MRS(C,\Omega,\chi)$, can be expressed as a function of C/Ω. Since we are also interested in analytical solutions, we employ the stronger assumption that $MRS(C,\Omega,\chi)$ depends linearly on C/Ω such that

$$MRS = \eta \times \frac{C}{\Omega} \qquad \eta > 0, \tag{40}$$

where η represents the intensity of the quest for status of the representative consumer. The sign of η follows from the already mentioned fact that $\partial MRS/\partial C > 0$ holds because of the normality of status. Equation (40) implies that the status-related extra returns ε^K and ε^Z are linear functions of C/Ω:

$$\varepsilon^K = \phi_K \eta \times \frac{C}{\Omega}, \qquad \varepsilon^Z = \phi_Z \eta \times \frac{C}{\Omega}. \tag{41}$$

The structure that we impose by (39) and (40) on $u(C,S)$ and $S\left(\Omega,\bar{\Omega}\right)$ is rather mild. In the Supplement we show that the quite general specification of relative wealth preferences given by

$$u(C,S) = V\left[g(C)\,h(S)\right], \qquad S = S\left(\Omega,\bar{\Omega}\right),$$

satisfies the properties (39) and (40) if i) the instantaneous utility function has the form

$$u(C,S) = \frac{1}{1-\theta}\left\{\left[C^{\xi}h(S)\right]^{1-\theta} - 1\right\}, \qquad \xi > 0, \quad \theta > 0, \quad 1 + \xi(\theta - 1) > 0 \tag{42}$$

where $h(S) > 0$ and $h'(S) > 0$, and ii) the status function exhibits the ratio specification

$$S\left(\Omega,\bar{\Omega}\right) = \varphi\left(\Omega/\bar{\Omega}\right), \qquad \varphi' > 0, \quad \varphi'' \leq 0. \tag{43}$$

These specifications of $u(C,S)$ and $S\left(\Omega,\bar{\Omega}\right)$ imply that $\chi = S(\Omega,\Omega) = \varphi(1)$ and

$$\sigma^S = \frac{1}{1+\xi(\theta-1)} > 0, \tag{44}$$

$$MRS = \eta \times \frac{C}{\Omega}, \qquad \eta \equiv \frac{\beta}{\xi} > 0, \quad \beta \equiv \frac{h'\left[\varphi(1)\right]\varphi'(1)}{h\left[\varphi(1)\right]} = \frac{h'(\chi)\,\varphi'(1)}{h(\chi)} > 0. \tag{45}$$

For $\xi = 1$ these results simplify to $\sigma^S = 1/\theta$ and $\eta = \beta$.

Substituting (41) into (35) and (36) yields

$$\frac{\dot{C}}{C} = \sigma^S\left\{\alpha^2\left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)} + \phi_K\eta\frac{C}{\Omega} - \rho\right\}, \tag{46}$$

$$\dot{L}_A = (L-L_A)\left\{-(1-\alpha)\left[\frac{K}{A(L-L_A)}\right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A - \lambda(L-L_A) + \frac{(\phi_K-\phi_Z)\,\eta}{\alpha}\frac{C}{\Omega}\right\}, \tag{47}$$

where C/Ω is given by (38).

In the following we analyze the system that consists of the differential equations (33), (34), (46), and (47). To determine the BGP, we replace $\dot{A}/A$, $\dot{K}/K$, and $\dot{C}/C$ by the common growth rate g and set $\dot{L}_A = 0$. From the differential equation (33) and the labor market equilibrium condition (27), it follows that

$$L_A^* = \frac{g^*}{\lambda}, \qquad L_Y^* = L - L_A^* = \frac{\lambda L - g^*}{\lambda}. \tag{48}$$

The production function for new blueprints and the labor market equilibrium condition imply that any rise in the common growth rate g^* requires a reallocation of labor from the final goods sector to the R&D sector, i.e., an increase in L_A^* and a corresponding decrease in L_Y^*.

The differential equation (34), which results from the equilibrium condition of the market for final goods, implies that

$$\left(\frac{Y}{K}\right)^* = \left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha} = g^* + \left(\frac{C}{K}\right)^*. \tag{49}$$

Substituting (48) and (49) into the differential equations (46) and (47) and taking into account Equation (38), we derive a two-dimensional system of equations in the two variables g^* and $(C/K)^*$ (see Appendix B):

$$-\left(1-\alpha^2\sigma^S\right) g^* + \sigma^S \left\{ \alpha^2 + \frac{\phi_K \eta}{\phi_K + \dfrac{\phi_Z (1-\alpha)}{\lambda L - g^*}\left[g^* + \left(\dfrac{C}{K}\right)^*\right]} \right\} \left(\frac{C}{K}\right)^* = \sigma^S \rho, \tag{50}$$

$$(1+\alpha) g^* + \frac{1}{\alpha} \left\{ \alpha^2 + \frac{(\phi_K - \phi_Z) \eta}{\phi_K + \dfrac{\phi_Z (1-\alpha)}{\lambda L - g^*}\left[g^* + \left(\dfrac{C}{K}\right)^*\right]} \right\} \left(\frac{C}{K}\right)^* = \lambda L. \tag{51}$$

Equation (50) is a representation of the steady-state version of the Euler equation for consumption,

$$g^* = \sigma^S \left[r^* + \left(\varepsilon^K\right)^* - \rho\right], \tag{52}$$

which is obtained by expressing the rental rate r^* and the status-related extra return of wealth accumulation in the form of real capital $(\varepsilon^K)^*$ as functions of g^* and $(C/K)^*$ and taking into account that $(\dot{C}/C)^* = g^*$. From a technical point of view, Equation (51) yields combinations of g^* and $(C/K)^*$ that exhibit the property that $\dot{L}_A = 0$. For the economic interpretations it will be of crucial importance that Equation (51) is equivalent to the steady-state version of the no-arbitrage condition

$$r^* + \left(\varepsilon^K\right)^* = (\pi/p_A)^* + \left(\varepsilon^Z\right)^*, \tag{53}$$

which is obtained by expressing r^*, $(\varepsilon^K)^*$, the dividend yield $(\pi/p_A)^*$, and the status-related extra return of wealth accumulation in the form of shares $(\varepsilon^Z)^*$ as functions of g^* and $(C/K)^*$ and taking into account that $(\dot{p}_A/p_A)^* = 0$.[8]

In the Supplement we analyze the existence, uniqueness, and stability properties of the BGP. If an economically meaningful equilibrium exists, it is unique. The saddle-point stability is proved analytically for the special cases in which i) shares and capital are equally status relevant or ii) shares are irrelevant for status. For the general case, we illustrate the saddle point stability numerically.

[8] The equivalence of (51) and (53) is not obvious at first glance. In the Supplement we show that i) the differential equation for p_A given by (31) is the starting point for the derivation of the differential equation for L_A given by (47) and ii) Equation (47) is equivalent to $\dot{L}_A = (L - L_A)\left\{1/\alpha\left[r + \varepsilon^K - \left(\pi/p_A + \varepsilon^Z\right)\right] - \dot{K}/K + \dot{A}/A\right\}$. Setting $\dot{L}_A = 0$ and taking into account that $(\dot{K}/K)^* = (\dot{A}/A)^* = g^*$, we obtain (53).

3 The long-run economic effects of the quest for status

In the following we analyze the dependence of the BGP on the status parameter η. This parameter is an important determinant of the status-related extra returns $\varepsilon^K = \phi_K \eta \times (C/\Omega)$ and $\varepsilon^Z = \phi_Z \eta \times (C/\Omega)$. More precisely, we consider modifications in the specification of status preferences that cause variations in η, but leave the effective elasticity of intertemporal substitution σ^S unchanged. In the context of the general CIES preferences (42) and (43) and the resulting expressions for σ^S and η as given by (44) and (45), respectively, this implies that we consider variations in β, while leaving ξ unchanged. This thought experiment should be interpreted as the comparison between the balanced growth paths of two economies that differ only with respect to the status parameter η (high-η-economy or more status conscious economy versus low-η-economy or less status conscious economy).

Before we start with the analysis, we want to stress a crucial feature of our framework that is important for the interpretation of our results but which is absent in the standard Romer (1990) model. From the steady-state versions of the Euler equation for consumption, $g^* = \sigma^S \left[r^* + (\varepsilon^K)^* - \rho\right]$, and the no-arbitrage relation, $r^* + (\varepsilon^K)^* = (\pi/p_A)^* + (\varepsilon^Z)^*$, it is obvious that a rise in the steady-state growth rate g^* requires an increase in both the effective rate of return on physical capital, $r^* + (\varepsilon^K)^*$, and the effective rate of return on shares, $(\pi/p_A)^* + (\varepsilon^Z)^*$. The most important aspect of the model with status preferences is that the *market* rates of return r^* and $(\pi/p_A)^*$ and the *effective* rates of return $r^* + (\varepsilon^K)^*$ and $(\pi/p_A)^* + (\varepsilon^Z)^*$ may move *in opposite directions* in response to variations in the status parameter η. The production function of the R&D sector, $\dot{A} = \lambda A L_A$, implies that $g^* = \lambda L_A^*$. Hence, a higher growth rate requires more scientists in the R&D sector. In any (stationary and non-stationary) equilibrium, the following positive relation between the dividend yield and employment in the final goods sector holds (for a proof see the Supplement):

$$\pi/p_A = \alpha\lambda L_Y = \alpha\lambda\left(L - L_A\right). \tag{54}$$

It follows that $(\pi/p_A)^* = \alpha\left(\lambda L - g^*\right)$, i.e., for given values of α, λ, and L, there is an inverse relation between the steady-state value of the dividend yield $(\pi/p_A)^*$ and the steady-state value of the common growth rate g^*. This result, together with the Euler equation for consumption and the no-arbitrage relation, implies the following: A stronger quest for status (higher η) can only be associated with a higher common growth rate g^* if there is an increase in the status-related extra return of shares $(\varepsilon^Z)^*$ that is only *partially* compensated by a decrease in the dividend yield $(\pi/p_A)^*$. In this case both effective rates of return (on physical capital and on shares) are higher and so is the growth rate.

First, we analyze the special case $\phi_K = \phi_Z = 1$ in which the status-relevant measure of wealth Ω equals the standard definition of wealth. In this case physical capital and shares are equally relevant for status such that status-related extra returns coincide at any time t. From the steady-state version of the no-arbitrage equation (53) it then follows that, along the BGP, the market rates of return on physical capital and shares are equal, i.e., $r^* = (\pi/p_A)^*$. Altogether, we are able to state the following proposition for this case.

Proposition 1. *If $\phi_Z = \phi_K = 1$, the BGP exhibits the following properties:*

i) The solutions for g^* and $(C/K)^*$ are given by

$$g^* = \frac{\sigma^S \left[(\alpha+\eta)\lambda L - \rho\right]}{1+\sigma^S\left[\alpha+\eta(1+\alpha)\right]},$$

$$\left(\frac{C}{K}\right)^* = \frac{\lambda L - (1+\alpha)g^*}{\alpha} = \frac{\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho}{\alpha\left\{1+\sigma^S\left[\alpha+\eta(1+\alpha)\right]\right\}}.$$

The solution for $(C/K)^*$ is economically sensible if

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1+\alpha} \Leftrightarrow \left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho > 0. \tag{55}$$

The steady-state growth rate g^* is strictly positive if and only if the representative household is sufficiently patient in the sense that

$$\rho < (\alpha+\eta)\lambda L. \tag{56}$$

ii) If (55) holds, then g^* depends positively on the status parameter η

$$\frac{\partial g^*}{\partial \eta} = \frac{\alpha\sigma^S}{1+\left[\alpha+\eta(1+\alpha)\right]\sigma^S}\left(\frac{C}{K}\right)^* > 0,$$

while the other endogenous variables exhibit the following dependence on η:

$$\frac{\partial v^*}{\partial \eta} < 0 \quad \textit{for} \quad v = \frac{C}{K}, \frac{C}{Y}, \frac{C}{\Omega}, L_Y, \frac{Y}{K}, r, p, \frac{\pi}{p_A},$$

$$\frac{\partial v^*}{\partial \eta} > 0 \quad \textit{for} \quad v = L_A, \frac{x}{L_Y}, x, \varepsilon^K, \varepsilon^Z, r+\varepsilon^K, \frac{\pi}{p_A}+\varepsilon^Z, p_A, \frac{w}{A},$$

$$sgn\left(\frac{\partial \pi^*}{\partial \eta}\right) = sgn(2\alpha - 1).$$

iii) The composition of wealth does not depend on the status parameter η:

$$\left(\frac{K}{K+p_A A}\right)^* = \alpha, \qquad \left(\frac{p_A A}{K+p_A A}\right)^* = 1-\alpha.$$

Proof. See Appendix C.2.

In the following we provide the economic interpretation of the results described in this proposition. If $\phi_Z = \phi_K = 1$, any economically sensible BGP exhibits the property that the common growth rate g^* (of consumption, physical capital, the number of shares, the mass of intermediate goods, output of final goods, the representative household's wealth, and of real wages) depends positively on the status parameter η. A crucial feature of the case $\phi_Z = \phi_K = 1$ is that the identical effective rates of return $r^* + (\varepsilon^K)^*$ and $(\pi/p_A)^* + (\varepsilon^Z)^*$ also depend positively on the status parameter η. As mentioned above, this results from the fact that a rise in η leads to an increase in the identical status-related components $(\varepsilon^K)^*$ and $(\varepsilon^Z)^*$ that is only partially offset by the fall in the identical market rates of return, r^* and $(\pi/p_A)^*$. According to the Euler equation for consumption, the rise in the effective rate of return $r^* + (\varepsilon^K)^*$ implies an increase in the growth rate of private consumption $(\dot{C}/C)^* = g^*$. In other words, the greater η, the steeper the consumption path chosen by the representative household, i.e., the higher the willingness to substitute future consumption for current consumption. The resulting changes in the saving behavior imply that the growth rate of the representative household's wealth increases. In contrast to its growth rate, the *composition* of wealth does not depend on η because α percent is held in the form of physical capital, while $1 - \alpha$ percent is held in the form of shares.

Since physical capital holdings grow at a higher rate, the capital input in the intermediate goods sector also has to grow at a higher rate. In the high-η-economy, firms in the intermediate goods sector face a lower rental rate of physical capital r^*, i.e., a lower marginal cost, and hence they will charge a lower price for their products as determined by $p^* = (1/\alpha) \times r^*$. The lower price of intermediate goods p^* induces the representative firm of the final goods sector to produce with a higher intensity of intermediate goods. This increase in $(x/L_Y)^*$ originates from both an increase in the common input of each existing variety of intermediate goods x^* and a fall in labor input L_Y^*. The latter effect allows for the sectoral reallocation of labor from final goods production to R&D that is necessary to achieve a faster rate of technological progress.

As explained above, the number of shares that are held by individuals and issued by the firms to finance the purchase of new technologies grows faster in the high-η-economy. The price of shares and therefore also the price of new technologies depends positively on η, while there is no growth in the price of new technologies along the BGP. The reason for the level effect is the following. Since the status-related extra returns of both savings vehicles are equal in case of $\phi_K = \phi_Z = 1$, the dividends financed by operating profits are discounted with the rental rate of physical capital, r^*. The rise in η implies a fall of the rental rate of physical capital, which guarantees a rise in the net present value of profits and therefore of the price of shares and patents, irrespective of the fact that the dependence of operating profits π^* upon η is ambiguous.[9] New entrants in the intermediate goods sector have to pay a higher price for the patents in the high-η-economy. However, since the effective rate of return on shares is higher in the high-η-economy, its inhabitants are more willing to acquire the associated shares, in spite of the lower dividend yield.

Now we show why the R&D sector has to charge a higher price for the blueprints in the high-η economy. The rise in the intermediate goods intensity in the final goods sector implies an increase in the marginal product of labor and, hence, a rise in the ratio of the real wage to the mass of varieties $(w/A)^*$. Since the technology of the R&D sector is linear and we have perfect competition, the equilibrium has to be characterized by $p_A^* = \lambda^{-1} (w/A)^*$ and profits in the R&D sector are zero. In the high-η-economy, each value of A is associated with a higher real wage. Moreover, the real wage also grows faster.

Finally, we show why the results of the proposition are consistent with the market clearing condition of the final goods sector, $Y = C + \dot{K}$, and the implied relation $g^* = (\dot{K}/K)^* = (Y/K)^* - (C/K)^* = [1 - (C/Y)^*] (Y/K)^*$. An increase in η raises the growth rate of physical capital because the reduction of $(Y/K)^*$ is smaller then the reduction of $(C/K)^*$. In other words, the rise in the economy-wide savings rate $[1 - (C/Y)^*]$ is only partially compensated

[9]Recall that an increase in η reduces p^* and raises x^*. Altogether we have: $\mathrm{sgn}(\partial \pi^*/\partial \eta) = \mathrm{sgn}(2\alpha - 1)$.

by the fall in $(Y/K)^*$. The reason for the fall of $(Y/K)^*$ is in turn given by the increase in the intermediate goods intensity of the final goods sector and the associated fall in the average product of the aggregate input of intermediate goods $[Y/(Ax)]^* = (Y/K)^*$.

Next we turn our attention to the special case $\phi_K = 1$ and $\phi_Z = 0$ in which wealth held in the form of shares is irrelevant for status, $\varepsilon^Z = 0$. The steady-state version of the no-arbitrage equation simplifies to $r^* + \left(\varepsilon^K\right)^* = (\pi/p_A)^*$. Hence, along the BGP, the rental rate of physical capital is less than the dividend yield, i.e., $r^* < (\pi/p_A)^*$. Moreover, to calculate the fundamental price of shares, future dividend payments are discounted by using the effective rate of return on physical capital, $r^* + (\varepsilon^K)^*$. Altogether, we are able to state the following proposition for this case.

Proposition 2. *If $\phi_Z = 0$ and $\phi_K = 1$, the BGP exhibits the following properties:*

i) The solutions for g^* and $(C/K)^*$ are given by

$$g^* = \frac{\sigma^S(\alpha\lambda L - \rho)}{1 + \alpha\sigma^S},$$
$$\left(\frac{C}{K}\right)^* = \frac{\alpha\left[\lambda L - (1+\alpha)g^*\right]}{\alpha^2 + \eta} = \frac{\alpha\left[\left(1 - \alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho\right]}{\left(1 + \alpha\sigma^S\right)\left(\alpha^2 + \eta\right)}.$$

The solution for $(C/K)^*$ is economically sensible if

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1+\alpha} \Leftrightarrow \left(1 - \alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho > 0. \tag{57}$$

The steady-state growth rate g^* is strictly positive if and only if the representative household is sufficiently patient in the sense that

$$\rho < \alpha\lambda L. \tag{58}$$

ii) The growth rate g^* is independent of the status parameter η,

$$\frac{\partial g^*}{\partial \eta} = 0$$

and this independence is also true for the following variables:

$$\frac{\partial v^*}{\partial \eta} = 0 \qquad \text{for} \qquad v = L_A, L_Y, r + \varepsilon^K, \frac{\pi}{p_A}, \varepsilon^Z.$$

The other endogenous variables exhibit the following dependence on η:

$$\frac{\partial v^*}{\partial \eta} < 0 \qquad \text{for} \qquad v = \frac{C}{K}, \frac{C}{\Omega}, \frac{C}{Y}, \frac{Y}{K}, r, p,$$
$$\frac{\partial v^*}{\partial \eta} > 0 \qquad \text{for} \qquad v = \frac{x}{L_Y}, x, \pi, \varepsilon^K, p_A, \frac{w}{A}.$$

iii) The composition of wealth exhibits the following properties:

$$\frac{\partial \left[K/\left(K+p_A A\right)\right]^*}{\partial \eta} > 0, \qquad \left.\left(\frac{K}{K+p_A A}\right)^*\right|_{\eta=0} = \alpha, \quad \lim_{\eta\to\infty}\left(\frac{K}{K+p_A A}\right)^* < 1.$$

Proof. See Appendix C.3.

The striking feature of the case $\phi_Z = 0$ and $\phi_K = 1$ is that the growth rate g^* is independent of the status parameter η. The technology used in the research sector and labor market clearing imply that employment in the R&D sector and in the final goods sector, L_A^* and L_Y^*, respectively, are also independent of η. This result does not come as a surprise. In interpreting Equation (54), $\pi/p_A = \alpha\lambda L_Y = \alpha\lambda\left(L - L_A\right)$, we already stressed that a stronger quest for status (higher η) can only be associated with a higher common growth rate g^* if there is an increase in the status-related extra return of shares $(\varepsilon^Z)^*$ that is only partially compensated by a decrease in the dividend yield $(\pi/p_A)^*$. However, if $\phi_Z = 0$, then $\varepsilon^Z = 0$, so that the required increase in $(\varepsilon^Z)^*$ cannot occur.

A rise in η leads to an increase in the status-related component of saving in the form of physical capital $(\varepsilon^K)^*$ that is perfectly offset by the fall in the rental rate r^* so that the effective rate of return on physical capital remains unchanged. From the Euler equation for consumption it follows that the growth rate of private consumption $(\dot{C}/C)^* = g^*$ remains unchanged, too. In other words, along the BGP, the willingness to substitute future consumption for current consumption is independent of the status parameter η. The no-arbitrage equation $r^* + (\varepsilon^K)^* = (\pi/p_A)^*$ implies that the dividend yield $(\pi/p_A)^*$ is also independent of η. Note that a rise in η leaves the growth rate of wealth unchanged, but alters the *composition* of wealth in favor of physical capital.

In the high-η-economy, firms in the intermediate goods sector are confronted with a lower rental rate of physical capital (similar to the case $\phi_K = \phi_Z = 1$). The resulting lower price of intermediate goods induces the representative firm of the final goods sector to choose a higher intermediate goods intensity. This increase of $(x/L_Y)^*$ originates in an increase of x^*, i.e., the input of each existing variety increases. In contrast to the case of $\phi_K = \phi_Z = 1$, employment in final goods production remains unchanged. For the aggregate physical capital input $K = Ak^* = Ax^*$ we have that each value of A is associated with a higher value of K in the high-η-economy but that the growth rate of physical capital is the same as in the low-η-economy.

In contrast to the case $\phi_K = \phi_Z = 1$, operating profits of an intermediate goods producing firm, $\pi^* = (1-\alpha)\, p^* x^*$, depend positively on η because the percentage change of x^* overcompensates the percentage change of p^*. For the fundamental price of shares we also have an unambiguous result: The assumptions $\phi_K = 1$ and $\phi_Z = 0$ imply that the stream of dividend payments is not discounted with the rental rate r^* but with the effective rate of return $r^* + (\varepsilon^K)^*$. Since $\partial[r^* + (\varepsilon^K)^*]/\partial\eta = 0$, the higher dividends are discounted at the same rate such that the fundamental value of shares increases. New entrants into the intermediate goods sector are therefore able to pay for the patents by issuing more expensive shares. In contrast to the case of $\phi_Z = \phi_K = 1$, the number of shares that are held by the individuals grows with the same rate in the high-η-economy and in the low-η economy.

Again we have argued that the price for patents that new entrants in the intermediate goods sector pay depends positively on η. Analogous to the previous case, firms in the R&D sector in the high-η-economy have to charge higher prices because they are confronted with higher real wages. Regarding the growth rate of real wages, g^*, there is, however, no difference between the high-η-economy and the low-η-economy. Analogous to the case $\phi_K = \phi_Z = 1$ we have that the ratio of the real wage to the mass of varieties $(w/A)^*$ increases because the intermediate goods intensity in the final goods sector $(x/L_Y)^*$ rises and the corresponding increase in the marginal product of labor implies a higher economy-wide wage.

From the market clearing condition of the final goods market, $Y = C + \dot{K}$, and the associated condition $g^* = (\dot{K}/K)^* = (Y/K)^* - (C/K)^* = [1 - (C/Y)^*]\,(Y/K)^*$, we get the following additional information: An increase of η does not affect the growth rate of physical capital because $(Y/K)^*$ and $(C/K)^*$ fall by the same amount. In other words, the increase in the economy-wide savings rate $[1 - (C/Y)^*]$ is fully compensated by a fall of $(Y/K)^*$. The reduction of $(Y/K)^*$ can be derived in analogy to the case $\phi_K = \phi_Z = 1$ because of the rise of the intermediate goods intensity in the final goods sector.

Next, we dwell more on the importance of ϕ_Z. In this context, we allow for both $0 < \phi_Z \leq \phi_K$ and $\phi_Z > \phi_K$, i.e., shares of entrants might be less or more status relevant than physical capital employed by incumbents (or the assets that were used to finance its purchase).

The effects of *ceteris paribus* changes in η are summarized in Proposition 4, which is stated and proven in the Supplement because most of the results given in Proposition 1 for the special case $\phi_Z = \phi_K$ carry over to situations in which either $0 < \phi_Z < \phi_K$ or $\phi_Z > \phi_K$ holds. In particular, the growth rate g^*, the average propensity to save $1 - (C/Y)^*$, and the effective rates of return $r^* + \left(\varepsilon^K\right)^*$ and $(\pi/p_A)^* + \left(\varepsilon^Z\right)^*$ depend positively on the status parameter η regardless of whether shares are less or more status relevant than physical capital. However, the results for $(Y/K)^*$ and the variables that depend crucially upon $(Y/K)^*$ cease to be valid if the relative status relevance of shares ϕ_Z/ϕ_K exceeds a critical value that is greater than unity. Finally, we show that a rise in η alters the composition of wealth in favor of shares if $\phi_Z > \phi_K$ and in favor of physical capital if $\phi_K > \phi_Z$, while $K/\left(K + p_A A\right) = \alpha$ holds for $\phi_Z = \phi_K$. Since these modifications do not affect the main result with respect to economic growth we do not discuss the details here. Instead, we proceed with the effects of *ceteris paribus* changes in the relative status importance of shares.

Proposition 3. *If $\phi_K > 0$ and $\phi_Z \geq 0$, then the growth rate g^* depends positively on the relative status weight of shares, i.e.,*

$$\frac{\partial g^*}{\partial \phi_Z} > 0.$$

Moreover, the following endogenous variables exhibit an unambiguous dependence on ϕ_Z:

$$\frac{\partial v^*}{\partial \phi_Z} > 0 \qquad \text{for} \qquad v = \frac{C}{K}, \frac{Y}{K}, p, r, L_A, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, \varepsilon^Z,$$

$$\frac{\partial v^*}{\partial \phi_Z} < 0 \qquad \text{for} \qquad v = \frac{x}{L_Y}, L_Y, x, \pi, \frac{w}{A}, p_A, \frac{\pi}{p_A}, \frac{K}{K + p_A A}.$$

Proof. See Appendix C.4.

We now turn to the economic interpretation of the results summarized in Proposition 3. A *ceteris paribus* rise in ϕ_Z implies that private households re-adjust their portfolio by shifting wealth from physical capital holdings toward shares. The market rate of return on physical capital, r^*, rises, while the market rate of return on shares, $(\pi/p_A)^*$, falls. However, the rise of the status-related extra return of shares, $(\varepsilon^Z)^* = \phi_Z \eta \times (C/\Omega)^*$, more than offsets the fall in the dividend yield so that the *effective* rate of return on shares, $(\pi/p_A)^* + (\varepsilon^Z)^*$, depends positively on ϕ_Z. The no-arbitrage condition implies that the effective rate of return on physical capital also depends positively on ϕ_Z. According to the Euler equation for consumption, the rise in the effective rate of return implies an increase in the growth rate of private consumption, $(\dot{C}/C)^* = g^*$. In other words, the consumption path chosen by the representative household becomes steeper. The common growth rate of total wealth and its components, physical capital and shares, increases. Hence, while the proportion of physical capital decreases, its growth rate, $(\dot{K}/K)^* = g^*$, increases.

In the high-ϕ_Z-economy firms in the intermediate goods sector face a higher rental rate of physical capital, r^*, and hence they will charge a higher price for their products according to the mark-up pricing rule, $p^* = (1/\alpha) \times r^*$. The higher value of p^* causes the representative firm of the final goods sector to choose a lower intensity of intermediate goods, $(x/L_Y)^*$. Since the high-ϕ_Z-economy exhibits a higher common growth rate, g^*, employment in the R&D sector, L_A^*, is also higher, while the opposite is true for employment in the final goods sector, L_Y^*. Since both $(x/L_Y)^*$ and L_Y^* depend negatively on ϕ_Z, the identical input of the different varieties x^* depends negatively on ϕ_Z, too. To put it differently: Production and employment of each variety is lower but the stock of varieties grows at a faster rate, $(\dot{A}/A)^* = g^*$. In the high-ϕ_Z-economy, firms in the intermediate goods sector face lower profits, $\pi^* = (1-\alpha)\, p^* x^*$, because the increase in the price p^* is overcompensated by a decrease in the number of units sold, x^*. The fall in x^* implies that each firm in the intermediate goods sector uses less physical capital, $k^* = x^*$. Since the stock of varieties grows faster, also the aggregate stock of physical capital grows at a higher rate.

In the high-ϕ_Z-economy, the lower ratio of intermediate goods to labor, $(x/L_Y)^*$, implies a lower marginal product of labor. Consequently, the ratio of the real wage to the mass of varieties, $(w/A)^*$, is also lower, while, by contrast, the growth rate of the real wage, $(\dot{w}/w)^* = g^*$, is higher. The lower level of $(w/A)^*$ together with the linear technology in the research sector and perfect competition imply that the price of blueprints, p_A^*, is lower. Since it holds that $p_Z = p_A$ in equilibrium, shares are also cheaper. An increase in ϕ_Z leads to a decrease of the dividend yield, $(\pi/p_A)^*$, because π^* decreases by a larger percentage value than p_A^*. As explained above, the rise in ϕ_Z implies that the composition of the household's portfolio shifts in favor of shares. Furthermore, the growth rate of the stock of shares, $(\dot{Z}/Z)^* = (\dot{A}/A)^* = g^*$, rises.

Finally, inspired by Corneo and Jeanne (1997, 2001a) we end our detailed analysis with a remark on the social optimality of the decentralized long-run growth rate. The standard Romer (1990) model exhibits the well-known property that the decentralized long-run economic growth rate is less than its socially optimal counterpart due to several distortions. In our paper the quest for status acts so as to increase the decentralized long-run economic growth rate provided that shares are status relevant, i.e., $\phi_Z > 0$. A necessary condition for the perfect replication of the socially optimal BGP is that the status relevance of shares exceeds that of physical capital, i.e., $\phi_Z > \phi_K > 0$. More specifically, we can show that if $\phi_K > 0$ and $(\eta, \phi_Z) = (\tilde{\eta}, \tilde{\varsigma}\phi_K)$, where $\tilde{\varsigma} > 1$ and $\tilde{\eta}$ are uniquely determined constants, then the decentralized BGP equals its socially optimal counterpart in the absence of any government intervention. For details see Proposition 5 given in the Supplement.[10] Please note that the condition $(\eta, \phi_Z) = (\tilde{\eta}, \tilde{\varsigma}\phi_K)$ can only be satisfied by pure coincidence, since η, ϕ_K, and ϕ_Z are exogenously given status parameters.

4 Conclusions

In this paper we introduced status preferences into an R&D-based economic growth model with three sectors of production (final goods, intermediate goods, and blueprints) to analyze the impact of status concerns on technological progress and on long-run economic growth. In contrast to the standard relative wealth approach used in the status literature, we allowed for the possibility that the components of household's wealth differ with respect to their status relevance. The introduction of the generalized relative wealth preferences implies that the effective rate of return of saving in the form of a particular asset is the sum of its standard market rate of return and its status-related extra return. In both the Euler equation for consumption growth and in the no-arbitrage condition, the rental rate of physical capital and the market rate of return of shares have to be replaced by the corresponding effective rates of return.

First, we analyzed the effects of an increase in the intensity of the quest for status, i.e., a rise in the marginal rate of substitution (MRS) of status-relevant own wealth for consumption. This rise affects the economy by raising the extra returns of all assets that are status relevant. As long as this impact effect is not perfectly offset by a decrease in the corresponding market rate of return, the common steady-state effective rate of return of all assets rises. The resulting stronger incentive to save causes the demand for shares and hence for new technologies to grow at a higher rate, which fosters technological progress. According to the underlying production technology in the R&D sector, the acceleration of technological progress is ultimately due to an increase in the employment of scientists. Altogether, these effects induce the common long-run growth rate to rise.

One of our main results is that the effects of an increase in the intensity of the quest for status on the common growth rate depend crucially upon the status relevance of shares. We started with two special cases in which explicit solutions for all variables can be easily calculated: i) if physical capital and shares are equally status relevant, then the status-related extra returns of these two assets are identical. A rise in the intensity of the quest for status causes the common growth rate to rise unambiguously. This result is due to the fact that the rise in the status-related extra return of physical capital and shares is only partially compensated by the decrease in the rental rate and the dividend yield so that the common effective rate of return of the two assets increases; ii) if wealth held in the form of shares is irrelevant for status, then the status-related extra return of shares equals zero. A rise in the MRS of status-relevant own wealth for consumption causes the extra return of physical capital to increase. But since this rise is perfectly offset by a fall in the rental rate of capital, the effective rate of return of real

[10] In the Supplement we derive the socially optimal solution, calculate the values of $\tilde{\eta}$ and $\tilde{\varsigma}$, and discuss the resulting properties of the decentralized BGP in detail.

capital and, hence, the common growth rate remain unchanged. While the growth rate of the representative household's wealth remains unchanged, the composition of wealth is altered in favor of physical capital. Finally, we considered the case in which wealth held in the form of shares and in the form of physical capital are relevant for status. In this (realistic) case an increase in the intensity of the quest for status causes the common growth rate to rise irrespective of the relative status relevance of shares.

Second, we kept the intensity of the quest for status constant and analyzed the implications of an increase in the relative status relevance of shares. Private households adjust their portfolio by shifting wealth from physical capital holdings (or bonds/loans) to shares. The rental rate of capital rises, while the dividend yield falls. However, this fall in the dividend yield is more than offset by the rise in the status-related extra return of shares so that the effective rate of return of shares rises. The no-arbitrage condition implies that the effective rate of return of physical capital rises, too. Consequently, the common growth rate along the BGP increases. The consumption path chosen by the representative household becomes steeper. While the proportion of shares increases, not only shares, but also physical capital are accumulated at a higher rate. Altogether the differential status effect of traditional physical capital versus those of shares is one potential channel to explain the superior growth patterns of countries in which entrants/startups have better access to new funds such as the United States.

A final interesting feature of our framework is that, while the standard R&D-based economic growth model of Romer (1990) exhibits the property that the decentralized long-run growth rate is unambiguously smaller than its socially optimal counterpart, the externality resulting from relative wealth preferences reduces the influence of the other distortions provided that shares matter for status: However, as long as the status relevance of shares does not significantly exceed that of physical capital, neither the perfect replication of the socially optimal growth rate nor excessive growth can occur.

With respect to further research we would like to mention two promising avenues: First, from a public economics point of view it would be interesting to analyze how the socially optimal taxation/subsidization is influenced by the introduction of relative wealth preferences and the possibility that the various assets differ with respect to their relative status relevance. Second, one could abandon the representative agent framework and allow for the heterogeneity of households. This could yield useful insights when analyzing the driving forces behind wealth disparities and assessing the conditions and policies under which the poor do not fall too far behind the rich.

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Appendix

A The representative household's optimization problem

We dismantle the flow budget constraint (1) into two differential equations for the state variables K and Z:

$$\dot{K} = rK + wL + DZ - p_Z Q - C, \tag{A.1}$$

$$\dot{Z} = Q. \tag{A.2}$$

The representative individual chooses time paths for C and Q so as to maximize lifetime utility given by

$$\int_0^{\infty} e^{-\rho t} u\left[C, S\left(\Omega, \bar{\Omega}\right)\right] dt,$$

where

$$\Omega \equiv \phi_K K + \phi_Z p_Z Z \qquad \text{and} \qquad \bar{\Omega} \equiv \phi_K \bar{K} + \phi_Z p_Z \bar{Z}, \tag{A.3}$$

subject to the differential equations (A.1) and (A.2) and the two initial conditions $K(0) = K_0$ and $Z(0) = Z_0$, where K_0 and Z_0 are exogenously given. The representative household takes the time paths of r, w, p_Z, D, $\bar{K}$, and $\bar{Z}$ as given. The current value Hamiltonian is

$$H = u\left[C, S\left(\phi_K K + \phi_Z p_Z Z, \phi_K \bar{K} + \phi_Z p_Z \bar{Z}\right)\right] + \mu_K\left(rK + wL + DZ - p_Z Q - C\right) + \mu_Z Q,$$

where the costate variables μ_K and μ_Z denote the shadow price of physical capital and shares, respectively. The necessary optimality conditions for an interior optimum are

$$\mu_K = \frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial C}, \tag{A.4}$$

$$\mu_Z = \mu_K p_Z, \tag{A.5}$$

$$\dot{\mu}_K = \rho\mu_K - \left[\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S}\frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega}\phi_K + \mu_K r\right], \tag{A.6}$$

$$\dot{\mu}_Z = \rho\mu_Z - \left[\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S}\frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega}\phi_Z p_Z + \mu_K D\right], \tag{A.7}$$

where Ω and $\bar{\Omega}$ are given by (A.3). The transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\mu_K K = 0 \qquad \text{and} \qquad \lim_{t\to\infty} e^{-\rho t}\mu_Z Z = 0. \tag{A.8}$$

Since the Hamiltonian is jointly concave in C, Q, K, and Z, the transversality conditions ensure that the necessary optimality conditions are also sufficient. Using (A.4) and (A.6) we obtain

$$\frac{\dot{\mu}_K}{\dot{\mu}_K} = -\left[r + \frac{\frac{\partial U\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S} \frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega} \phi_K}{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial C}} - \rho \right]. \tag{A.9}$$

From (A.5) it follows that

$$\dot{\mu}_Z = \dot{\mu}_K p_Z + \mu_K \dot{p}_Z. \tag{A.10}$$

Substituting (A.4), (A.5), and (A.10) into (A.7) it follows that

$$\frac{\dot{\mu}_K}{\mu_K} = -\left[\frac{\dot{p}_Z}{p_Z} + \frac{D}{p_Z} + \frac{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S} \frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega} \phi_Z}{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial C}} - \rho \right]. \tag{A.11}$$

Equations (A.9) and (A.11) yield two alternative representations of $\dot{\mu}_K/\mu_K$. The required equality of the right-hand sides of (A.9) and (A.11) yields the no-arbitrage relation of the economy with relative wealth preferences:

$$r + \frac{\frac{\partial U\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S} \frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega} \phi_K}{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial C}} = \frac{\dot{p}_Z}{p_Z} + \frac{D}{p_Z} + \frac{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial S} \frac{\partial S\left(\Omega, \bar{\Omega}\right)}{\partial \Omega} \phi_Z}{\frac{\partial u\left(C, S\left(\Omega, \bar{\Omega}\right)\right)}{\partial C}}. \tag{A.12}$$

In any symmetric situation, $\Omega = \bar{\Omega}$ holds. Moreover, due to Assumption (7), we also have $S(\Omega, \bar{\Omega}) = S\left(\Omega, \Omega\right) = \chi$ for all $\Omega > 0$, where χ is an exogenously given constant. Equations (A.4), (A.9), and (A.12) simplify to

$$\mu_K = \frac{\partial u\left(C, \chi\right)}{\partial C}, \tag{A.13}$$

$$\frac{\dot{\mu}_K}{\mu_K} = -\left[r + \varepsilon^K\left(C, \Omega, \chi, \phi_K\right) - \rho\right], \tag{A.14}$$

$$r + \varepsilon^K\left(C, \Omega, \chi, \phi_K\right) = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} + \varepsilon^Z\left(C, \Omega, \chi, \phi_Z\right), \tag{A.15}$$

where

$$\varepsilon^K\left(C, \Omega, \chi, \phi_K\right) \equiv MRS\left(C, \Omega, \chi\right) \times \phi_K, \tag{A.16}$$

$$\varepsilon^Z\left(C, \Omega, \chi, \phi_Z\right) \equiv MRS\left(C, \Omega, \chi\right) \times \phi_Z, \tag{A.17}$$

$$MRS\left(C, \Omega, \chi\right) \equiv \frac{\partial u\left(C, \chi\right)}{\partial S} \frac{\partial S\left(\Omega, \Omega\right)}{\partial \Omega} \left[\frac{\partial u\left(C, \chi\right)}{\partial C}\right]^{-1}. \tag{A.18}$$

Note that (A.15) is equal to the no-arbitrage relation (11) as given in the main text, while the definitions (A.16)-(A.18) coincide with the definitions ε^K, ε^Z, and MRS [see Equations

(12)–(14)]. From (A.13) it follows that

$$\frac{\dot{\mu}_K}{\mu_K} = C \frac{\partial^2 u(C,\chi)}{\partial C^2} \left[\frac{\partial u(C,\chi)}{\partial C} \right]^{-1} \times \frac{\dot{C}}{C}. \tag{A.19}$$

Using (A.14) and (A.19), we obtain the Euler equation for consumption of a decentralized economy populated by households with relative wealth preferences:

$$\frac{\dot{C}}{C} = \sigma^S(C,\chi)\left[r + \varepsilon^K(C,\Omega,\chi,\phi_K) - \rho\right], \tag{A.20}$$

where

$$\sigma^S(C,\chi) \equiv -\frac{\partial u(C,\chi)}{\partial C}\left[C \frac{\partial^2 u(C,\chi)}{\partial C^2} \right]^{-1}. \tag{A.21}$$

Note that (A.20) and (A.21) are equivalent to (15) and (16) as given in the main text. Using (A.5), the transversality conditions (A.8) can be written as

$$\lim_{t\to\infty} e^{-\rho t}\mu_K K = 0, \qquad \lim_{t\to\infty} e^{-\rho t}\mu_K p_Z Z = 0. \tag{A.22}$$

Integration of (A.14) yields

$$\mu_K(t) = \mu_K(0)\, e^{\rho t} \exp\left[-\int_0^t \left[r(v) + \varepsilon^K(C(v),\Omega(v),\chi,\phi_K)\right] dv \right].$$

The assumption that $\partial u(C,\chi)/\partial C > 0$ together with (A.13) implies that $\mu_K(t) > 0$ for $t \geq 0$. Hence, the transversality conditions (A.22) are equivalent to

$$\lim_{t\to\infty}\left\{ \exp\left[-\int_0^t \left[r(v) + \varepsilon^K(v)\right] dv \right] K \right\} = 0,$$
$$\lim_{t\to\infty}\left\{ \exp\left[-\int_0^t \left[r(v) + \varepsilon^K(v)\right] dv \right] p_Z Z \right\} = 0,$$

where $\varepsilon^K(v) = \varepsilon^K[C(v),\Omega(v),\chi,\phi_K]$. Note that these conditions are identical to the conditions (18) as given in the main text.

B The derivation of (50) and (51)

The dynamic evolution of the variables K, C, A, and L_A is governed by the four differential equations (33), (34), (46), (47), where C/Ω is given by (38). It is easily verified from these five equations that a BGP exhibits the following properties:

$$L_A = \text{constant}, \quad \frac{A}{K} = \text{constant}, \quad \frac{C}{K} = \text{constant}, \quad \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \text{constant}.$$

Denote the steady-state value of a variable x by x^* and the common steady-state growth rate of C, K, and A by $g^* = (\dot{A}/A)^* = (\dot{K}/K)^* = (\dot{C}/C)^*$. Using (33), (34), (46), (47), and (38), we can show that g^*, L_A^*, $(A/K)^*$, $(C/K)^*$, and $(C/\Omega)^*$ are determined by the following system of

equations:

$$g^* = \lambda L_A^*, \tag{B.1}$$

$$g^* = \left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha} - \left(\frac{C}{K}\right)^*, \tag{B.2}$$

$$g^* = \sigma^S \left\{\alpha^2 \left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha} + \phi_K \eta \left(\frac{C}{\Omega}\right)^* - \rho\right\}, \tag{B.3}$$

$$\begin{aligned} 0 = &- (1-\alpha) \left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha} + \left(\frac{C}{K}\right)^* + \lambda L_A^* \\ &- \lambda (L - L_A^*) + \frac{(\phi_K - \phi_Z)\eta}{\alpha} \times \left(\frac{C}{\Omega}\right)^*, \end{aligned} \tag{B.4}$$

$$\left(\frac{C}{\Omega}\right)^* = \frac{\left(\frac{C}{K}\right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda (L - L_A^*)} \left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha}}. \tag{B.5}$$

From (B.1) and (B.2) it follows that

$$L_A^* = \frac{g^*}{\lambda}, \qquad L - L_A^* = L_Y^* = \frac{\lambda L - g^*}{\lambda}, \tag{B.6}$$

$$\left[\left(\frac{A}{K}\right)^* (L - L_A^*)\right]^{1-\alpha} = \left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^*. \tag{B.7}$$

Substituting (B.6) and (B.7) into (B.5) yields

$$\left(\frac{C}{\Omega}\right)^* = \frac{\left(\frac{C}{K}\right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]}. \tag{B.8}$$

Substituting (B.6)-(B.8) into (B.3) and (B.4) and applying simple transformations, we obtain the following system of equations that determines g^* and $(C/K)^*$:

$$-\left(1 - \alpha^2 \sigma^S\right) g^* + \sigma^S \left\{\alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]}\right\} \left(\frac{C}{K}\right)^* = \sigma^S \rho,$$

$$(1+\alpha) g^* + \frac{1}{\alpha} \left\{\alpha^2 + \frac{(\phi_K - \phi_Z)\eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]}\right\} \left(\frac{C}{K}\right)^* = \lambda L.$$

These two equations are identical to (50) and (51). ■

C Proofs of Propositions 1 – 3

C.1 The steady-state values of the endogenous variables

First, g^* and $(C/K)^*$ are obtained by solving (50)–(51). Second, L_A^*, L_Y^*, and $(Y/K)^*$ are determined by substituting the solutions for g^* and $(C/K)^*$ into (48)–(49):

$$L_A^* = \frac{g^*}{\lambda}, \qquad L_Y^* = L - L_A^* = \frac{\lambda L - g^*}{\lambda}, \tag{C.1}$$

$$\left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^*. \tag{C.2}$$

Third, using (19), (21), (22), (23), (24), (26), (27), (28), (38), (41), (52), (53), (54), and various equations given in the supplement, the steady-state values of the other endogenous variables can be expressed as functions of g^*, $(C/K)^*$, L_A^*, L_Y^*, and $(Y/K)^*$:

$$\left(\frac{K}{A}\right)^* \frac{1}{L_Y^*} = \left(\frac{K}{A}\right)^* \frac{1}{L - L_A^*} = \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)}, \tag{C.3}$$

$$r^* = \alpha^2 \left[\left(\frac{K}{A}\right)^* \frac{1}{L - L_A^*}\right]^{-(1-\alpha)} = \alpha^2 \left(\frac{Y}{K}\right)^*, \tag{C.4}$$

$$p^* = \frac{1}{\alpha} \times r^* = \alpha \left(\frac{Y}{K}\right)^*, \tag{C.5}$$

$$x^* = k^* = \left(\frac{\alpha^2}{r^*}\right)^{1/(1-\alpha)} L_Y^* = \frac{\lambda L - g^*}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)}, \tag{C.6}$$

$$\left(\frac{x}{L_Y}\right)^* = \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)}, \tag{C.7}$$

$$\pi^* = (1-\alpha)\,\alpha L_Y^* \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)} = (1-\alpha)\,\alpha \frac{\lambda L - g^*}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)}, \tag{C.8}$$

$$p_A^* = \frac{(1-\alpha)}{\lambda} \left(\frac{\alpha^2}{r^*}\right)^{\alpha/(1-\alpha)} = \frac{1-\alpha}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)}, \tag{C.9}$$

$$\left(\frac{\pi}{p_A}\right)^* = \alpha\lambda L_Y^* = \alpha\left(\lambda L - g^*\right), \tag{C.10}$$

$$\left(\frac{w}{A}\right)^* = \lambda p_A^* = (1-\alpha) \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)}, \tag{C.11}$$

$$\left(\frac{C}{\Omega}\right)^* = \frac{(C/K)^*}{\phi_K + \phi_Z \dfrac{1-\alpha}{\lambda L - g^*} \left(\dfrac{Y}{K}\right)^*}, \tag{C.12}$$

$$\left(\frac{C}{Y}\right)^* = \frac{(C/K)^*}{(Y/K)^*} = \frac{(C/K)^*}{g^* + (C/K)^*}, \tag{C.13}$$

$$r^* + \left(\varepsilon^K\right)^* = \alpha^2 \left(\frac{Y}{K}\right)^* + \frac{\phi_K \eta\,(C/K)^*}{\phi_K + \phi_Z \dfrac{1-\alpha}{\lambda L - g^*} \left(\dfrac{Y}{K}\right)^*} = \frac{1}{\sigma^S} g^* + \rho, \tag{C.14}$$

$$\left(\frac{\pi}{p_A}\right)^* + \left(\varepsilon^Z\right)^* = \alpha\left(\lambda L - g^*\right) + \frac{\phi_Z \eta \left(C/K\right)^*}{\phi_K + \phi_Z \dfrac{1-\alpha}{\lambda L - g^*}\left(\dfrac{Y}{K}\right)^*} = \frac{1}{\sigma^S} g^* + \rho, \tag{C.15}$$

$$\left(\varepsilon^K\right)^* = \frac{\phi_K \eta \left(C/K\right)^*}{\phi_K + \phi_Z \dfrac{1-\alpha}{\lambda L - g^*}\left(\dfrac{Y}{K}\right)^*} = \frac{1}{\sigma^S} g^* + \rho - \alpha^2 \left(\frac{Y}{K}\right)^*, \tag{C.16}$$

$$\left(\varepsilon^Z\right)^* = \frac{\phi_Z \eta \left(C/K\right)^*}{\phi_K + \phi_Z \dfrac{1-\alpha}{\lambda L - g^*}\left(\dfrac{Y}{K}\right)^*} = \frac{1+\alpha\sigma^S}{\sigma^S} g^* + \rho - \alpha\lambda L, \tag{C.17}$$

$$\left(\frac{K}{K+p_A A}\right)^* = \frac{1}{1 + \dfrac{1-\alpha}{\lambda L_Y^*}\left(\dfrac{Y}{K}\right)^*} = \frac{1}{1 + \dfrac{1-\alpha}{\lambda L - g^*}\left(\dfrac{Y}{K}\right)^*}. \tag{C.18}$$

C.2 Proof of Proposition 1

Proof of i): Setting $\phi_Z = \phi_K = 1$ in (50) and (51) yields

$$\begin{aligned} -\left(1-\alpha^2\sigma^S\right) g^* + \sigma^S \left\{ \alpha^2 + \frac{\eta}{1 + \dfrac{(1-\alpha)}{\lambda L - g^*}\left[g^* + \left(\dfrac{C}{K}\right)^*\right]} \right\} \left(\frac{C}{K}\right)^* &= \sigma^S \rho, \\ (1+\alpha)\, g^* + \alpha \left(\frac{C}{K}\right)^* &= \lambda L. \end{aligned}$$

Solving this system of two equations for g^* and $(C/K)^*$, we obtain:

$$g^* = \frac{\sigma^S \left[(\alpha+\eta)\lambda L - \rho\right]}{1 + \sigma^S \left[\alpha + \eta(1+\alpha)\right]}, \tag{C.19}$$

$$\left(\frac{C}{K}\right)^* = \frac{\lambda L - (1+\alpha)\, g^*}{\alpha} = \frac{\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\,\sigma^S \rho}{\alpha\left\{1 + \sigma^S\left[\alpha + \eta(1+\alpha)\right]\right\}}. \tag{C.20}$$

From (C.19) and (C.20) the validity of the conditions (55) and (56) is immediately clear.

Proof of ii): The partial derivative of g^* with respect to η is given by

$$\frac{\partial g^*}{\partial \eta} = \frac{\sigma^S \left\{\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\,\sigma^S\rho\right\}}{\left\{1+\sigma^S\left[\alpha+\eta(1+\alpha)\right]\right\}^2} = \frac{\alpha\sigma^S}{1 + \left[\alpha + \eta(1+\alpha)\right]\sigma^S}\left(\frac{C}{K}\right)^*, \tag{C.21}$$

where the second representation is obtained by using (C.20). If (55) holds, then $(C/K)^* > 0$ and

$$\frac{\partial g^*}{\partial \eta} > 0 \qquad \text{and} \qquad \frac{\partial \left(C/K\right)^*}{\partial \eta} = -\frac{1+\alpha}{\alpha}\frac{\partial g^*}{\partial \eta} < 0. \tag{C.22}$$

From (C.19), (C.20), and (C.1)–(C.2) it follows that

$$L_A^* = \frac{g^*}{\lambda}, \qquad L_Y^* = \frac{\lambda L - g^*}{\lambda}, \qquad \left(\frac{Y}{K}\right)^* = \frac{\lambda L - g^*}{\alpha}. \tag{C.23}$$

Using (C.20), (C.23), and (C.4)–(C.17) the steady-state values of the remaining endogenous variables can be expressed as functions of g^* solely:

$$r^* = \alpha\left(\lambda L - g^*\right), \qquad p^* = \lambda L - g^*,$$

$$x^* = \frac{\alpha^{1/(1-\alpha)}}{\lambda}\left(\lambda L - g^*\right)^{-\alpha/(1-\alpha)}, \qquad \left(\frac{x}{L_Y}\right)^* = \left(\frac{\lambda L - g^*}{\alpha}\right)^{-1/(1-\alpha)},$$

$$p_A^* = \frac{1-\alpha}{\lambda}\left(\frac{\lambda L - g^*}{\alpha}\right)^{-\alpha/(1-\alpha)}, \qquad \left(\frac{w}{A}\right)^* = (1-\alpha)\left(\frac{\lambda L - g^*}{\alpha}\right)^{-\alpha/(1-\alpha)},$$

$$\pi^* = \frac{(1-\alpha)\,\alpha^{1/(1-\alpha)}}{\lambda}\left(\lambda L - g^*\right)^{-(2\alpha-1)/(1-\alpha)}, \qquad \left(\frac{\pi}{p_A}\right)^* = \alpha\left(\lambda L - g^*\right),$$

$$\left(\frac{C}{\Omega}\right)^* = \lambda L - (1+\alpha)\,g^*, \qquad \left(\frac{C}{Y}\right)^* = \frac{\lambda L - (1+\alpha)\,g^*}{\lambda L - g^*},$$

$$r^* + \left(\varepsilon^K\right)^* = \left(\frac{\pi}{p_A}\right)^* + \left(\varepsilon^Z\right)^* = \frac{1}{\sigma^S}g^* + \rho,$$

$$\left(\varepsilon^K\right)^* = \left(\varepsilon^Z\right)^* = \frac{1+\alpha\sigma^S}{\sigma^S}g^* - \alpha\lambda L + \rho.$$

Taking into account that $\partial g^*/\partial\eta > 0$ holds [see (C.22)], it is obvious from the equations given above that

$$\frac{\partial v^*}{\partial \eta} < 0 \qquad \text{for} \qquad v = \frac{C}{K}, \frac{C}{Y}, \frac{C}{\Omega}, L_Y, \frac{Y}{K}, r, p, \frac{\pi}{p_A},$$

$$\frac{\partial v^*}{\partial \eta} > 0 \qquad \text{for} \qquad v = L_A, \frac{x}{L_Y}, x, \varepsilon^K, \varepsilon^Z, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, p_A, \frac{w}{A},$$

$$\operatorname{sgn}\left(\frac{\partial \pi^*}{\partial \eta}\right) = \operatorname{sgn}\left(2\alpha - 1\right).$$

These results prove the validity of the assertions made in part ii) of Proposition 1.

Proof of iii): Using (C.23) and (C.18), we finally obtain

$$\left(\frac{K}{K + p_A A}\right)^* = \left[1 + \frac{1-\alpha}{\lambda L - g^*}\frac{\lambda L - g^*}{\alpha}\right]^{-1} = \alpha \Rightarrow \left(\frac{p_A A}{K + p_A A}\right)^* = 1 - \alpha. \quad \blacksquare$$

C.3 Proof of Proposition 2

Proof of i): Setting $\phi_Z = 0$ in (50) and (51) yields

$$\begin{aligned} -\left(1 - \alpha^2\sigma^S\right)g^* + \sigma^S\left(\alpha^2 + \eta\right)\left(\frac{C}{K}\right)^* &= \sigma^S\rho, \\ (1+\alpha)\,g^* + \frac{1}{\alpha}\left(\alpha^2 + \eta\right)\left(\frac{C}{K}\right)^* &= \lambda L. \end{aligned}$$

Solving this system of two equations for g^* and $(C/K)^*$, we obtain

$$g^* = \frac{\sigma^S (\alpha\lambda L - \rho)}{1 + \alpha\sigma^S}, \tag{C.24}$$

$$\left(\frac{C}{K}\right)^* = \frac{\alpha\left[\left(1 - \alpha^2\sigma^S\right)\lambda L + (1+\alpha)\,\sigma^S\rho\right]}{\left(1 + \alpha\sigma^S\right)\left(\alpha^2 + \eta\right)}. \tag{C.25}$$

The validity of the conditions (57) and (58) is immediately clear.

Proof of ii): Equations (C.24) and (C.25) imply that

$$\frac{\partial g^*}{\partial \eta} = 0 \qquad \text{and} \qquad \frac{\partial (C/K)^*}{\partial \eta} < 0. \tag{C.26}$$

From (C.1)–(C.17) it follows that the variables L_A^*, L_Y^*, $(\pi/p_A)^*$, $r^* + \left(\varepsilon^K\right)^*$, $(\pi/p_A)^* + \left(\varepsilon^Z\right)^*$, and $\left(\varepsilon^Z\right)^*$ can be expressed as functions of g^* solely. Hence, taking into account that $\partial g^*/\partial\eta = 0$ [see (C.26)] we obtain

$$\frac{\partial v^*}{\partial \eta} = 0 \qquad \text{for} \qquad v = L_A, L_Y, r + \varepsilon^K, \pi/p_A, \varepsilon^Z, (\pi/p_A) + \varepsilon^Z.$$

The result with respect to ε^Z can also be inferred directly from the fact that $\varepsilon^Z = 0$ for $\phi_Z = 0$.

Using (C.2) and (C.26) we obtain

$$\frac{\partial (Y/K)^*}{\partial \eta} = \frac{\partial g^*}{\partial \eta} + \frac{\partial (C/K)^*}{\partial \eta} = \frac{\partial (C/K)^*}{\partial \eta} < 0. \tag{C.27}$$

From (C.1)–(C.17) it also follows that the variables r^*, p^*, $(x/L_Y)^*$, p_A^*, and $(w/A)^*$ can be expressed as functions of $(Y/K)^*$ solely. It is verified at a glance that

$$\frac{\partial v^*}{\partial (Y/K)^*} \begin{cases} > 0, & \text{for} \qquad v = r, p, \\ < 0, & \text{for} \qquad v = x/L_Y, p_A, w/A. \end{cases}$$

Hence, using the fact that $\partial (Y/K)^* / \partial\eta < 0$ [see (C.27)], we obtain

$$\frac{\partial v^*}{\partial \eta} \begin{cases} < 0, & \text{for} \qquad v = r, p, \\ > 0, & \text{for} \qquad v = x/L_Y, p_A, w/A. \end{cases}$$

The variables x^*, π^*, and $\left(\varepsilon^K\right)^*$ can be expressed as functions of both g^* and $(Y/K)^*$, where

$$\frac{\partial v^*}{\partial (Y/K)^*} < 0 \qquad \text{for} \qquad v = x, \pi, \varepsilon^K.$$

Taking into account that $\partial g^*/\partial\eta = 0$ [see (C.26)] and $\partial (Y/K)^* / \partial\eta < 0$ [see (C.27)] we get

$$\frac{\partial v^*}{\partial \eta} > 0 \qquad \text{for} \qquad v = x, \pi, \varepsilon^K.$$

Setting $\phi_Z = 0$ in (C.12) yields $(C/\Omega)^* = (1/\phi_K)(C/K)^*$. Hence,

$$\frac{\partial (C/\Omega)^*}{\partial \eta} = \frac{1}{\phi_K} \frac{\partial (C/K)^*}{\partial \eta} < 0.$$

Finally, using (C.13) and taking into account that $\partial g^*/\partial \eta = 0$ and $\partial (C/K)^* /\partial \eta < 0$ [see (C.26)], we obtain

$$\left(\frac{C}{Y}\right)^* = \frac{(C/K)^*}{g^* + (C/K)^*} \Rightarrow \frac{\partial (C/Y)^*}{\partial \eta} = \frac{g^*}{[g^* + (C/K)^*]^2} \frac{\partial (C/K)^*}{\partial \eta} < 0.$$

The validity of the assertions made in part ii) of proposition 2 is now easily verified by summarizing the results derived above in the following compact way:

$$\begin{aligned}
\frac{\partial v^*}{\partial \eta} &= 0 \qquad \text{for} \qquad v = L_A, L_Y, r + \varepsilon^K, \frac{\pi}{p_A}, \varepsilon^Z, \\
\frac{\partial v^*}{\partial \eta} &< 0 \qquad \text{for} \qquad v = \frac{C}{K}, \frac{C}{\Omega}, \frac{C}{Y}, \frac{Y}{K}, r, p, \\
\frac{\partial v^*}{\partial \eta} &> 0 \qquad \text{for} \qquad v = \frac{x}{L_Y}, x, \pi, \varepsilon^K, p_A, \frac{w}{A}.
\end{aligned}$$

Proof of iii) Using (C.24), (C.25), (C.1), and (C.2) we obtain

$$L_Y^* = L - L_A^* = \frac{\lambda L + \sigma^S \rho}{\lambda (1 + \alpha \sigma^S)}, \qquad \left(\frac{Y}{K}\right)^* = \frac{(1 + \eta \sigma^S) \alpha \lambda L + (\alpha - \eta) \sigma^S \rho}{(1 + \alpha \sigma^S)(\alpha^2 + \eta)}.$$

Substituting these results into (C.18) yields

$$\left(\frac{K}{K + p_A A}\right)^* = \left[1 + \frac{1 - \alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*\right]^{-1} \tag{C.28}$$

$$= \frac{(\alpha^2 + \eta)(\lambda L + \sigma^S \rho)}{\alpha (\lambda L + \sigma^S \rho) + \eta \{[1 + \alpha \sigma^S (1 - \alpha)] \lambda L + \alpha \sigma^S \rho\}}. \tag{C.29}$$

Differentiating (C.28) with respect to η and taking into account that $\partial (Y/K)^* /\partial \eta < 0$ and $\partial L_Y^*/\partial \eta = 0$, we obtain

$$\frac{\partial [K/(K + p_A A)]^*}{\partial \eta} = - \left[1 + \frac{1 - \alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*\right]^{-2} \frac{1 - \alpha}{\lambda L_Y^*} \times \frac{\partial (Y/K)^*}{\partial \eta} > 0.$$

It is obvious from (C.29) that

$$\left.\left(\frac{K}{K + p_A A}\right)^*\right|_{\eta = 0} = \alpha.$$

Moreover, recalling that g^* is strictly positive if and only if $\rho < \alpha \lambda L$ holds, we obtain

$$\lim_{\eta \to \infty} \left(\frac{K}{K + p_A A}\right)^* = \frac{\lambda L + \sigma^S \rho}{\lambda L + \sigma^S \rho + (1 - \alpha) \sigma^S (\alpha \lambda L - \rho)} < 1.$$

These results prove the validity of all assertions made in part iii) of Proposition 2. ■

C.4 Proof of Proposition 3

Introducing the definition $c \equiv C/K$, equations (50) and (51) can be written as

$$M_1\left(g^*, c^*, \eta, \phi_K, \phi_Z\right) = 0 \qquad \text{and} \qquad M_2\left(g^*, c^*, \eta, \phi_K, \phi_Z\right) = 0,$$

where

$$\begin{aligned} M_1 &\equiv -\left(1-\alpha^2\sigma^S\right)g^* + \sigma^S\left[\alpha^2 + \frac{\phi_K \eta}{\phi_K + \dfrac{\phi_Z\left(1-\alpha\right)}{\lambda L - g^*}\left(g^* + c^*\right)}\right]c^* - \sigma^S\rho, \\ M_2 &\equiv \left(1+\alpha\right)g^* + \frac{1}{\alpha}\left[\alpha^2 + \frac{\left(\phi_K - \phi_Z\right)\eta}{\phi_K + \dfrac{\phi_Z\left(1-\alpha\right)}{\lambda L - g^*}\left(g^* + c^*\right)}\right]c^* - \lambda L. \end{aligned}$$

In the Supplement we show that quite weak assumptions are sufficient for the existence of a unique solution

$$g^* = \Pi^g\left(\eta, \phi_K, \phi_Z\right), \qquad c^* = \Pi^c\left(\eta, \phi_K, \phi_Z\right),$$

that is economically meaningful in the sense that $g^* > 0$ and $c^* > 0$. The partial derivatives of g^* and c^* with respect to the status parameters can be derived by means of implicit differentiation:

$$\begin{aligned} \frac{\partial g^*}{\partial par} &= \frac{1}{\Psi}\left(\frac{\partial M_2}{\partial c^*}\frac{\partial M_1}{\partial par} - \frac{\partial M_1}{\partial c^*}\frac{\partial M_2}{\partial par}\right), \qquad par = \eta, \phi_K, \phi_Z, \\ \frac{\partial c^*}{\partial par} &= \frac{1}{\Psi}\left(-\frac{\partial M_2}{\partial g^*}\frac{\partial M_1}{\partial par} + \frac{\partial M_1}{\partial g^*}\frac{\partial M_2}{\partial par}\right), \qquad par = \eta, \phi_K, \phi_Z, \end{aligned}$$

where

$$\Psi \equiv \frac{\partial M_1}{\partial c^*}\frac{\partial M_2}{\partial g^*} - \frac{\partial M_1}{\partial g^*}\frac{\partial M_2}{\partial c^*}. \tag{C.30}$$

Note that the partial derivatives of M_1 and M_2 are evaluated at

$$\left(g^*, c^*, \eta, \phi_K, \phi_Z\right) = \left(\Pi^g\left(\eta, \phi_K, \phi_Z\right), \Pi^c\left(\eta, \phi_K, \phi_Z\right), \eta, \phi_K, \phi_Z\right).$$

In other words, we consider the following expressions: $\partial M_j / \partial \omega|_{M_1 = M_2 = 0}$, $\omega = g^*$, c^*, and par.

It can be shown by tedious calculations that

$$\frac{\partial M_1}{\partial g^*} = -\left(1-\alpha^2\sigma^S\right) - \frac{\eta\phi_K\phi_Z\left(1-\alpha\right)\sigma^S c^*\left(\lambda L + c^*\right)}{\left[\phi_K(\lambda L - g^*) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}, \tag{C.31}$$

$$\frac{\partial M_1}{\partial c^*} = \alpha^2\sigma^S + \frac{\eta\phi_K\sigma^S\left(\lambda L - g^*\right)\left[\phi_K(\lambda L - g^*) + \phi_Z\left(1-\alpha\right)g^*\right]}{\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}, \tag{C.32}$$

$$\frac{\partial M_2}{\partial g^*} = \left(1+\alpha\right) - \frac{\eta\left(\phi_K - \phi_Z\right)\phi_Z\left(1-\alpha\right)\left(\lambda L + c^*\right)c^*}{\alpha\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}, \tag{C.33}$$

$$\frac{\partial M_2}{\partial c^*} = \alpha + \frac{\eta\left(\phi_K - \phi_Z\right)\left(\lambda L - g^*\right)\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)g^*\right]}{\alpha\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}. \tag{C.34}$$

Unfortunately, if $\phi_Z > \phi_K$, then the sign of $\partial M_2/\partial c^*$ cannot be determined immediately. However, using the fact that

$$M_2 = 0 \Leftrightarrow \frac{\eta\left(\phi_K - \phi_Z\right)\left(\lambda L - g^*\right)}{\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)} = \frac{\alpha\left[\lambda L - \left(1+\alpha\right)g^* - \alpha c^*\right]}{c^*}$$

we can show that

$$\frac{\partial M_2}{\partial c^*} = \frac{\phi_Z\alpha\left(1-\alpha\right)\left(c^*\right)^2 + \left[\phi_K(\lambda L - g^*) + \phi_Z\left(1-\alpha\right)g^*\right]\left[\lambda L - \left(1+\alpha\right)g^*\right]}{c^*\left[\phi_K(\lambda L - g^*) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]} \tag{C.35}$$

holds at the steady state. From (C.31), (C.32), and (C.35) it follows that

$$\frac{\partial M_1}{\partial g^*} < 0, \qquad \frac{\partial M_1}{\partial c^*} > 0, \qquad \frac{\partial M_2}{\partial c^*} > 0 \tag{C.36}$$

holds at the steady state, regardless of whether $0 < \phi_Z \leq \phi_K$ or $0 < \phi_K < \phi_Z$.

From (C.30), (C.33), and (C.36) it is obvious that

$$\phi_Z \geq \phi_K \Rightarrow \frac{\partial M_2}{\partial g^*} > 0 \Rightarrow \Psi > 0. \tag{C.37}$$

To determine the sign of Ψ for the case $\phi_Z < \phi_K$ we make use of the fact that Ψ can be expressed as

$$\begin{aligned}\Psi \quad = \quad & \frac{\eta\left[\phi_K\left(1+\alpha\sigma^S\right) - \phi_Z\left(1-\alpha^2\sigma^S\right)\right]\left(\lambda L - g^*\right)\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)g^*\right]}{\alpha\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2} \\ & + \alpha\left(1+\sigma^S\alpha\right) + \frac{\eta\phi_Z^2\left(1-\alpha\right)\alpha^2\sigma^S c^*\left(\lambda L + c^*\right)}{\alpha\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}.\end{aligned} \tag{C.38}$$

Equation (C.38) implies that

$$\phi_Z < \phi_K \Rightarrow \phi_K\left(1+\alpha\sigma^S\right) - \phi_Z\left(1-\alpha^2\sigma^S\right) > 0 \Rightarrow \Psi > 0. \tag{C.39}$$

From (C.37) and (C.39) it follows that

$$\Psi > 0 \qquad \text{for } \phi_Z \geq 0. \tag{C.40}$$

To analyze the effects of changes in ϕ_Z, we substitute

$$\begin{aligned}
\frac{\partial M_1}{\partial \phi_Z} &= -\frac{\eta\phi_K (1-\alpha)\sigma^S c^* (\lambda L - g^*)(g^* + c^*)}{[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^2} < 0, \\
\frac{\partial M_2}{\partial \phi_Z} &= -\frac{\eta\phi_K c^* (\lambda L - g^*)[(\lambda L - g^*) + (1-\alpha)(g^* + c^*)]}{\alpha[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^2} < 0
\end{aligned}$$

into the general representation of the solutions. This yields

$$\begin{aligned}
\frac{\partial g^*}{\partial \phi_Z} &= \frac{1}{\Psi}\frac{\phi_K \eta\sigma^S c^* (\lambda L - g^*)^2 \left\{\phi_K \left(\alpha^2 + \eta\right)(\lambda L - g^*) + \phi_Z (1-\alpha)\left[\alpha^2 (g^* + c^*) + \eta g^*\right]\right\}}{\alpha[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^3}, \\
\frac{\partial c^*}{\partial \phi_Z} &= \frac{1}{\Psi}\frac{\phi_K \eta c^* (\lambda L - g^*)\left[\left(1 + \sigma^S\alpha\right)(1-\alpha)(g^* + c^*) + \left(1 - \alpha^2\sigma^S\right)(\lambda L - g^*)\right]}{\alpha[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^2} \\
&\quad + \frac{1}{\Psi}\frac{\phi_K \phi_Z \eta^2 (1-\alpha)\sigma^S (c^*)^2 (\lambda L - g^*)(\lambda L + c^*)}{\alpha[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^3}.
\end{aligned}$$

These solutions imply that

$$\phi_K > 0 \qquad \Rightarrow \qquad \frac{\partial g^*}{\partial \phi_Z} > 0 \quad \text{and} \quad \frac{\partial c^*}{\partial \phi_Z} > 0. \tag{C.41}$$

From (C.1), (C.9), (C.13), (C.14), and (C.16) it follows that the variables L_A^*, L_Y^*, $(\pi/p_A)^*$, $r^* + \left(\varepsilon^K\right)^*$, $(\pi/p_A)^* + \left(\varepsilon^Z\right)^*$, and $\left(\varepsilon^Z\right)^*$ can be expressed as functions of g^* solely, where

$$\frac{\partial v^*}{\partial g^*}\begin{cases} > 0 & \text{for} \quad v = L_A, r + \varepsilon^K, (\pi/p_A) + \varepsilon^Z, \varepsilon^Z, \\ < 0 & \text{for} \quad v = L_Y, \pi/p_A. \end{cases}$$

Hence, using the fact that $\partial g^*/\partial \phi_Z > 0$ [see (C.41)], we obtain

$$\frac{\partial v^*}{\partial \phi_Z}\begin{cases} > 0 & \text{for} \quad v = L_A, r + \varepsilon^K, (\pi/p_A) + \varepsilon^Z, \varepsilon^Z, \\ < 0 & \text{for} \quad v = L_Y, \pi/p_A. \end{cases}$$

From (C.41) and (C.2) it follows that

$$\frac{\partial (Y/K)^*}{\partial \phi_Z} = \frac{\partial g^*}{\partial \phi_Z} + \frac{\partial c^*}{\partial \phi_Z} > 0. \tag{C.42}$$

From (C.1)–(C.17) it follows that the variables r^*, p^*, $(x/L_Y)^*$, p_A^*, $(w/A)^*$ can be expressed as functions of $(Y/K)^*$ solely, where

$$\frac{\partial v^*}{\partial (Y/K)^*}\begin{cases} > 0 & \text{for} \quad v = r, p, \\ < 0 & \text{for} \quad v = x/L_Y, p_A, w/A. \end{cases}$$

Hence, using the fact that $\partial (Y/K)^* / \partial \phi_Z > 0$ [see (C.42)], we obtain

$$\frac{\partial v^*}{\partial \phi_Z}\begin{cases} > 0 & \text{for} \quad v = r, p, \\ < 0 & \text{for} \quad v = x/L_Y, p_A, w/A. \end{cases}$$

Equations (C.6), (C.8), and (C.18) imply that the variables x^*, π^*, and $[K/(K+p_A A)]^*$ can be expressed as functions of both g^* and $(Y/K)^*$, where

$$\frac{\partial v^*}{\partial g^*} < 0 \qquad \text{for} \qquad v = x, \pi, \frac{K}{K+p_A A},$$
$$\frac{\partial v^*}{\partial (Y/K)^*} < 0 \qquad \text{for} \qquad v = x, \pi, \frac{K}{K+p_A A}.$$

Taking into account that $\partial g^*/\partial \phi_Z > 0$ and $\partial (Y/K)^* / \partial \phi_Z > 0$, we obtain

$$\frac{\partial v^*}{\partial \phi_Z} < 0 \qquad \text{for} \qquad v = x, \pi, \frac{K}{K+p_A A}.$$

The results derived above prove the validity of the assertions made in Proposition 3. ■

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Supplement

D The decentralized equilibrium: further results

D.1 Derivation of Equation (32)

From the flow budget constraint of the representative household (1) it follows that

$$\dot{K} = rK + wL + DZ - C - p_Z\dot{Z}.$$

Using i) the labor market equilibrium condition, $L = L_Y + L_A$, ii) the normalization of the number of shares, $Z = A$ ($\Rightarrow \dot{Z} = \dot{A}$), iii) the equilibrium condition of the market for blueprints, $p_A = p_Z$, and iv) the assumption that the operating profit of firms in the intermediate goods sector is fully distributed in the form of dividends at any time t, $D(t) = \pi(t)$, we obtain

$$\dot{K} = rK + w(L_Y + L_A) - C + \pi A - p_A\dot{A}.$$

Employing i) the equilibrium condition of the rental market of real capital, $K = Ak$, ii) the fact that the identical operating profit of the firms in the intermediate goods sector is equal to $\pi = px - rk$, and iii) the production function for blueprints of the representative firm in the R&D sector, $\dot{A} = \lambda AL_A$, we get

$$\begin{aligned} \dot{K} &= rAk + wL_Y + wL_A - C + (px - rk)A - p_A\lambda AL_A \\ &= (wL_Y + Apx) - C - (p_A\lambda AL_A - wL_A). \end{aligned}$$

Perfect competition in the R&D sector and in the final goods sector implies $p_A\dot{A} - wL_A = p_A\lambda AL_A - wL_A = 0$ and $wL_Y + Apx = Y$, such that

$$\dot{K} = Y - C. \tag{D.1}$$

Obviously, (D.1) is equivalent to (32) as given in the main text. ■

D.2 Derivation of Equation (34)

Using that firms in the intermediate goods sector produce identical quantities in Equation (19) implies that

$$Y = L_Y^{1-\alpha}\int_0^A x^\alpha\, di = L_Y^{1-\alpha}Ax^\alpha = (AL_Y)^{1-\alpha}(Ax)^\alpha. \tag{D.2}$$

Taking into account that $x_i = k_i$ for $i \in [0, A]$, we obtain $k_i = k$ for $i \in [0, A]$, where $k = x$. From $k = x$ and $K = Ak$ it then follows that $Ax = Ak = K$. Substituting this into (D.2) and using $L = L_Y + L_A$, we obtain

$$Y = K^\alpha (AL_Y)^{1-\alpha} = K^\alpha [A(L - L_A)]^{1-\alpha}. \tag{D.3}$$

Using (D.3) and $Y = \dot{K} + C$, we finally get

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} = \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} - \frac{C}{K}. \tag{D.4}$$

Obviously, (D.4) is equivalent to (34) as given in the main text. ■

D.3 Derivation of Equation (35)

Solving (28) for r and taking into account that $L = L_Y + L_A$, we obtain

$$r = \alpha^2 \left(\frac{K}{AL_Y}\right)^{-(1-\alpha)} = \alpha^2 \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)}. \tag{D.5}$$

Substituting (D.5) into (15) yields

$$\frac{\dot{C}}{C} = \sigma^S\left(C, \chi\right) \left\{\alpha^2 \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} + \varepsilon^K\left(C, \Omega, \chi, \phi_K\right) - \rho\right\}. \tag{D.6}$$

The differential equation (D.6) is identical to (35) as given in the main text. ■

D.4 Derivation of Equation (37)

Taking into account that $Z = A$ and $p_A = p_Z$ hold in equilibrium, (6) implies

$$\Omega = \phi_K K + \phi_Z p_Z Z = \phi_K K + \phi_Z p_A A. \tag{D.7}$$

Equation (26) and the labor market equilibrium condition (27) imply that

$$p_A = \frac{1}{\lambda}\frac{w}{A}.$$

The first order condition of the representative firm in the final goods sector with respect to the choice of labor input (20) together with the labor market equilibrium condition (27) and the fact that$x_i = x$ for $i \in [0, A]$ holds, implies that

$$w = (1-\alpha) \int_0^A \left(\frac{x_i}{L_Y}\right)^\alpha di = (1-\alpha)\, A \left(\frac{x}{L_Y}\right)^\alpha.$$

Using $x = k$ and $K = Ak$, we obtain

$$\frac{x}{L_Y} = \frac{k}{L_Y} = \frac{Ak}{AL_Y} = \frac{K}{AL_Y} = \frac{K}{A\left(L - L_A\right)}. \tag{D.8}$$

The last three results imply

$$\frac{w}{A} = (1-\alpha) \left[\frac{K}{A\left(L - L_A\right)}\right]^\alpha, \tag{D.9}$$

and

$$p_A = \frac{1-\alpha}{\lambda} \left[\frac{K}{A\left(L - L_A\right)}\right]^\alpha. \tag{D.10}$$

Substituting (D.10) into (D.7) yields

$$\Omega = \phi_K K + \phi_Z \frac{(1-\alpha) A}{\lambda} \left[\frac{K}{A(L-L_A)} \right]^{\alpha}. \tag{D.11}$$

Obviously, (D.11) is equivalent to (37) as given in the main text. ■

D.5 Derivation of Equations (54) and (36)

Using $p_A = p_Z$ and $D(t) = \pi(t)$, we obtain from (11) that

$$r + \varepsilon^K (C, \Omega, \chi, \phi_K) = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A} + \varepsilon^Z (C, \Omega, \chi, \phi_Z). \tag{D.12}$$

From (24) and (D.5) it follows that

$$\pi = (1-\alpha)\, \alpha^{(1+\alpha)/(1-\alpha)} r^{-\alpha/(1-\alpha)} L_Y = (1-\alpha)\, \alpha \left[\frac{K}{A(L-L_A)} \right]^{\alpha} L_Y. \tag{D.13}$$

Using (D.13), (D.10), and $L = L_Y + L_A$, we get

$$\frac{\pi}{p_A} = \alpha\lambda L_Y = \alpha\lambda (L - L_A). \tag{D.14}$$

Note that (D.14) equals (54) as given in the main text.

From (D.10) it follows that

$$\frac{\dot{p}_A}{p_A} = \alpha \frac{\dot{K}}{K} - \alpha \frac{\dot{A}}{A} + \alpha \frac{\dot{L}_A}{L - L_A}. \tag{D.15}$$

Substituting (D.15) into (D.12) yields

$$\dot{L}_A = (L - L_A) \left\{ \frac{1}{\alpha} \left[r + \varepsilon^K - \left(\frac{\pi}{p_A} + \varepsilon^Z \right) \right] - \frac{\dot{K}}{K} + \frac{\dot{A}}{A} \right\}, \tag{D.16}$$

where $\varepsilon^K = \varepsilon^K (C, \Omega, \chi, \phi_K)$ and $\varepsilon^Z = \varepsilon^Z (C, \Omega, \chi, \phi_Z)$. Substituting (D.5), (D.14), (34) [= (D.4)], and (33) into (D.16), we obtain

$$\begin{aligned} \dot{L}_A = (L - L_A) \Bigg\{ & -(1-\alpha) \left[\frac{K}{A(L-L_A)} \right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A \\ & -\lambda (L - L_A) + \frac{\varepsilon^K (C, \Omega, \chi, \phi_K) - \varepsilon^Z (C, \Omega, \chi, \phi_K)}{\alpha} \Bigg\}. \end{aligned} \tag{D.17}$$

The differential equation (D.17) is identical to (36) as given in the main text. ■

D.6 Derivation of (44) and (45)

The instantaneous utility function as given by (42),

$$u(C, S) = \frac{1}{1-\theta} \left\{ \left[C^{\xi} h(S) \right]^{1-\theta} - 1 \right\}, \qquad \xi > 0, \quad \theta > 0, \quad 1 + \xi(\theta - 1) > 0,$$

where $h(S) > 0$ and $h'(S) > 0$, and the ratio specification of the status function as given by (43),

$$S\left(\Omega,\bar{\Omega}\right) = \varphi\left(\Omega/\bar{\Omega}\right), \qquad \varphi' > 0, \quad \varphi'' \leq 0,$$

exhibit the following properties:

$$\frac{\partial u(C,S)}{\partial C} = \xi C^{\xi(1-\theta)-1}\left[h(S)\right]^{1-\theta} > 0, \qquad \frac{\partial u(C,S)}{\partial S} = C^{\xi(1-\theta)}\left[h(S)\right]^{-\theta} h'(S) > 0,$$

$$\frac{\partial^2 u(C,S)}{\partial C^2} = -\left[1+\xi(\theta-1)\right]\xi C^{\xi(1-\theta)-2}\left[h(S)\right]^{1-\theta} < 0,$$

$$S(\Omega,\Omega) = \varphi(\Omega/\Omega) = \varphi(1) \equiv \chi,$$

$$\frac{\partial S\left(\Omega,\bar{\Omega}\right)}{\partial \Omega} = \varphi'\left(\Omega/\bar{\Omega}\right)\times\left(1/\bar{\Omega}\right), \qquad \frac{\partial S(\Omega,\Omega)}{\partial \Omega} = \varphi'(1)(1/\Omega).$$

Evaluating the partial derivatives of the instantaneous utility function U at $(C,S) = (C,\chi) = (C,\varphi(1))$ and substituting the resulting expressions as well as the result for $\partial S(\Omega,\Omega)/\partial\Omega$ into the definitions of $\sigma^S(C,\chi)$ and $MRS(C,\Omega,\chi)$ as given by (14) and (16), respectively, we obtain:

$$\sigma^S(C,\chi) \equiv -\frac{\partial u(C,\chi)}{\partial C}\left[C\frac{\partial^2 u(C,\chi)}{\partial C^2}\right]^{-1} = \frac{1}{1+\xi(\theta-1)}, \tag{D.18}$$

$$MRS(C,\Omega,\chi) \equiv \frac{\partial u(C,\chi)}{\partial S}\frac{\partial S(\Omega,\Omega)}{\partial \Omega}\left[\frac{\partial u(C,\chi)}{\partial C}\right]^{-1} = \frac{1}{\xi}\frac{h'(\varphi(1))\varphi'(1)}{h(\varphi(1))}\times\frac{C}{\Omega}. \tag{D.19}$$

Equation (D.18) implies that the symmetric effective elasticity of intertemporal substitution under relative wealth preferences does not depend on C, i.e., $\partial\sigma^S(C,\chi)/\partial C = 0$. From (D.19) it follows that the symmetric marginal rate of substitution of status-relevant own wealth Ω for consumption C, $MRS(C,\Omega,\chi)$, depends linearly on (C/Ω):

$$MRS(C,\Omega,\chi) = \eta\times\frac{C}{\Omega}, \qquad \text{where} \quad \eta \equiv \frac{\beta}{\xi} > 0, \quad \beta \equiv \frac{h'\left[\varphi(1)\right]\varphi'(1)}{h\left[\varphi(1)\right]} = \frac{h'(\chi)\varphi'(1)}{h(\chi)} > 0.$$

Consequently, ε^K and ε^Z depend linearly on (C/Ω), too:

$$\varepsilon^K = \phi_K\eta\times\frac{C}{\Omega}, \qquad \text{and} \qquad \varepsilon^Z = \phi_Z\eta\times\frac{C}{\Omega}.$$

The results given above prove the validity of (44) and (45). ■

E The BGP – Existence, Uniqueness and its Dependence on the Status Parameters

E.1 Existence and Uniqueness of the Steady State

E.1.1 Preliminaries

From (50) and (51) it follows that the steady state values g^* and c^*, where

$$c \equiv C/K, \tag{E.1}$$

satisfy the equations

$$M_1\left(g^*, c^*, \eta, \phi_K, \phi_Z, \sigma^S, \rho, \lambda, L, \alpha\right) = 0, \tag{E.2}$$

$$M_2\left(g^*, c^*, \eta, \phi_K, \phi_Z, \sigma^S, \rho, \lambda, L, \alpha\right) = 0, \tag{E.3}$$

where

$$M_1 \equiv -\left(1-\alpha^2\sigma^S\right) g^* + \sigma^S \left[\alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z(1-\alpha)}{\lambda L - g^*}\left(g^* + c^*\right)}\right] c^* - \sigma^S \rho, \tag{E.4}$$

$$M_2 \equiv (1+\alpha) g^* + \frac{1}{\alpha}\left[\alpha^2 + \frac{\left(\phi_K - \phi_Z\right)\eta}{\phi_K + \frac{\phi_Z(1-\alpha)}{\lambda L - g^*}\left(g^* + c^*\right)}\right] c^* - \lambda L. \tag{E.5}$$

We analyze the conditions for the existence of a unique, economically meaningful solution (g^*, c^*). From (E.2)–(E.5) it follows that

$$g^* = \Pi^g\left(\eta, \phi_K, \phi_Z, \sigma^S, \rho, \lambda, L, \alpha\right),$$

$$c^* = \Pi^c\left(\eta, \phi_K, \phi_Z, \sigma^S, \rho, \lambda, L, \alpha\right).$$

Moreover, we study the effects of changes in the status parameters η, ϕ_K, and ϕ_Z on g^* and c^*. For this reason, we will use a simplified general representation of the steady state values:

$$g^* = \Pi^g\left(\eta, \phi_K, \phi_Z\right), \tag{E.6}$$

$$c^* = \Pi^c\left(\eta, \phi_K, \phi_Z\right). \tag{E.7}$$

These results are the starting point for a complete analysis of the properties of the BGP.

E.1.2 Special Case $\phi_Z = 0$

First, we analyze the special case in which shares issued by entrants – in contrast to physical capital used by incumbents – do not matter for status, i.e., $\phi_Z = 0$ and $\phi_K > 0$. Setting $\phi_Z = 0$ in (E.4) and (E.5) it is easily verified that both $M_1 = 0$ and $M_2 = 0$ imply a linear relation between c^* and g^*:

$$\begin{aligned} c^*|_{M_1=0} &= \frac{\sigma^S \rho + \left(1-\alpha^2\sigma^S\right) g^*}{\sigma^S\left(\alpha^2+\eta\right)}, \\ c^*|_{M_2=0} &= \frac{\alpha\left[\lambda L - (1+\alpha) g^*\right]}{\alpha^2+\eta}. \end{aligned} \tag{E.8}$$

The graphical representation of these results in the (g^*, c^*)-plane (see Figure 1) exhibits the following properties:

i) The straight line that corresponds to $M_1 = 0$ (henceforth, the $M_1 = 0$-line) is positively sloped if and only if

$$1 - \alpha^2\sigma^S > 0 \Leftrightarrow \sigma^S < 1/\alpha^2. \tag{E.9}$$

In this paper we assume that (E.9) holds. Note that this assumption is quite weak. For instance, if $\alpha = 1/3$, then $1/\alpha^2 = 9$.

ii) The straight line that corresponds to $M_2 = 0$ (henceforth, the $M_2 = 0$-line) is negatively sloped. From $L_Y^* = (\lambda L - g^*)/\lambda > 0$ and $L_A^* = g^*/\lambda > 0$ it follows that economically meaningful rates of growth satisfy the condition

$$0 < g^* < \lambda L. \tag{E.10}$$

In order to ensure that not only $L_Y^* > 0$ and $L_A^* > 0$, but also $c^* > 0$ holds, we have to restrict the analysis to situations in which

$$0 < g^* < \frac{\lambda L}{1+\alpha} \tag{E.11}$$

holds. Assumption (E.11) is slightly stronger than assumption (E.10). However, we can easily show that it is not at all restrictive. From $L_A^* = g^*/\lambda > 0$ it follows that condition (E.11) is equivalent to

$$0 < \frac{L_A^*}{L} < \frac{1}{1+\alpha} \tag{E.12}$$

Condition (E.12) requires that the labor input of the R&D sector is less than $100/(1+\alpha)$ percent of total labor input. From $\alpha \in (0,1)$ it follows that $100/(1+\alpha) > 50$. For instance, if $\alpha = 1/3$, then $100/(1+\alpha) = 75$. Hence, condition (E.12) is not at all restrictive because it allows for an unrealistically high employment share of the R&D sector and hence for an excessive growth rate.

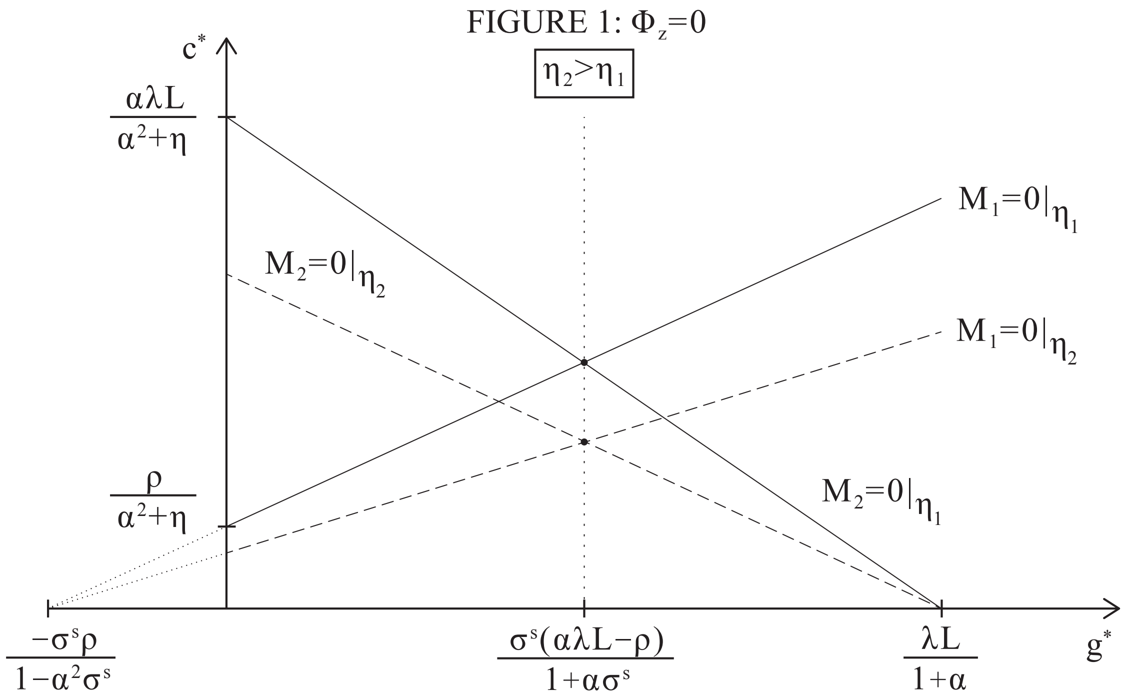

Figure 1: BGP determination of c^* and g^* in case of $\phi_Z = 0$

To ensure that there exists an economically meaningful steady state, i.e., a point of intersection of the $M_1 = 0$-line and the $M_2 = 0$-line in which $g^* > 0$ and $c^* > 0$, we have to introduce a further assumption. Taking into account that

$$c^*|_{M_1=0} = \begin{cases} 0 & \text{for} \quad g^* = -\dfrac{\sigma^S \rho}{1-\alpha^2\sigma^S} \\ \dfrac{\rho}{\alpha^2+\eta} & \text{for} \quad g^* = 0 \\ \dfrac{(1+\alpha)\,\sigma^S\rho + \left(1-\alpha^2\sigma^S\right)\lambda L}{(1+\alpha)\,(\alpha^2+\eta)\,\sigma^S} & \text{for} \quad g^* = \dfrac{\lambda L}{1+\alpha} \end{cases}$$

$$c^*|_{M_2=0} = \begin{cases} \dfrac{\alpha\lambda L}{\alpha^2+\eta} & \text{for} \quad g^* = 0 \\ 0 & \text{for} \quad g^* = \dfrac{\lambda L}{1+\alpha} \end{cases}$$

it can be verified at first glance from a graphical representation (see Figure 1) that this missing assumption is given by

$$\frac{\rho}{\alpha^2+\eta} < \frac{\alpha\lambda L}{\alpha^2+\eta} \Leftrightarrow \alpha\lambda L - \rho > 0. \tag{E.13}$$

Assumption (E.13) implies that the negatively sloped $M_2 = 0$-line intersects the vertical axis at a point that is above the point at which the positively sloped $M_1 = 0$-line intersects. At $g^* = \lambda L/\,(1+\alpha)$ the $M_2 = 0$-line intersects the horizontal axis, while the $M_1 = 0$-line assumes a strictly positive value.

The unique steady-state values that correspond to $\phi_Z = 0$ are given by

$$g^* = \frac{\sigma^S\,(\alpha\lambda L - \rho)}{1+\alpha\sigma^S}, \qquad c^* = \frac{\alpha\left[\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\,\sigma^S\rho\right]}{\left(1+\alpha\sigma^S\right)(\alpha^2+\eta)}. \tag{E.14}$$

E.1.3 General Case $\phi_Z > 0$

It can be shown by tedious calculations that $M_1 = 0$ implies the following quadratic equation:

$$\Omega_2\,(c^*)^2 + \Omega_1 c^* + \Omega_0 = 0,$$

where

$$\Omega_2 \equiv \alpha^2\sigma^S\phi_Z\,(1-\alpha)\,, \tag{E.15}$$

$$\Omega_1 \equiv \phi_K\left(\alpha^2+\eta\right)\sigma^S\,(\lambda L - g^*) - \phi_Z\,(1-\alpha)\left[\left(1-2\alpha^2\sigma^S\right)g^* + \sigma^S\rho\right], \tag{E.16}$$

$$\Omega_0 \equiv -\left[\left(1-\alpha^2\sigma^S\right)g^* + \sigma^S\rho\right]\left[\phi_K\,(\lambda L - g^*) + \phi_Z\,(1-\alpha)\,g^*\right]. \tag{E.17}$$

It is obvious that $\Omega_2 > 0$. From (E.10) and (E.9) it follows that $\Omega_0 < 0$. Hence, it is clear that the quadratic equation has a negative and a positive root. (The validity of this assertion is, for instance, verified at first glance by a graphical representation of the quadratic equation). Since only the positive root makes sense from an economic point of view, we obtain

$$c^*|_{M_1=0} = \frac{-\Omega_1 + \sqrt{\Omega_1^2 - 4\Omega_2\Omega_0}}{2\Omega_2}. \tag{E.18}$$

From (E.15)–(E.17) it follows that

$$\Omega_2 = \Omega_2 (\phi_Z, ...) > 0, \qquad \Omega_1 = \Omega_1 (g^*, \phi_K, \phi_Z, \eta, ...), \qquad \Omega_0 = \Omega_0 (g^*, \phi_K, \phi_Z, ...) < 0.$$

Hence, it is clear that

$$c^*|_{M_1=0} = \Xi^1 (g^*, \phi_K, \phi_Z, \eta, ...) > 0, \tag{E.19}$$

where Ξ^1 is defined as the right-hand side of (E.18). The graphical representation of (E.19) in the (g^*, c^*)-plane will be called $M_1 = 0$-curve.

In the special case in which physical capital is irrelevant for status ($\phi_K = 0$) and $\phi_Z > 0$, we obtain the following linear relation between c^* and g^* that is unaffected by changes in η or ϕ_Z:

$$c^*|_{M_1=0} = \frac{\left(1-\alpha^2\sigma^S\right) g^* + \sigma^S \rho}{\alpha^2 \sigma^S}.$$

Hence, if $\phi_K = 0$, then the $M_1 = 0$-line is positively sloped and its position depends neither on η nor on ϕ_Z.

The analysis of the general case in which $\phi_K > 0$ and $\phi_Z > 0$ holds, is much more complicated. In order to derive the signs of the partial derivatives of (E.18) with respect to g^*, ϕ_K, ϕ_Z, and η we can either differentiate (E.18) with respect to these variables and thereby taking into account (E.15)–(E.17) or make use of the fact that

$$M_1 \left(g^*, \Xi^1 (g^*, \phi_K, \phi_Z, \eta, ...), \eta, \phi_K, \phi_Z\right) = 0$$

holds and apply the implicit function theorem. It can be shown by tedious calculations that

$$\frac{\partial M_1}{\partial g^*} = -\left(1-\alpha^2\sigma^S\right) - \frac{\eta\phi_K\phi_Z (1-\alpha)\sigma^S c^* (\lambda L + c^*)}{\left[\phi_K(\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)\right]^2} < 0, \tag{E.20}$$

$$\frac{\partial M_1}{\partial c^*} = \alpha^2\sigma^S + \frac{\eta\phi_K\sigma^S (\lambda L - g^*)\left[\phi_K(\lambda L - g^*) + \phi_Z (1-\alpha) g^*\right]}{\left[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)\right]^2} > 0, \tag{E.21}$$

$$\frac{\partial M_1}{\partial \eta} = \frac{\phi_K \sigma^S c^* (\lambda L - g^*)}{\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)} > 0, \tag{E.22}$$

$$\frac{\partial M_1}{\partial \phi_K} = \frac{\eta\phi_Z\sigma^S (1-\alpha) c^* (\lambda L - g^*)(g^* + c^*)}{\left[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)\right]^2} > 0, \tag{E.23}$$

$$\frac{\partial M_1}{\partial \phi_Z} = -\frac{\eta\phi_K (1-\alpha)\sigma^S c^* (\lambda L - g^*)(g^* + c^*)}{\left[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)\right]^2} < 0. \tag{E.24}$$

Using these results we obtain

$$\left.\frac{\partial c^*}{\partial g^*}\right|_{M_1=0} = -\frac{\left.\dfrac{\partial M_1}{\partial g^*}\right|_{M_1=0}}{\left.\dfrac{\partial M_1}{\partial c^*}\right|_{M_1=0}} > 0, \qquad \left.\frac{\partial c^*}{\partial \eta}\right|_{M_1=0} = -\frac{\left.\dfrac{\partial M_1}{\partial \eta}\right|_{M_1=0}}{\left.\dfrac{\partial M_1}{\partial c^*}\right|_{M_1=0}} < 0,$$

$$\left.\frac{\partial c^*}{\partial \phi_K}\right|_{M_1=0} = -\frac{\left.\frac{\partial M_1}{\partial \phi_K}\right|_{M_1=0}}{\left.\frac{\partial M_1}{\partial c^*}\right|_{M_1=0}} < 0, \qquad \left.\frac{\partial c^*}{\partial \phi_Z}\right|_{M_1=0} = -\frac{\left.\frac{\partial M_1}{\partial \phi_Z}\right|_{M_1=0}}{\left.\frac{\partial M_1}{\partial c^*}\right|_{M_1=0}} > 0.$$

The notation $\partial M_1/\partial\omega|_{M_1=0}$ expresses the fact that the partial derivative of M_1 with respect to ω is evaluated at $(g^*, c^*, \eta, \phi_K, \phi_Z) = \left(g^*, \Xi^1\left(g^*, \phi_K, \phi_Z, \eta, ...\right), \eta, \phi_K, \phi_Z\right)$. Hence, the $M_1 = 0$-curve is positively sloped. A rise in η or ϕ_K causes the $M_1 = 0$-curve to shift downwards, while a rise in ϕ_Z leads to an upward shift.

Next, we will discuss the properties of the $M_2 = 0$-curve. It can be shown that $M_2 = 0$ implies the following quadratic equation:

$$\Lambda_2 \left(c^*\right)^2 + \Lambda_1 c^* + \Lambda_0 = 0, \tag{E.25}$$

where

$$\Lambda_2 \equiv \alpha^2 \phi_Z \left(1-\alpha\right), \tag{E.26}$$

$$\Lambda_1 \equiv \left[\left(\phi_K - \phi_Z\right)\left(\alpha^2 + \eta\right) - \phi_Z \alpha \left(1 - 2\alpha\right)\right]\left(\lambda L - g^*\right) + 2\alpha^2 \phi_Z \left(1-\alpha\right) g^*, \tag{E.27}$$

$$\Lambda_0 \equiv -\alpha \left[\phi_K(\lambda L - g^*) + \phi_Z \left(1-\alpha\right) g^*\right]\left[\lambda L - \left(1+\alpha\right) g^*\right]. \tag{E.28}$$

It is obvious that $\Lambda_2 > 0$. From (E.10) and (E.9) it follows that $\Lambda_0 < 0$. Hence, it is clear that the quadratic equation has a negative and a positive root. Since only the positive root is economically meaningful we obtain

$$c^*|_{M_2=0} = \frac{-\Lambda_1 + \sqrt{\Lambda_1^2 - 4\Lambda_2\Lambda_0}}{2\Lambda_2}. \tag{E.29}$$

From (E.26)–(E.28) it follows that

$$\Lambda_2 = \Lambda_2\left(\phi_Z, ...\right) > 0, \qquad \Lambda_1 = \Lambda_1\left(g^*, \phi_K, \phi_Z, \eta, ...\right), \qquad \Lambda_0 = \Lambda_0\left(g^*, \phi_K, \phi_Z, ...\right) < 0.$$

Hence, it is clear that

$$c^*|_{M_2=0} = \Xi^2\left(g^*, \phi_K, \phi_Z, \eta, ...\right) > 0, \tag{E.30}$$

where Ξ^2 is defined as the right-hand side of (E.29). The graphical representation of (E.30) in the (g^*, c^*)-plane will be called $M_2 = 0$-curve.

In the special case in which $\phi_K = \phi_Z$ (see Figure 2) we obtain the following linear relation between c^* and g^*:

$$c^*|_{M_2=0} = \frac{\lambda L - \left(1+\alpha\right) g^*}{\alpha}. \tag{E.31}$$

Obviously, the $M_2 = 0$-line is negatively sloped and its position does not depend on η. The $M_2 = 0$-line intersects the positively sloped $M_1 = 0$-curve at

$$g^* = \frac{\sigma^S\left[\left(\alpha+\eta\right)\lambda L - \rho\right]}{1 + \sigma^S\left[\alpha + \eta\left(1+\alpha\right)\right]}, \qquad c^* = \frac{\left(1-\alpha^2\sigma^S\right)\lambda L + \left(1+\alpha\right)\sigma^S\rho}{\alpha\left\{1 + \sigma^S\left[\alpha + \eta\left(1+\alpha\right)\right]\right\}}. \tag{E.32}$$

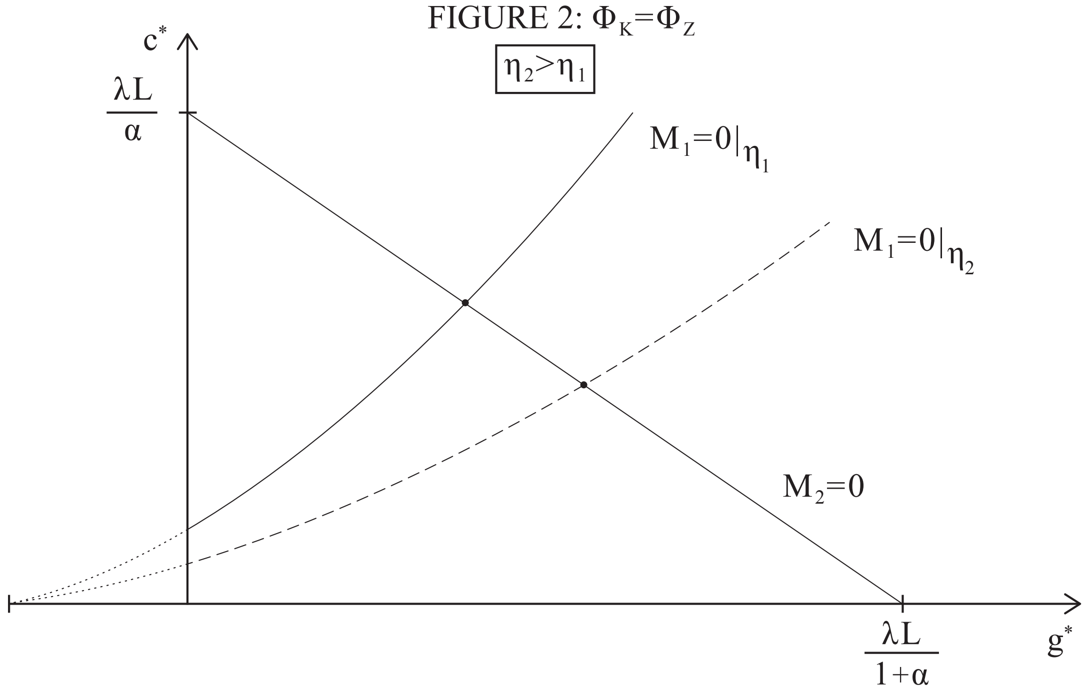

Figure 2: BGP determination of c^* and g^* in case of $\phi_Z = \phi_K$

In the general case in which $\phi_K \neq \phi_Z$ holds, the analysis of the $M_2 = 0$-curve is much more complicated. To derive the signs of the partial derivatives of (E.30), $c^*|_{M_2=0} = \Xi^2(g^*, \phi_K, \phi_Z, \eta, ...)$, with respect to g^*, ϕ_K, ϕ_Z, and η we can either differentiate (E.29) with respect to these variables and thereby taking into account (E.26)–(E.28) or make use of the fact that

$$M_2\left(g^*, \Xi^2(g^*, \phi_K, \phi_Z, \eta, ...), \eta, \phi_K, \phi_Z\right) = 0$$

holds and apply the implicit function theorem. It can be shown by tedious calculations that

$$\frac{\partial M_2}{\partial g^*} = (1+\alpha) - \frac{\eta(\phi_K - \phi_Z)\phi_Z(1-\alpha)(\lambda L + c^*)c^*}{\alpha\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)\right]^2}, \tag{E.33}$$

$$\frac{\partial M_2}{\partial c^*} = \alpha + \frac{\eta(\phi_K - \phi_Z)(\lambda L - g^*)\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)g^*\right]}{\alpha\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)\right]^2}, \tag{E.34}$$

$$\frac{\partial M_2}{\partial \eta} = \frac{(\phi_K - \phi_Z)c^*(\lambda L - g^*)}{\alpha\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)\right]}, \tag{E.35}$$

$$\frac{\partial M_2}{\partial \phi_K} = \frac{\eta\phi_Z c^*(\lambda L - g^*)\left[(\lambda L - g^*) + (1-\alpha)(g^* + c^*)\right]}{\alpha\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)\right]^2} > 0, \tag{E.36}$$

$$\frac{\partial M_2}{\partial \phi_Z} = -\frac{\eta\phi_K c^*(\lambda L - g^*)\left[(\lambda L - g^*) + (1-\alpha)(g^* + c^*)\right]}{\alpha\left[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)\right]^2} < 0. \tag{E.37}$$

It is easily verified that

$$M_2 = 0 \Leftrightarrow \frac{\eta(\phi_K - \phi_Z)(\lambda L - g^*)}{\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^* + c^*)} = \frac{\alpha\left[\lambda L - (1+\alpha)g^* - \alpha c^*\right]}{c^*}. \tag{E.38}$$

Using this result we obtain

$$\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0} = \frac{\phi_Z \alpha (1-\alpha)(c^*)^2 + [\phi_K(\lambda L - g^*) + \phi_Z (1-\alpha) g^*][\lambda L - (1+\alpha) g^*]}{c^* [\phi_K(\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]}. \tag{E.39}$$

The notation $\partial M_2/\partial\omega|_{M_2=0}$ expresses the fact that the partial derivative of M_2 with respect to ω is evaluated at $(g^*, c^*, \eta, \phi_K, \phi_Z) = \left(g^*, \Xi^2(g^*, \phi_K, \phi_Z, \eta, ...), \eta, \phi_K, \phi_Z\right)$. This transformation shows that

$$\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0} > 0$$

regardless of whether $\phi_Z \leq \phi_K$ or $\phi_Z > \phi_K$ holds.

The properties of the $M_2 = 0$-curve are given by the following partial derivatives of $c^*|_{M_2=0} = \Xi^2(g^*, \phi_K, \phi_Z, \eta, ...)$:

$$\left.\frac{\partial c^*}{\partial g^*}\right|_{M_2=0} = -\frac{\left.\frac{\partial M_2}{\partial g^*}\right|_{M_2=0}}{\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0}} \Rightarrow \operatorname{sgn}\left(\left.\frac{\partial c^*}{\partial g^*}\right|_{M_2=0}\right) = -\operatorname{sgn}\left(\left.\frac{\partial M_2}{\partial g^*}\right|_{M_2=0}\right),$$

$$\left.\frac{\partial c^*}{\partial \eta}\right|_{M_2=0} = -\frac{\left.\frac{\partial M_2}{\partial \eta}\right|_{M_2=0}}{\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0}} \Rightarrow \operatorname{sgn}\left(\left.\frac{\partial c^*}{\partial \eta}\right|_{M_2=0}\right) = -\operatorname{sgn}\left(\left.\frac{\partial M_2}{\partial \eta}\right|_{M_2=0}\right) = -\operatorname{sgn}(\phi_K - \phi_Z),$$

$$\left.\frac{\partial c^*}{\partial \phi_K}\right|_{M_2=0} = -\frac{\left.\frac{\partial M_2}{\partial \phi_K}\right|_{M_2=0}}{\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0}} < 0, \qquad \left.\frac{\partial c^*}{\partial \phi_Z}\right|_{M_2=0} = -\frac{\left.\frac{\partial M_2}{\partial \phi_Z}\right|_{M_2=0}}{\left.\frac{\partial M_2}{\partial c^*}\right|_{M_2=0}} > 0.$$

A rise in ϕ_Z causes the $M_2 = 0$-curve to shift upwards, while a rise in ϕ_K leads to a downward shift. A rise in η causes the $M_2 = 0$-curve to shift downwards if $\phi_K > \phi_Z$, while an upward shift obtains if $\phi_K < \phi_Z$.

With respect to the slope of the $M_2 = 0$-curve things are more complicated. The $M_2 = 0$-curve is negatively sloped if and only if $\partial M_2/\partial g^*|_{M_2=0} > 0$. From (E.33) it follows that $\phi_Z \geq \phi_K$ is sufficient for $\partial M_2/\partial g^*|_{M_2=0} > 0$. Moreover, it is obvious that $\partial M_2/\partial g^*|_{M_2=0} > 0$ also holds for $\phi_Z < \phi_K$ as long as η is sufficiently small. Unfortunately, we were not able to give an analytical proof that this property also holds for "large" values of η. We tried several illustrations. Irrespective of the magnitude of η we obtained a negatively sloped $M_2 = 0$-curve. Figure 3 supports the general validity of this result. In Figure 3 we depicted the $M_2 = 0$-curves that correspond to the special cases $\phi_Z = 0$ and $\phi_Z = \phi_K > 0$ [see (E.8) and (E.31)]:

$$c^*|_{M_2=0;\phi_Z=0} = \frac{\alpha[\lambda L - (1+\alpha) g^*]}{\alpha^2 + \eta},$$

$$c^*|_{M_2=0;\phi_Z=\phi_K} = \frac{\lambda L - (1+\alpha) g^*}{\alpha}.$$

Both curves intersect the horizontal axis at $g^* = \lambda L/(1+\alpha)$. Since

$$c^*|_{M_2=0;\phi_Z=0} = \frac{\alpha^2}{\alpha^2+\eta} c^*|_{M_2=0;\phi_Z=\phi_K}$$

holds, the $M_2 = 0$-curve that corresponds to $\phi_Z = 0$ is flatter than its $\phi_Z = \phi_K$-counterpart. From $\partial c^*/\partial\phi_Z|_{M_2=0} > 0$ it follows that if $0 < \phi_Z < \phi_K$, then the resulting $M_2 = 0|_{0<\phi_Z<\phi_K}$-curve (dashed line) lies above the $M_2 = 0|_{\phi_Z=0}$-curve and below the $M_2 = 0|_{\phi_Z=\phi_K}$-curve.

FIGURE 3

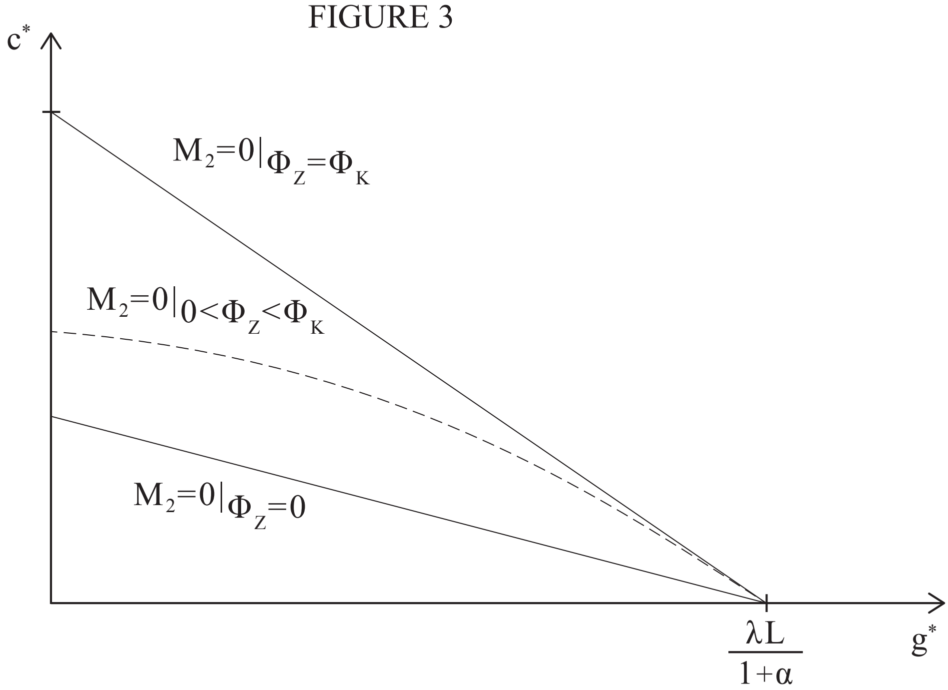

Figure 3: BGP determination of c^* and g^* in the general case

Figures 1–3 might give the erroneous impression that the $M_2 = 0$-curve always intersects the horizontal axis at $g^* = \lambda L/(1+\alpha)$. This property is true for $0 \leq \phi_Z \leq \phi_K$, but might cease to be valid if $\phi_Z > \phi_K$ holds provided that η is sufficiently large. More specifically, for the limiting case $g^* \to \lambda L/(1+\alpha)$ we obtain

$$\Lambda_0 \to 0, \qquad \Lambda_1 \to \frac{\alpha\lambda L}{1+\alpha}\left[(\phi_K - \phi_Z)\left(\alpha^2+\eta\right) + \alpha\phi_Z\right]$$

so that the quadratic equation (E.25) simplifies to

$$\alpha^2\phi_Z(1-\alpha)(c^*)^2 + \frac{\alpha\lambda L}{1+\alpha}\left[(\phi_K - \phi_Z)\left(\alpha^2+\eta\right) + \alpha\phi_Z\right]c^* = 0.$$

Solving for c^* we obtain the following two roots:

$$c^* = 0, \qquad c^* = -\frac{\left[(\phi_K - \phi_Z)\left(\alpha^2+\eta\right) + \alpha\phi_Z\right]\alpha\lambda L}{(1+\alpha)\,\alpha^2\phi_Z(1-\alpha)}.$$

The economically meaningful solution of the quadratic equation (E.25) exhibits the property that $c^* > 0$ holds for $g^* < \lambda L/(1+\alpha)$, and that c^* converges to a nonnegative value for $g^* \to \lambda L/(1+\alpha)$. This, in turn, implies that the function (E.29) has the following properties:

$$\lim_{g^* \to \lambda L/(1+\alpha)} c^*|_{M_2=0} = \begin{cases} 0 & \text{for} \quad \Theta > 0, \\ -\dfrac{\left[(\phi_K - \phi_Z)\left(\alpha^2+\eta\right) + \alpha\phi_Z\right]\alpha\lambda L}{(1+\alpha)\,\alpha^2\phi_Z(1-\alpha)} > 0 & \text{for} \quad \Theta < 0, \end{cases}$$

where

$$\Theta \equiv (\phi_K - \phi_Z)\left(\alpha^2+\eta\right) + \alpha\phi_Z.$$

Hence, if

$$\eta > \alpha(1-\alpha) \qquad \text{and} \qquad \phi_Z > \frac{\phi_K\left(\alpha^2+\eta\right)}{\eta - \alpha(1-\alpha)}$$

hold, then $\Theta < 0$ so that the $M_2 = 0$-curve does not intersect the horizontal axis at $g^* = \lambda L/(1+\alpha)$.

The main message of the considerations made above can be expressed as follows: If the positively sloped $M_1 = 0$-curve and the negatively sloped $M_2 = 0$-curve intersect, then the point of intersection is the unique steady state. In other words, if a steady state exists, then it is unique.

E.2 The effect of changes in η and ϕ_Z

E.2.1 The effect of changes in η in the special cases $\phi_Z = 0$ and $\phi_Z = \phi_K$

Special case $\phi_Z = 0$: The steady-state values g^* and c^* are given by (E.14),

$$g^* = \frac{\sigma^S(\alpha\lambda L - \rho)}{1+\alpha\sigma^S}, \qquad c^* = \frac{\alpha\left[\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho\right]}{\left(1+\alpha\sigma^S\right)\left(\alpha^2+\eta\right)}.$$

It is obvious that a rise in the status parameter η causes c^* to decrease, but leaves g^* unchanged:

$$\frac{\partial g^*}{\partial \eta} = 0, \qquad \frac{\partial c^*}{\partial \eta} = -\frac{\alpha\left[\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho\right]}{\left(1+\alpha\sigma^S\right)\left(\alpha^2+\eta\right)^2} < 0.$$

In graphical terms (see Figure 1), a rise in η (from η_1 to η_2) causes the negatively sloped $M_2 = 0$-line to rotate counterclockwise and the positively sloped $M_1 = 0$-line to rotate clockwise around the corresponding points of intersection with the horizontal axis. The rotating $M_1 = 0$- and $M_2 = 0$-lines always intersect at the same growth rate g^* irrespective of the magnitude of the status parameter η. The effects of changes in η on the steady-state values of the other endogenous variables is summarized in Proposition 2.

Special case $\phi_Z = \phi_K$: The steady-state values g^* and c^* are given by (E.32),

$$g^* = \frac{\sigma^S\left[(\alpha+\eta)\lambda L - \rho\right]}{1+\sigma^S\left[\alpha+\eta(1+\alpha)\right]}, \qquad c^* = \frac{\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho}{\alpha\left\{1+\sigma^S\left[\alpha+\eta(1+\alpha)\right]\right\}}.$$

Assumption (E.13) ensures that $g^* > 0$. Assumption (E.9) is sufficient for $c^* > 0$. According to (C.22) a rise in the status parameter η causes g^* to increase and c^* to decrease. The validity of this result is easily confirmed by the graphical analysis (see Figure 2). A rise in η (from η_1 to η_2) causes the positively sloped $M_1 = 0$-curve to shift downwards, while the position of the negatively sloped $M_2 = 0$-line remains unchanged. Consequently, g^* rises, while c^* falls. The effects of changes in η on the steady-state values of the other endogenous variables is summarized in Proposition 1.

E.2.2 The effects of changes in η and ϕ_Z in the general case $\phi_Z > 0$

The solutions (E.6) and (E.7), $g^* = \Pi^g(\eta, \phi_K, \phi_Z)$ and $c^* = \Pi^c(\eta, \phi_K, \phi_Z)$, satisfy (E.2)–(E.5) so that

$$M_1\left(\Pi^g(\eta, \phi_K, \phi_Z), \Pi^c(\eta, \phi_K, \phi_Z), \eta, \phi_K, \phi_Z\right) = 0,$$

$$M_2\left(\Pi^g(\eta, \phi_K, \phi_Z), \Pi^c(\eta, \phi_K, \phi_Z), \eta, \phi_K, \phi_Z\right) = 0.$$

It is obvious that the partial derivatives are determined by the following system of equations:

$$\frac{\partial M_1}{\partial g^*}\frac{\partial g^*}{\partial var} + \frac{\partial M_1}{\partial c^*}\frac{\partial c^*}{\partial var} + \frac{\partial M_1}{\partial var} = 0, \qquad var = \eta, \phi_K, \phi_Z,$$

$$\frac{\partial M_2}{\partial g^*}\frac{\partial g^*}{\partial var} + \frac{\partial M_2}{\partial c^*}\frac{\partial c^*}{\partial var} + \frac{\partial M_2}{\partial var} = 0, \qquad var = \eta, \phi_K, \phi_Z.$$

Please note that the partial derivatives of M_1 and M_2 are evaluated at $(g^*, c^*, \eta, \phi_K, \phi_Z) = (\Pi^g(\eta, \phi_K, \phi_Z), \Pi^c(\eta, \phi_K, \phi_Z), \eta, \phi_K, \phi_Z)$. In other words, we consider the following expressions: $\partial M_j/\partial\omega|_{M_1=M_2=0}$, $\omega = g^*$, c^*, η, ϕ_K, and ϕ_Z. Solving for $\partial g^*/\partial var$ and $\partial c^*/\partial var$ we obtain

$$\frac{\partial g^*}{\partial var} = \frac{1}{\Psi}\left(\frac{\partial M_2}{\partial c^*}\frac{\partial M_1}{\partial var} - \frac{\partial M_1}{\partial c^*}\frac{\partial M_2}{\partial var}\right), \tag{E.40}$$

$$\frac{\partial c^*}{\partial var} = \frac{1}{\Psi}\left(-\frac{\partial M_2}{\partial g^*}\frac{\partial M_1}{\partial var} + \frac{\partial M_1}{\partial g^*}\frac{\partial M_2}{\partial var}\right), \tag{E.41}$$

where

$$\Psi \equiv \frac{\partial M_1}{\partial c^*}\frac{\partial M_2}{\partial g^*} - \frac{\partial M_1}{\partial g^*}\frac{\partial M_2}{\partial c^*}. \tag{E.42}$$

The partial derivatives of M_1 and M_2 are given by (E.20)–(E.24) and (E.33)–(E.37). Recall that in (E.40), (E.41), and (E.42) these partial derivatives of M_1 and M_2 are evaluated at $(g^*, c^*, \eta, \phi_K, \phi_Z) = (\Pi^g(\eta, \phi_K, \phi_Z), \Pi^c(\eta, \phi_K, \phi_Z), \eta, \phi_K, \phi_Z)$.

It follows from (E.20), (E.21), and (E.39) that

$$\frac{\partial M_1}{\partial g^*} < 0, \qquad \frac{\partial M_1}{\partial c^*} > 0, \qquad \frac{\partial M_2}{\partial c^*} > 0 \tag{E.43}$$

hold at the steady state. From (E.33) it is obvious that

$$\phi_Z \geq \phi_K \Rightarrow \frac{\partial M_2}{\partial g^*} > 0. \tag{E.44}$$

Using (E.42)–(E.44) we obtain

$$\phi_Z \geq \phi_K \Rightarrow \Psi > 0. \tag{E.45}$$

In order to determine the sign of Ψ for the case $\phi_Z < \phi_K$ we make use of the fact that Ψ can be expressed as

$$\begin{aligned} \Psi &= \frac{\eta\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\left(1-\alpha^2\sigma^S\right)\right](\lambda L-g^*)\left[\phi_K(\lambda L-g^*)+\phi_Z(1-\alpha)g^*\right]}{\alpha\left[\phi_K(\lambda L-g^*)+\phi_Z(1-\alpha)(g^*+c^*)\right]^2} \\ &\quad+\alpha\left(1+\sigma^S\alpha\right)+\frac{\eta\phi_Z^2(1-\alpha)\alpha^2\sigma^S c^*(\lambda L+c^*)}{\alpha\left[\phi_K(\lambda L-g^*)+\phi_Z(1-\alpha)(g^*+c^*)\right]^2}. \end{aligned} \tag{E.46}$$

Equation (E.46) implies that

$$\phi_Z < \phi_K \Rightarrow \phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\left(1-\alpha^2\sigma^S\right) > 0 \Rightarrow \Psi > 0. \tag{E.47}$$

From (E.45) and (E.47) it follows that

$$\Psi > 0 \qquad \text{for } \phi_Z \geq 0. \tag{E.48}$$

The effects of changes in ϕ_Z on the steady state are summarized in Proposition 3. The effects of changes in η on the BGP are described by the following proposition that is not included in the main text.

Proposition 4. *If $\phi_Z > 0$, then changes in the intensity of the quest for status as measured by the parameter η affect the BGP as follows:*

$$\frac{\partial g^*}{\partial \eta} > 0,$$

$$\frac{\partial v^*}{\partial \eta} > 0 \qquad \textit{for} \qquad v = L_A, \varepsilon^K, \varepsilon^Z, r+\varepsilon^K, \frac{\pi}{p_A}+\varepsilon^Z,$$

$$\frac{\partial v^*}{\partial \eta} < 0 \qquad \textit{for} \qquad v = L_Y, \frac{C}{Y}, \frac{\pi}{p_A},$$

$$sgn\left[\frac{\partial (C/K)^*}{\partial \eta}\right] = -sgn\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\left(1-\alpha^2\sigma^S\right)\right],$$

$$sgn\left(\frac{\partial v^*}{\partial \eta}\right) = -sgn\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\right] \qquad \textit{for} \qquad v = \frac{Y}{K}, r, p,$$

$$sgn\left(\frac{\partial v^*}{\partial \eta}\right) = sgn\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\right] \qquad \textit{for} \qquad v = \frac{x}{L_Y}, p_A, \frac{w}{A},$$

$$sgn\left(\frac{\partial\left[K/(K+p_A A)\right]^*}{\partial \eta}\right) = sgn\left(\phi_K-\phi_Z\right).$$

Proof: Substitution of (E.20), (E.21), (E.22), (E.33), (E.34), and (E.35) into (E.40) and (E.41) yields

$$\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi}\frac{\phi_Z\alpha\sigma^S c^*(\lambda L-g^*)}{\phi_K(\lambda L-g^*)+\phi_Z(1-\alpha)(g^*+c^*)}, \tag{E.49}$$

$$\frac{\partial c^*}{\partial \eta} = -\frac{1}{\Psi}\frac{\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\left(1-\alpha^2\sigma^S\right)\right]c^*(\lambda L-g^*)}{\alpha\left[\phi_K(\lambda L-g^*)+\phi_Z(1-\alpha)(g^*+c^*)\right]}. \tag{E.50}$$

Using the last two equations and (C.2) we obtain

$$\frac{\partial (Y/K)^*}{\partial \eta} = \frac{\partial g^*}{\partial \eta} + \frac{\partial c^*}{\partial \eta} = -\frac{1}{\Psi} \frac{\left[\phi_K \left(1+\alpha\sigma^S\right) - \phi_Z\right] c^* \left(\lambda L - g^*\right)}{\alpha \left[\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)\right]}. \tag{E.51}$$

From (E.49), (E.50), and (E.51) it follows that

$$\frac{\partial g^*}{\partial \eta} > 0 \qquad \text{for} \qquad \phi_Z > 0,$$

$$\operatorname{sgn}\left(\frac{\partial c^*}{\partial \eta}\right) = -\operatorname{sgn}\left[\phi_K \left(1+\alpha\sigma^S\right) - \phi_Z \left(1-\alpha^2\sigma^S\right)\right],$$

$$\operatorname{sgn}\left[\frac{\partial (Y/K)^*}{\partial \eta}\right] = -\operatorname{sgn}\left[\phi_K \left(1+\alpha\sigma^S\right) - \phi_Z\right].$$

The last results together with (C.1)–(C.17) imply that

$$\frac{\partial L_A^*}{\partial \eta} = \frac{1}{\lambda}\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_Z \alpha \sigma^S c^* \left(\lambda L - g^*\right)}{\lambda \left[\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)\right]} > 0,$$

$$\frac{\partial L_Y^*}{\partial \eta} = -\frac{\partial L_A^*}{\partial \eta} = -\frac{1}{\Psi} \frac{\phi_Z \alpha \sigma^S c^* \left(\lambda L - g^*\right)}{\lambda \left[\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)\right]} < 0,$$

$$\frac{\partial r^*}{\partial \eta} = \alpha^2 \frac{\partial (Y/K)^*}{\partial \eta} = -\frac{1}{\Psi} \frac{\alpha \left[\phi_K \left(1+\alpha\sigma^S\right) - \phi_Z\right] c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)},$$

$$\frac{\partial p^*}{\partial \eta} = \alpha \frac{\partial (Y/K)^*}{\partial \eta} = -\frac{1}{\Psi} \frac{\left[\phi_K \left(1+\alpha\sigma^S\right) - \phi_Z\right] c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)},$$

$$\frac{\partial (x/L_Y)^*}{\partial \eta} = -\frac{1}{1-\alpha} \left[\left(\frac{Y}{K}\right)^*\right]^{-(2-\alpha)/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta},$$

$$\frac{\partial (\pi/p_A)^*}{\partial \eta} = -\alpha \frac{\partial g^*}{\partial \eta} < 0,$$

$$\frac{\partial p_A^*}{\partial \eta} = -\frac{\alpha}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta},$$

$$\frac{\partial (w/A)^*}{\partial \eta} = -\alpha \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta},$$

$$\frac{\partial \left[r^* + \left(\varepsilon^K\right)^*\right]}{\partial \eta} = \frac{1}{\sigma^S}\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_Z \alpha c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)} > 0,$$

$$\frac{\partial \left[(\pi/p_A)^* + \left(\varepsilon^Z\right)^*\right]}{\partial \eta} = \frac{1}{\sigma^S}\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_Z \alpha c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)} > 0,$$

$$\frac{\partial \left(\varepsilon^K\right)^*}{\partial \eta} = \frac{1}{\sigma^S}\frac{\partial g^*}{\partial \eta} - \alpha^2 \frac{\partial (Y/K)^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_K \left(1+\alpha\sigma^S\right) \alpha c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)} > 0,$$

$$\frac{\partial \left(\varepsilon^Z\right)^*}{\partial \eta} = \frac{1+\alpha\sigma^S}{\sigma^S}\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_Z \left(1+\alpha\sigma^S\right) \alpha c^* \left(\lambda L - g^*\right)}{\phi_K \left(\lambda L - g^*\right) + \phi_Z \left(1-\alpha\right)\left(g^* + c^*\right)} > 0.$$

To determine the effects of changes in η on the wealth share of physical capital $[K/(K+p_A A)]^*$

we use (C.2), (C.18), (E.49), and (E.51):

$$\begin{aligned}
\frac{\partial\left[K/\left(K+p_A A\right)\right]^*}{\partial\eta} &= -\frac{\dfrac{1-\alpha}{\left(\lambda L-g^*\right)^2}\left(\dfrac{Y}{K}\right)^*\dfrac{\partial g^*}{\partial\eta}+\dfrac{1-\alpha}{\lambda L-g^*}\dfrac{\partial\left(Y/K\right)^*}{\partial\eta}}{\left[1+\dfrac{1-\alpha}{\lambda L-g^*}\left(\dfrac{Y}{K}\right)^*\right]^2} \\
&= \frac{1}{\Psi}\frac{\left(1-\alpha\right)c^*\left(\lambda L-g^*\right)}{\left[\lambda L-g^*+\left(1-\alpha\right)\left(c^*+g^*\right)\right]^2}\times \\
&\quad\times\frac{\left\{\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\right]\left(\lambda L-g^*\right)-\phi_Z\alpha^2\sigma^S\left(c^*+g^*\right)\right\}}{\alpha\left[\phi_K\left(\lambda L-g^*\right)+\phi_Z\left(1-\alpha\right)\left(g^*+c^*\right)\right]}. \quad \text{(E.52)}
\end{aligned}$$

Since this representation of the result is not very informative, we rewrite it. Since the steady state values g^* and c^* satisfy both $M_1 = 0$ and $M_2 = 0$, where M_1 and M_2 are defined by (E.4) and (E.5), the following equations can be easily derived for $\eta > 0$, $\phi_K > 0$, $\phi_Z > 0$, and $\phi_Z \neq \phi_K$:

$$\begin{aligned}
\frac{c^*}{\phi_K+\dfrac{\phi_Z\left(1-\alpha\right)}{\lambda L-g^*}\left(g^*+c^*\right)} &= \frac{\left(1-\alpha^2\sigma^S\right)g^*-\alpha^2\sigma^S c^*+\sigma^S\rho}{\eta\phi_K\sigma^S}, \\
\frac{c^*}{\phi_K+\dfrac{\phi_Z\left(1-\alpha\right)}{\lambda L-g^*}\left(g^*+c^*\right)} &= \frac{\alpha\left[\lambda L-\left(1+\alpha\right)g^*-\alpha c^*\right]}{\left(\phi_K-\phi_Z\right)\eta}.
\end{aligned}$$

From (C.2) and (C.12) it follows that the identical left-hand sides of these two equations equal $(C/\Omega)^*$. Using the fact that the right-hand sides have to be identical, too, we obtain the following relations:

$$c^* = \frac{-\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\left(1-\alpha^2\sigma^S\right)\right]g^*+\alpha\phi_K\sigma^S\lambda L-\left(\phi_K-\phi_Z\right)\sigma^S\rho}{\alpha^2\sigma^S\phi_Z}, \quad \text{(E.53)}$$

$$c^*+g^* = \frac{-\left[\phi_K\left(1+\alpha\sigma^S\right)-\phi_Z\right]g^*+\alpha\phi_K\sigma^S\lambda L-\left(\phi_K-\phi_Z\right)\sigma^S\rho}{\alpha^2\sigma^S\phi_Z}. \quad \text{(E.54)}$$

If $\phi_Z = \phi_K > 0$ holds, then $M_2 = 0$ implies that

$$c^* = \frac{\lambda L-\left(1+\alpha\right)g^*}{\alpha}, \qquad c^*+g^* = \frac{\lambda L-g^*}{\alpha}.$$

If $\phi_K = 0$, then it follows from $M_1 = 0$ that

$$c^* = \frac{\left(1-\alpha^2\sigma^S\right)g^*+\sigma^S\rho}{\alpha^2\sigma^S}, \qquad c^*+g^* = \frac{g^*+\sigma^S\rho}{\alpha^2\sigma^S}.$$

These results show that (E.53) and (E.54) also hold for $\phi_K = 0$ and $\phi_Z = \phi_K > 0$. Using

(E.54), Equation (E.52) can be rewritten as

$$\begin{aligned}\frac{\partial\left[K/\left(K+p_{A}A\right)\right]^{*}}{\partial\eta} &= \frac{1}{\Psi}\frac{\left(\phi_{K}-\phi_{Z}\right)\left(1-\alpha\right)}{\left[\lambda L-g^{*}+\left(1-\alpha\right)\left(c^{*}+g^{*}\right)\right]^{2}}\times\\ &\quad\times\frac{c^{*}\left(\lambda L-g^{*}\right)\left(\lambda L+\sigma^{S}\rho\right)}{\alpha\left[\phi_{K}\left(\lambda L-g^{*}\right)+\phi_{Z}\left(1-\alpha\right)\left(g^{*}+c^{*}\right)\right]}.\end{aligned}\tag{E.55}$$

It is obvious that

$$\operatorname{sgn}\left(\frac{\partial\left[K/\left(K+p_{A}A\right)\right]^{*}}{\partial\eta}\right)=\operatorname{sgn}\left(\phi_{K}-\phi_{Z}\right).\tag{E.56}$$

To determine the effects of changes in η on the average propensity to consume $(C/Y)^{*}$, we use (C.2), (C.12), (E.49), and (E.51):

$$\begin{aligned}\frac{\partial\left(C/Y\right)^{*}}{\partial\eta} &= \left[\left(Y/K\right)^{*}\right]^{-2}\left[\frac{\partial c^{*}}{\partial\eta}\left(\frac{Y}{K}\right)^{*}-c^{*}\frac{\partial\left(Y/K\right)^{*}}{\partial\eta}\right]\\ &= -\frac{1}{\Psi}\frac{\left\{\left[\phi_{K}\left(1+\alpha\sigma^{S}\right)-\phi_{Z}\right]g^{*}+\phi_{Z}\alpha^{2}\sigma^{S}\left(c^{*}+g^{*}\right)\right\}c^{*}\left(\lambda L-g^{*}\right)}{\alpha\left[\phi_{K}\left(\lambda L-g^{*}\right)+\phi_{Z}\left(1-\alpha\right)\left(g^{*}+c^{*}\right)\right]\left(c^{*}+g^{*}\right)^{2}}.\end{aligned}\tag{E.57}$$

Using (E.54), Equation (E.57) can be rewritten as

$$\frac{\partial\left(C/Y\right)^{*}}{\partial\eta}=-\frac{1}{\Psi}\frac{\left[\phi_{K}\sigma^{S}\left(\alpha\lambda L-\rho\right)+\phi_{Z}\sigma^{S}\rho\right]c^{*}\left(\lambda L-g^{*}\right)}{\alpha\left[\phi_{K}\left(\lambda L-g^{*}\right)+\phi_{Z}\left(1-\alpha\right)\left(g^{*}+c^{*}\right)\right]\left(c^{*}+g^{*}\right)^{2}}<0.\tag{E.58}$$

Note that Assumption (E.13), $\alpha\lambda L-\rho>0$, is sufficient for the positive sign of the numerator so that $\partial\left(C/Y\right)^{*}/\partial\eta<0$ holds. Equation (E.58), in turn, implies that the average propensity to save depends positively on η:

$$\frac{\partial\left[1-\left(C/Y\right)^{*}\right]}{\partial\eta}=\frac{1}{\Psi}\frac{\left[\phi_{K}\sigma^{S}\left(\alpha\lambda L-\rho\right)+\phi_{Z}\sigma^{S}\rho\right]c^{*}\left(\lambda L-g^{*}\right)}{\alpha\left[\phi_{K}\left(\lambda L-g^{*}\right)+\phi_{Z}\left(1-\alpha\right)\left(g^{*}+c^{*}\right)\right]\left(c^{*}+g^{*}\right)^{2}}>0.$$

The results given above can be summarized as follows:

$$\begin{aligned}\frac{\partial v^{*}}{\partial\eta}&>0\qquad\text{for}\qquad v=g,L_{A},\varepsilon^{K},\varepsilon^{Z},r+\varepsilon^{K},\frac{\pi}{p_{A}}+\varepsilon^{Z},\\ \frac{\partial v^{*}}{\partial\eta}&<0\qquad\text{for}\qquad v=L_{Y},\frac{C}{Y},\frac{\pi}{p_{A}},\\ \operatorname{sgn}\left(\frac{\partial c^{*}}{\partial\eta}\right)&=-\operatorname{sgn}\left[\phi_{K}\left(1+\alpha\sigma^{S}\right)-\phi_{Z}\left(1-\alpha^{2}\sigma^{S}\right)\right],\\ \operatorname{sgn}\left(\frac{\partial v^{*}}{\partial\eta}\right)&=-\operatorname{sgn}\left[\phi_{K}\left(1+\alpha\sigma^{S}\right)-\phi_{Z}\right]\qquad\text{for}\qquad v=\frac{Y}{K},r,p,\\ \operatorname{sgn}\left(\frac{\partial v^{*}}{\partial\eta}\right)&=\operatorname{sgn}\left[\phi_{K}\left(1+\alpha\sigma^{S}\right)-\phi_{Z}\right]\qquad\text{for}\qquad v=\frac{x}{L_{Y}},p_{A},\frac{w}{A},\\ \operatorname{sgn}\left(\frac{\partial\left[K/\left(K+p_{A}A\right)\right]^{*}}{\partial\eta}\right)&=\operatorname{sgn}\left(\phi_{K}-\phi_{Z}\right).\quad\blacksquare\end{aligned}$$

F The stability properties of the steady state

F.1 The derivation of the Jacobian and the conditions for saddlepoint stability

The dynamic evolution of the variables K, C, A, and L_A is governed by the four differential equations (33), (34), (46), (47), where C/Ω is given by (38). For convenience, we restate these equations here:

$$\frac{\dot{A}}{A} = \lambda L_A, \tag{F.1}$$

$$\frac{\dot{K}}{K} = \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} - \frac{C}{K}, \tag{F.2}$$

$$\frac{\dot{C}}{C} = \sigma^S \left\{\alpha^2 \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} + \phi_K \eta \frac{C}{\Omega} - \rho\right\}, \tag{F.3}$$

$$\dot{L}_A = \left(L - L_A\right) \left\{-\left(1-\alpha\right) \left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A - \lambda\left(L - L_A\right) + \frac{\left(\phi_K - \phi_Z\right)\eta}{\alpha}\frac{C}{\Omega}\right\} \tag{F.4}$$

with

$$\frac{C}{\Omega} = \frac{\frac{C}{K}}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda\left(L - L_A\right)}\left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)}}. \tag{F.5}$$

Introducing the definitions

$$c \equiv \frac{C}{K}, \qquad a \equiv \frac{A}{K}, \tag{F.6}$$

we obtain

$$\left[\frac{K}{A\left(L - L_A\right)}\right]^{-(1-\alpha)} = a^{1-\alpha}\left(L - L_A\right)^{1-\alpha},$$

$$\frac{C}{\Omega} = \frac{c}{\phi_K + \phi_Z\left(1-\alpha\right)\lambda^{-1}a^{1-\alpha}\left(L - L_A\right)^{-\alpha}}.$$

Using the last two equations, the differential equations (F.2)–(F.4) can be written as follows:

$$\frac{\dot{K}}{K} = a^{1-\alpha}\left(L - L_A\right)^{1-\alpha} - c, \tag{F.7}$$

$$\frac{\dot{C}}{C} = \sigma^S\left[\alpha^2 a^{1-\alpha}\left(L - L_A\right)^{1-\alpha} + \frac{\phi_K \eta c}{\phi_K + \phi_Z\left(1-\alpha\right)\lambda^{-1}a^{1-\alpha}\left(L - L_A\right)^{-\alpha}} - \rho\right], \tag{F.8}$$

$$\begin{aligned} \dot{L}_A = \left(L - L_A\right)\Big\{&-\left(1-\alpha\right)a^{1-\alpha}\left(L - L_A\right)^{1-\alpha} + 2\lambda L_A - \lambda L \\ &+ \left[1 + \frac{\left(\phi_K - \phi_Z\right)\eta\alpha^{-1}}{\phi_K + \phi_Z\left(1-\alpha\right)\lambda^{-1}a^{1-\alpha}\left(L - L_A\right)^{-\alpha}}\right]c\Big\}. \end{aligned} \tag{F.9}$$

Substituting (F.1), (F.7), and (F.8) into

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} \qquad \text{and} \qquad \frac{\dot{a}}{a} = \frac{\dot{A}}{A} - \frac{\dot{K}}{K}$$

yields the following differential equations for c and a:

$$\begin{aligned} \dot{c} = c\Big\{ & -\left(1-\sigma^S\alpha^2\right) a^{1-\alpha}\left(L-L_A\right)^{1-\alpha} \\ & + \left[1+\frac{\sigma^S\phi_K\eta}{\phi_K+\phi_Z\left(1-\alpha\right)\lambda^{-1}a^{1-\alpha}\left(L-L_A\right)^{-\alpha}}\right]c - \sigma^S\rho\Big\}, \end{aligned} \tag{F.10}$$

$$\dot{a} = a\left[\lambda L_A - a^{1-\alpha}\left(L-L_A\right)^{1-\alpha} + c\right]. \tag{F.11}$$

The differential equations (F.10), (F.9), and (F.11) have the following general form:

$$\dot{c} = \dot{c}\left(c, L_A, a\right), \qquad \dot{L}_A = \dot{L}_A\left(c, L_A, a\right), \qquad \dot{a} = \dot{a}\left(c, L_A, a\right).$$

The steady-state values c^*, L_A^*, and a^* satisfy the following equations:

$$0 = \dot{c}\left(c^*, L_A^*, a^*\right), \qquad 0 = \dot{L}_A\left(c^*, L_A^*, a^*\right), \qquad 0 = \dot{a}\left(c^*, L_A^*, a^*\right).$$

To discuss the stability properties of the steady state, we have to calculate the Jacobian

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \equiv \left.\begin{pmatrix} \frac{\partial\dot{c}}{\partial c} & \frac{\partial\dot{c}}{\partial L_A} & \frac{\partial\dot{c}}{\partial a} \\ \frac{\partial\dot{L}_A}{\partial c} & \frac{\partial\dot{L}_A}{\partial L_A} & \frac{\partial\dot{L}_A}{\partial a} \\ \frac{\partial\dot{a}}{\partial c} & \frac{\partial\dot{a}}{\partial L_A} & \frac{\partial\dot{a}}{\partial a} \end{pmatrix}\right|_{(c,L_A,a)=\left(c^*,L_A^*,a^*\right)}.$$

The characteristic polynomial of the Jacobian $\mathbf{M}$ is given by

$$\begin{aligned} 0 \;=\; & z^3 - \mathrm{trace}\left(\mathbf{M}\right) z^2 \\ & + \left(m_{11}m_{22} - m_{12}m_{21} + m_{11}m_{33} - m_{13}m_{31} + m_{22}m_{33} - m_{23}m_{32}\right) z \\ & - \det\left(\mathbf{M}\right), \end{aligned} \tag{F.12}$$

where

$$\mathrm{trace}\left(\mathbf{M}\right) = m_{11} + m_{22} + m_{33}, \tag{F.13}$$

$$\begin{aligned} \det\left(\mathbf{M}\right) \;=\; & m_{11}m_{22}m_{33} - m_{11}m_{23}m_{32} + m_{12}m_{23}m_{31} \\ & - m_{12}m_{21}m_{33} + m_{13}m_{21}m_{32} - m_{13}m_{22}m_{31}. \end{aligned} \tag{F.14}$$

It is well known that the roots of the characteristic polynomial z_1, z_2, and z_3 satisfy the following

equations:

$$\text{trace}\left(\mathbf{M}\right)=z_1+z_3+z_2, \tag{F.15}$$

$$\det\left(\mathbf{M}\right)=z_1z_2z_3. \tag{F.16}$$

Since the system of the three differential equations involves two jump variables ($c\equiv C/K$ and L_A) and one state variable ($a\equiv A/K$), the steady state (c^*,L_A^*,a^*) exhibits saddlepoint stability if one of the three roots is a negative real number, while the two other roots are either positive real numbers or complex numbers with positive real parts. From (F.15) and (F.16) it follows that a sufficient (but not necessary) condition for saddlepoint stability is given by

$$\det\left(\mathbf{M}\right)<0 \qquad \text{and} \qquad \text{trace}\left(\mathbf{M}\right)>0. \tag{F.17}$$

Proof: i) If all three roots are real numbers, then the condition $\det\left(\mathbf{M}\right)=z_1z_2z_3<0$ implies either that one root is strictly negative, while the other two roots are strictly positive ($z_1<0$, $z_2>0$, and $z_3>0$) or that all three roots are strictly negative ($z_1<0$, $z_2<0$, and $z_3<0$). Obviously, the condition trace$(\mathbf{M})=z_1+z_3+z_2>0$ rules out the latter case. ii) If only one out of the three roots is real, while the two other roots are a complex number $z_2=\delta_1+\delta_2 i$ and its complex conjugate $z_3=\delta_1-\delta_2 i$, $\delta_2\neq 0$, then the condition $\det\left(\mathbf{M}\right)=z_1\left(\delta_1^2+\delta_2^2\right)<0$ implies that the real number is strictly negative ($z_1<0$), while the real part of the complex number and its complex conjugate, $\text{Re}(z_2)=\text{Re}(z_3)=\delta_1$, may be of either sign. In this case, the condition trace$(\mathbf{M})=z_1+2\delta_1>0$ rules out that $\text{Re}(z_2)=\text{Re}(z_3)=\delta_1\leq 0$. iii) From i) and ii) it follows that if the condition "$\det\left(\mathbf{M}\right)<0$ and trace$(\mathbf{M})>0$" is satisfied, then one of the three roots is a negative real number ($z_1<0$), while the two other roots are either positive real numbers ($z_2>0$ and $z_3>0$) or a complex number $z_2=\delta_1+\delta_2 i$ and its complex conjugate $z_3=\delta_1-\delta_2 i$, with $\text{Re}(z_2)=\text{Re}(z_3)=\delta_1>0$. This, in turn, implies that "$\det\left(\mathbf{M}\right)<0$ and trace$(\mathbf{M})>0$" is a sufficient condition for saddlepoint stability. ■

The stable arm of the linearized system is given by

$$\begin{pmatrix} c\left(t\right) \\ L_A\left(t\right) \\ a\left(t\right) \end{pmatrix}=\begin{pmatrix} c^* \\ L_A^* \\ a^* \end{pmatrix}+D_1\begin{pmatrix} e_{11} \\ e_{21} \\ 1 \end{pmatrix}e^{z_1 t},$$

where $\left(e_{11},e_{21},1\right)^T$ denotes an eigenvector of the Jacobian $\mathbf{M}$ that corresponds to the eigenvalue $z_1<0$, and where D_1 is an undetermined coefficient. From the initial condition $a\left(0\right)=a_0\equiv A\left(0\right)/K\left(0\right)$ it follows that $D_1=a_0-a^*$.

The eigenvector $\left(e_{11},e_{21},1\right)^T$ satisfies the following system of equations expressed in matrix form:

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}\begin{pmatrix} e_{11} \\ e_{21} \\ 1 \end{pmatrix}=z_1\begin{pmatrix} e_{11} \\ e_{21} \\ 1 \end{pmatrix}.$$

Simple transformations yield the following *three* equations for the determination of *two* variables:

$$\begin{aligned}
(m_{11}-z_1)\,e_{11}+m_{12}e_{21} &= -m_{13},\\
m_{21}e_{11}+(m_{22}-z_1)\,e_{21} &= -m_{23},\\
m_{31}e_{11}+m_{32}e_{21} &= -(m_{33}-z_1)\,.
\end{aligned}$$

There exists a unique solution for e_{11} and e_{21} that can be expressed in *three equivalent ways*:

$$\begin{aligned}
e_{11} &= \frac{-m_{13}\,(m_{22}-z_1)+m_{12}m_{23}}{(m_{11}-z_1)\,(m_{22}-z_1)-m_{12}m_{21}}\\
&= \frac{m_{12}\,(m_{33}-z_1)-m_{13}m_{32}}{(m_{11}-z_1)\,m_{32}-m_{12}m_{31}}\\
&= \frac{m_{23}m_{32}-(m_{22}-z_1)\,(m_{33}-z_1)}{(m_{22}-z_1)\,m_{31}-m_{21}m_{32}},
\end{aligned}$$

$$\begin{aligned}
e_{21} &= \frac{-m_{23}\,(m_{11}-z_1)+m_{13}m_{21}}{(m_{11}-z_1)\,(m_{22}-z_1)-m_{12}m_{21}}\\
&= \frac{m_{13}m_{31}-(m_{11}-z_1)\,(m_{33}-z_1)}{(m_{11}-z_1)\,m_{32}-m_{12}m_{31}}\\
&= \frac{(m_{33}-z_1)\,m_{21}-m_{23}m_{31}}{(m_{22}-z_1)\,m_{31}-m_{21}m_{32}}.
\end{aligned}$$

The stable arm exhibits the following properties:

$$a\,(t)-a^*=(a_0-a^*)\,e^{z_1t},$$

$$\begin{aligned}
c\,(t)-c^* &= (a_0-a^*)\,e_{11}e^{z_1t}=e_{11}\,[a\,(t)-a^*]\,,\\
L_A\,(t)-L_A^* &= (a_0-a^*)\,e_{21}e^{z_1t}=e_{21}\,[a\,(t)-a^*]\,.
\end{aligned}$$

The last three equations imply that

$$c\,(t)-c^*=\frac{e_{11}}{e_{21}}\,[L_A\,(t)-L_A^*]\,.$$

This equation describes the comovement of $L_A\,(t)$ and $c\,(t)$ during the transitional dynamics.

By tedious calculations it can be shown that the elements of the Jacobian are given by the following expressions:

$$m_{11}=\left[1+\frac{\sigma^S\phi_K\eta}{\phi_K+\phi_Z\,(1-\alpha)\,\lambda^{-1}\,(a^*)^{1-\alpha}\,(L-L_A^*)^{-\alpha}}\right]c^*,$$

$$m_{12} = (1-\alpha) c^* (a^*)^{1-\alpha} (L - L_A^*)^{-\alpha-1} \times$$

$$\times \left\{ \left(1-\sigma^S \alpha^2\right)(L - L_A^*) - \frac{\eta \phi_K \phi_Z \alpha \sigma^S \lambda^{-1} c^*}{\left[\phi_K + \phi_Z (1-\alpha) \lambda^{-1} (a^*)^{1-\alpha} \left(L - L_A^*\right)^{-\alpha}\right]^2} \right\},$$

$$m_{13} = -(1-\alpha) c^* (a^*)^{-\alpha} (L - L_A^*)^{-\alpha} \times$$

$$\times \left\{ \left(1-\sigma^S \alpha^2\right)(L - L_A^*) + \frac{\eta \phi_K \phi_Z (1-\alpha) \sigma^S \lambda^{-1} c^*}{\left[\phi_K + \phi_Z (1-\alpha) \lambda^{-1} (a^*)^{1-\alpha} \left(L - L_A^*\right)^{-\alpha}\right]^2} \right\},$$

$$m_{21} = (L - L_A^*) \left[1 + \frac{(\phi_K - \phi_Z) \eta \alpha^{-1}}{\phi_K + \phi_Z (1-\alpha) \lambda^{-1} (a^*)^{1-\alpha} \left(L - L_A^*\right)^{-\alpha}}\right],$$

$$m_{22} = (L - L_A^*) \left[(1-\alpha)^2 (a^*)^{1-\alpha} (L - L_A^*)^{-\alpha} + 2\lambda\right]$$

$$- \frac{(\phi_K - \phi_Z) \phi_Z \eta (1-\alpha) \lambda^{-1} c^* (a^*)^{1-\alpha} (L - L_A^*)^{-\alpha}}{\left[\phi_K + \phi_Z (1-\alpha) \lambda^{-1} (a^*)^{1-\alpha} \left(L - L_A^*\right)^{-\alpha}\right]^2},$$

$$m_{23} = -(1-\alpha)^2 (a^*)^{-\alpha} (L - L_A^*)^{2-\alpha}$$

$$- \frac{(\phi_K - \phi_Z) \phi_Z \eta \alpha^{-1} (1-\alpha)^2 \lambda^{-1} c^* (a^*)^{-\alpha} (L - L_A^*)^{1-\alpha}}{\left[\phi_K + \phi_Z (1-\alpha) \lambda^{-1} (a^*)^{1-\alpha} \left(L - L_A^*\right)^{-\alpha}\right]^2},$$

$$m_{31} = a^*,$$

$$m_{32} = a^* \left[\lambda + (1-\alpha) (a^*)^{1-\alpha} (L - L_A^*)^{-\alpha}\right],$$

$$m_{33} = -(1-\alpha) (a^*)^{1-\alpha} (L - L_A^*)^{1-\alpha}.$$

From (50) and (51) or (E.2)–(E.5) it follows that the steady state values g^* and $c^* \equiv (C/K)^*$ satisfy the equations:

$$-\left(1-\alpha^2 \sigma^S\right) g^* + \sigma^S \left[\alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} (g^* + c^*)}\right] c^* = \sigma^S \rho, \tag{F.18}$$

$$(1+\alpha) g^* + \frac{1}{\alpha} \left[\alpha^2 + \frac{(\phi_K - \phi_Z) \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} (g^* + c^*)}\right] c^* = \lambda L. \tag{F.19}$$

Since this system of equations determines only the steady state values g^* and c^*, it will be useful to express the elements of the Jacobian not as functions of c^*, L_A^*, and a^*, but as functions of g^* and c^* only. According to (F.11), the equation $0 = \dot{a}\left(c^*, L_A^*, a^*\right)$ implies that

$$(a^*)^{1-\alpha}\left(L - L_A^*\right)^{1-\alpha} = \lambda L_A^* + c^*.$$

From the steady-state version of (F.1) it follows that

$$g^* = \left(\dot{A}/A\right)^* = \lambda L_A^*.$$

Using the last two equations we obtain the following representations for L_A^* and a^*:

$$\begin{aligned} L_A^* &= \frac{g^*}{\lambda}, \\ a^* &= \frac{\lambda\left(g^* + c^*\right)^{1/(1-\alpha)}}{\lambda L - g^*}. \end{aligned}$$

Using the last two equations, the elements of the Jacobian can be rewritten as:

$$m_{11} = \left[1 + \frac{\sigma^S \phi_K \eta\left(\lambda L - g^*\right)}{\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)}\right] c^*,$$

$$m_{12} = \frac{\left(1-\alpha\right)\lambda c^*\left(g^* + c^*\right)}{\lambda L - g^*}\left\{1 - \sigma^S\alpha^2 - \frac{\eta\phi_K\phi_Z\alpha\sigma^S c^*\left(\lambda L - g^*\right)}{\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}\right\},$$

$$m_{13} = -\frac{\left(1-\alpha\right)c^*\left(\lambda L - g^*\right)}{\lambda\left(g^* + c^*\right)^{\alpha/(1-\alpha)}}\left\{1 - \sigma^S\alpha^2 + \frac{\eta\phi_K\phi_Z\left(1-\alpha\right)\sigma^S c^*\left(\lambda L - g^*\right)}{\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}\right\},$$

$$m_{21} = \frac{\lambda L - g^*}{\lambda}\left[1 + \frac{\left(\phi_K - \phi_Z\right)\eta\alpha^{-1}\left(\lambda L - g^*\right)}{\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)}\right],$$

$$m_{22} = \left(1-\alpha\right)^2\left(g^* + c^*\right) + 2\left(\lambda L - g^*\right) - \frac{\left(\phi_K - \phi_Z\right)\phi_Z\eta\left(1-\alpha\right)c^*\left(g^* + c^*\right)\left(\lambda L - g^*\right)}{\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2},$$

$$m_{23} = -\frac{\left(\lambda L - g^*\right)^2}{\lambda^2\left(g^* + c^*\right)^{\alpha/(1-\alpha)}}\left\{\left(1-\alpha\right)^2 + \frac{\left(\phi_K - \phi_Z\right)\phi_Z\eta\alpha^{-1}\left(1-\alpha\right)^2 c^*\left(\lambda L - g^*\right)}{\left[\phi_K\left(\lambda L - g^*\right) + \phi_Z\left(1-\alpha\right)\left(g^* + c^*\right)\right]^2}\right\},$$

$$m_{31} = \frac{\lambda\left(g^* + c^*\right)^{1/(1-\alpha)}}{\lambda L - g^*},$$

$$m_{32} = \frac{\lambda^2\left(g^* + c^*\right)^{1/(1-\alpha)}}{\left(\lambda L - g^*\right)^2}\left[\left(\lambda L - g^*\right) + \left(1-\alpha\right)\left(g^* + c^*\right)\right],$$

$$m_{33} = -\left(1-\alpha\right)\left(g^* + c^*\right).$$

Using these expressions it can be shown by tedious calculations that

$$\begin{aligned}
\operatorname{trace}(\mathbf{M}) &= c^* + 2(\lambda L - g^*) - (1-\alpha)\alpha(g^* + c^*) \\
&\quad + \frac{\phi_K \eta \sigma^S c^* (\lambda L - g^*)}{\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)} \\
&\quad + \frac{\phi_Z (\phi_Z - \phi_K) \eta (1-\alpha) c^* (\lambda L - g^*)(g^* + c^*)}{[\phi_K (\lambda L - g^*) + \phi_Z (1-\alpha)(g^* + c^*)]^2},
\end{aligned}$$

$$\begin{aligned}
\det(\mathbf{M}) &= -(1-\alpha) c^* (g^* + c)(\lambda L - g^*) \times \\
&\quad \times \left\{ \alpha\left(1+\alpha\sigma^S\right) + \frac{\left[\left(1+\alpha\sigma^S\right)\phi_K - \left(1-\alpha^2\sigma^S\right)\phi_Z\right]\eta(\lambda L - g^*)}{\alpha[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^*+c^*)]} \right. \\
&\quad - \frac{\phi_Z \eta (1-\alpha) c^* \phi_K \left(1+\alpha\sigma^S\right)(\lambda L - g^*)}{\alpha[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^*+c^*)]^2} \\
&\quad \left. + \frac{\phi_Z^2 \eta (1-\alpha) c^* \left[(\lambda L - g^*) + \alpha^2\sigma^S (g^* + c^*)\right]}{\alpha[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^*+c^*)]^2} \right\}.
\end{aligned}$$

Using (E.54),

$$c^* + g^* = \frac{-\left[\phi_K\left(1+\alpha\sigma^S\right) - \phi_Z\right]g^* + \alpha\phi_K\sigma^S\lambda L - (\phi_K - \phi_Z)\sigma^S\rho}{\alpha^2\sigma^S\phi_Z},$$

the last equation can be rewritten as

$$\begin{aligned}
\det(\mathbf{M}) &= -(1-\alpha) c^* (g^* + c)(\lambda L - g^*) \times \\
&\quad \times \left\{ \alpha\left(1+\alpha\sigma^S\right) + \frac{\left[\left(1+\alpha\sigma^S\right)\phi_K - \left(1-\alpha^2\sigma^S\right)\phi_Z\right]\eta(\lambda L - g^*)}{\alpha[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^*+c^*)]} \right. \\
&\quad \left. - \frac{(\phi_K - \phi_Z)\phi_Z \eta (1-\alpha) c^* \left(\lambda L + \sigma^S\rho\right)}{\alpha[\phi_K(\lambda L - g^*) + \phi_Z(1-\alpha)(g^*+c^*)]^2} \right\}.
\end{aligned}$$

For the two special cases $\phi_K = \phi_Z$ on the one hand and $\phi_Z = 0$, $\phi_K > 0$ on the other hand, the saddlepoint stability of the steady state can be shown analytically. The stability analysis of the remaining cases requires the use of numerical calculations.

F.2 The special case $\phi_K = \phi_Z$

From Proposition 1 it follows that if $\phi_K = \phi_Z$, then the steady state values g^* and c^* are given by

$$g^* = \frac{\sigma^S[(\alpha+\eta)\lambda L - \rho]}{1+\sigma^S[\alpha+\eta(1+\alpha)]},$$

$$c^* = \frac{\lambda L - (1+\alpha)g^*}{\alpha} = \frac{\left(1-\alpha^2\sigma^S\right)\lambda L + (1+\alpha)\sigma^S\rho}{\alpha\{1+\sigma^S[\alpha+\eta(1+\alpha)]\}}.$$

The last equation implies that

$$c^* + g^* = \alpha^{-1} (\lambda L - g^*) .$$

Using these results, the expressions for the elements of the Jacobian in the special case $\phi_K = \phi_Z$ can be written as:

$$m_{11} = \left(1 + \eta\alpha\sigma^S\right) c^*,$$

$$m_{12} = \frac{(1-\alpha)\,\lambda c^*}{\alpha}\left[1 - \sigma^S\alpha^2 - \frac{\eta\alpha^3\sigma^S c^*}{\lambda L - g^*}\right],$$

$$m_{13} = -\frac{(1-\alpha)\,\alpha^{\alpha/(1-\alpha)} c^* (\lambda L - g^*)^{(1-2\alpha)/(1-\alpha)}}{\lambda}\left[1 - \sigma^S\alpha^2 + \frac{\eta(1-\alpha)\,\alpha^2\sigma^S c^*}{\lambda L - g^*}\right],$$

$$m_{21} = \frac{\lambda L - g^*}{\lambda},$$

$$m_{22} = \frac{1+\alpha^2}{\alpha}(\lambda L - g^*) ,$$

$$m_{23} = -\frac{(1-\alpha)^2\,\alpha^{\alpha/(1-\alpha)} (\lambda L - g^*)^{(2-3\alpha)/(1-\alpha)}}{\lambda^2},$$

$$m_{31} = \frac{\lambda (\lambda L - g^*)^{\alpha/(1-\alpha)}}{\alpha^{1/(1-\alpha)}},$$

$$m_{32} = \frac{\lambda^2 (\lambda L - g^*)^{\alpha/(1-\alpha)}}{\alpha^{(2-\alpha)/(1-\alpha)}},$$

$$m_{33} = -\frac{(1-\alpha)(\lambda L - g^*)}{\alpha}.$$

These expressions imply that

$$\operatorname{trace}(\mathbf{M}) = \left(1 + \eta\alpha\sigma^S\right) c^* + (1+\alpha)(\lambda L - g^*) ,$$

$$\det(\mathbf{M}) = -(1-\alpha)\left[\left(1 + \alpha\sigma^S\right) + \eta(1+\alpha)\,\sigma^S\right] c^* (\lambda L - g^*)^2 .$$

Taking into account that $\lambda L - g^* = \lambda (L - L_A^*) = \lambda L_Y^* > 0$, it is obvious that trace($\mathbf{M}$) > 0 and $\det(\mathbf{M}) < 0$. Hence, the sufficient condition for the saddlepoint stability of the steady state (c^*, L_A^*, a^*) given by (F.17) is satisfied.

F.3 The special case $\phi_Z = 0$ and $\phi_K > 0$

From Proposition 2 or the equations given in (E.14) it follows that

$$g^* = \frac{\sigma^S(\alpha\lambda L - \rho)}{1 + \alpha\sigma^S},$$

$$c^* = \frac{\alpha\left[\lambda L - (1+\alpha)\,g^*\right]}{\alpha^2 + \eta} = \frac{\alpha\left[\left(1 - \alpha^2\sigma^S\right)\lambda L + (1+\alpha)\,\sigma^S\rho\right]}{(1+\alpha\sigma^S)(\alpha^2+\eta)}.$$

These two results imply that

$$
\begin{aligned}
c^* + g^* &= \frac{\alpha\lambda L - (\alpha - \eta)\, g^*}{\alpha^2 + \eta} = \frac{\left(1 + \eta\sigma^S\right)\alpha\lambda L + (\alpha - \eta)\,\sigma^S \rho}{\left(\alpha^2 + \eta\right)\left(1 + \alpha\sigma^S\right)}, \\
\lambda L - g^* &= \frac{\lambda L + \sigma^S \rho}{1 + \alpha\sigma^S}.
\end{aligned}
$$

Using the results given above, the elements of the Jacobian corresponding to the special case $\phi_Z = 0$ and $\phi_K > 0$ can be expressed as follows:

$$
m_{11} = \left(1 + \eta\sigma^S\right) c^*,
$$

$$
\begin{aligned}
m_{12} &= \frac{\left(1 - \sigma^S\alpha^2\right)(1 - \alpha)\,\lambda c^* \left(g^* + c^*\right)}{\lambda L - g^*}, \\
m_{12} &= \frac{\left(1 - \sigma^S\alpha^2\right)(1 - \alpha)\,\lambda c^* \left(g^* + c^*\right)}{\lambda L - g^*}, \\
m_{13} &= -\frac{\left(1 - \sigma^S\alpha^2\right)(1 - \alpha)\, c^* \left(\lambda L - g^*\right)}{\lambda \left(g^* + c^*\right)^{\alpha/(1-\alpha)}}, \\
m_{21} &= \frac{\alpha + \eta}{\alpha\lambda}\left(\lambda L - g^*\right), \\
m_{22} &= (1 - \alpha)^2 \left(g^* + c^*\right) + 2\left(\lambda L - g^*\right), \\
m_{23} &= -\frac{\left(\lambda L - g^*\right)^2}{\lambda^2 \left(g^* + c^*\right)^{\alpha/(1-\alpha)}} (1 - \alpha)^2, \\
m_{31} &= \frac{\lambda \left(g^* + c^*\right)^{1/(1-\alpha)}}{\lambda L - g^*}, \\
m_{32} &= \frac{\lambda^2 \left(g^* + c^*\right)^{1/(1-\alpha)}}{\left(\lambda L - g^*\right)^2} \left[\left(\lambda L - g^*\right) + (1 - \alpha)\left(g^* + c^*\right)\right], \\
m_{33} &= -(1 - \alpha)\left(g^* + c^*\right).
\end{aligned}
$$

It can be shown that

$$
\begin{aligned}
\operatorname{trace}(\mathbf{M}) &= \frac{\left\{\alpha\left[\left(1 - \alpha^2\sigma^S\right) + \alpha(1 + \alpha)\right] + \eta\left[\left(1 + \alpha\sigma^S\right)\left(1 - \alpha^2\sigma^S\right) + 1 + \alpha^3\sigma^S\right]\right\}\lambda L}{\left(1 + \alpha\sigma^S\right)\left(\alpha^2 + \eta\right)} \\
&\quad + \frac{\left\{4\alpha^2 + (1 - \alpha)^2\alpha + \eta\left[2 + \alpha(1 - \alpha) + \sigma^S\alpha(1 + \alpha)\right]\right\}\sigma^S\rho}{\left(1 + \alpha\sigma^S\right)\left(\alpha^2 + \eta\right)}, \\
\det(\mathbf{M}) &= -\frac{(1 - \alpha)\left(\alpha^2 + \eta\right)\left(1 + \alpha\sigma^S\right)}{\alpha} c^* \left(g^* + c^*\right)\left(\lambda L - g^*\right) < 0.
\end{aligned}
$$

It is obvious that $\det(\mathbf{M}) < 0$. Assumption (E.9), $1 - \alpha^2\sigma^S > 0$, is sufficient for $\operatorname{trace}(\mathbf{M}) > 0$. Hence, the sufficient condition for the saddlepoint stability of the steady state (c^*, L_A^*, a^*) given by (F.17) is satisfied.

F.4 The general case $\phi_Z > 0$

We now analyze also situations in which neither $\phi_Z = 0$ nor $\phi_Z = 1$ holds. More specifically, in the following illustrations we set $\phi_K = 1$ and consider alternative values of ϕ_Z allowing for both $0 \leq \phi_Z \leq \phi_K$ and $\phi_Z > \phi_K$. In addition, we also consider alternative values of η. In contrast to ϕ_Z and η we do not alter the rest of the parameters. The following parameter settings are used for all illustrations:

$$\alpha = 1/3, \quad \xi = 1 \Rightarrow \sigma^S = 1/\theta = 1/2, \quad \rho = 0.02, \quad L = 300 \times 10^6, \quad \lambda = 5.0 \times 10^{-10}.$$

The values in the following table correspond to the case $\eta = 0.1$:

ϕ_Z	g^*	z_1	z_2	z_3
0.0	1.29%	−0.06640	0.12200	0.38946
0.1	1.37%	−0.06560	0.12638	0.39522
0.2	1.46%	−0.06535	0.13024	0.40329
0.3	1.53%	−0.06567	0.13352	0.41331
0.4	1.59%	−0.06653	0.13622	0.42464
0.5	1.65%	−0.06778	0.13840	0.43659
0.6	1.70%	−0.06930	0.14015	0.44856
0.7	1.74%	−0.07096	0.14156	0.46013
0.8	1.77%	−0.07266	0.14271	0.47104
0.9	1.80%	−0.07434	0.14366	0.48117
1.0	1.82%	−0.07595	0.14445	0.49049
2.0	1.95%	−0.08736	0.14835	0.54946
10.0	2.06%	−0.10297	0.15182	0.62142
100.0	2.08%	−0.10739	0.15264	0.64091

In the next table we set $\eta = 0.5$:

ϕ_Z	g^*	z_1	z_2	z_3
0.0	1.29%	−0.04738	0.08198	0.31131
0.1	1.45%	−0.04795	0.08713	0.30888
0.2	1.64%	−0.04851	0.09295	0.30680
0.3	1.84%	−0.04909	0.09952	0.30553
0.4	2.07%	−0.04977	0.10680	0.30587
0.5	2.33%	−0.05071	0.11463	0.30908
0.6	2.59%	−0.05217	0.12256	0.31694
0.7	2.86%	−0.05453	0.12994	0.33126
0.8	3.10%	−0.05809	0.13618	0.35291
0.9	3.32%	−0.06286	0.14108	0.38092
1.0	3.50%	−0.06856	0.14484	0.41288
2.0	4.16%	−0.11842	0.15902	0.66893
10.0	4.46%	−0.17608	0.16759	0.95593
100.0	4.50%	−0.18988	0.16921	1.02504

It is obvious that all cases considered in the two tables exhibit saddlepoint stability. Moreover, both tables illustrate the statement of Proposition 3 that g^* depends positively on ϕ_Z. Finally, a comparison of the two tables illustrates the statement of Proposition 4 that g^* depends positively on η, regardless of whether $0 < \phi_Z \leq \phi_K$ or $\phi_Z > \phi_K$.

G Decentralized versus socially planned solution of an economy with relative wealth preferences

In this subsection we analyze whether there exists the theoretical possibility that the well-known distortions of the standard R&D-based growth model of the Romer (1990) type are perfectly offset by the externalities that result from relative wealth preferences so that the decentralized solution coincides with its socially optimal counterpart. Our results can be summarized by the following proposition:

Proposition 5. *Suppose that the lifetime utility of the representative household is given by*

$$\int_0^\infty e^{-\rho t}\left(\frac{C^{1-\theta}-1}{1-\theta}\right)dt, \qquad \theta, \rho > 0,$$

in the standard economy and by

$$\int_0^\infty e^{-\rho t}\left(\frac{[Ch(S)]^{1-\theta}-1}{1-\theta}\right)dt, \qquad S\left(\Omega,\bar{\Omega}\right) = \varphi\left(\frac{\Omega}{\bar{\Omega}}\right), \quad \varphi' > 0, \varphi'' \leq 0,$$

in the economy with relative wealth preferences. Then the following results hold:

i) The socially optimal solution of the economy with relative wealth preferences coincides *with the socially optimal solution of the economy with standard preferences.*

ii) If $\phi_K > 0$ and if the status parameters ϕ_Z and η happen to satisfy the conditions

$$\phi_Z = \tilde{\varsigma}\phi_K, \qquad \tilde{\varsigma} \equiv 1 + \frac{\alpha\,(\lambda L - \rho)}{\theta\,(1-\alpha)\,\lambda L} > 1,$$

$$\eta = \tilde{\eta} \equiv \frac{(1-\alpha)\,(\theta\lambda L)^2}{[(\theta-\alpha)\,\lambda L + \alpha\rho]\,[(\theta-1)\,\lambda L + \rho]},$$

then the BGP of the decentralized economy with relative wealth preferences coincides with the BGP of the socially planned economy. More specifically, $(\eta, \phi_Z) = (\tilde{\eta}, \tilde{\varsigma}\phi_K)$ implies that

$$g^* = \frac{\lambda L - \rho}{\theta} = g^P,$$

$$\left(\frac{C}{K}\right)^* = \frac{(\theta-\alpha)\,\lambda L + \alpha\rho}{\alpha\theta} = \left(\frac{C}{K}\right)^P,$$

$$\left(\frac{Y}{K}\right)^* = \frac{\lambda L}{\alpha} = \left(\frac{Y}{K}\right)^P,$$

$$L_A^* = \frac{\lambda L - \rho}{\theta\lambda} = L_A^P,$$

$$L_Y^* = \frac{(\theta-1)\,\lambda L + \rho}{\theta\lambda} = L_Y^P,$$

$$r^* = \alpha\lambda L > \frac{\alpha\,[(\theta-1)\,\lambda L + \rho]}{\theta} = (\pi/p_A)^*,$$

$$\left(\varepsilon^K\right)^* = (1-\alpha)\,\lambda L < \frac{(1-\alpha)\,\theta\lambda L + \alpha\,(\lambda L - \rho)}{\theta} = \left(\varepsilon^Z\right)^*,$$

$$r^* + \left(\varepsilon^K\right)^* = (\pi/p_A)^* + \left(\varepsilon^Z\right)^* = \lambda L,$$

holds, where x^* denotes the steady-state value of x in the decentralized economy with relative wealth preferences, while x^P denotes the common steady state value of x in the two socially planned economies.

Proof of i)

First, we analyze the socially optimal solution of the *standard model without status preferences.* The benevolent social planner maximizes lifetime utility of the representative household given by

$$\int_0^\infty e^{-\rho t}\left(\frac{C^{1-\theta}-1}{1-\theta}\right)dt \tag{G.1}$$

by optimally choosing the time paths of C, $x_i, i \in [0, A]$, $k_i, i \in [0, A]$, L_Y, and L_A by taking into account the production functions for the final good, the intermediate goods and new technologies

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha \, di, \tag{G.2}$$

$$x_i = k_i, \qquad i \in [0, A], \tag{G.3}$$

$$\dot{A} = \lambda L_A A, \tag{G.4}$$

the three resource constraints

$$L_Y + L_A = L, \tag{G.5}$$

$$K = \int_0^A k_i \, di, \tag{G.6}$$

$$\dot{K} = Y - C, \tag{G.7}$$

and the two initial conditions

$$A(0) = A_0, \qquad K(0) = K_0. \tag{G.8}$$

The symmetry assumptions that are employed in the production function for final goods (G.2) and the production functions for the intermediate goods (G.3) imply that the optimal plan of the social planner exhibits the following property:

$$x_i = x, \qquad i \in [0, A],$$

$$k_i = k = x, \qquad i \in [0, A].$$

Substituting these equations into the resource constraint for aggregate capital input (G.6) we obtain

$$K = Ak = Ax.$$

The last equations together with the resource constraint for employment (G.5) imply the following results for output of the final good:

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha \, di = L_Y^{1-\alpha} A x^\alpha = (L - L_A)^{1-\alpha} A k^\alpha = K^\alpha \left[A(L - L_A)\right]^{1-\alpha}. \tag{G.9}$$

The results given above imply that the socially optimal paths of C, L_A, K, and A can be determined by means of the following relatively simple optimization problem: Maximize lifetime utility of the representative household given by (G.1) by optimally choosing the time paths of C and L_A subject to the aggregate resource constraint

$$\dot{K} = K^\alpha \left[A(L - L_A)\right]^{1-\alpha} - C, \tag{G.10}$$

the production function for blueprints (G.4), and the two initial conditions (G.8).

The current value Hamiltonian is given by

$$H=\frac{C^{1-\theta}-1}{1-\theta}+\mu_K\left\{K^{\alpha}\left[A\left(L-L_A\right)\right]^{1-\alpha}-C\right\}+\mu_A\lambda L_A A.$$

The necessary optimality conditions for an interior solution are given by

$$\frac{\partial H}{\partial C}=C^{-\theta}-\mu_K=0, \tag{G.11}$$

$$\frac{\partial H}{\partial L_A}=-\mu_K\left(1-\alpha\right)A\left[\frac{K}{A\left(L-L_A\right)}\right]^{\alpha}+\mu_A\lambda A=0, \tag{G.12}$$

$$\dot{\mu}_K=\rho\mu_K-\frac{\partial H}{\partial K}=-\mu_K\left\{\alpha\left[\frac{K}{A\left(L-L_A\right)}\right]^{-(1-\alpha)}-\rho\right\}, \tag{G.13}$$

$$\dot{\mu}_A=\rho\mu_A-\frac{\partial H}{\partial A}=-\mu_A\left\{\frac{\mu_K}{\mu_A}\left(1-\alpha\right)\left[\frac{K}{A\left(L-L_A\right)}\right]^{\alpha}\left(L-L_A\right)+\lambda L_A-\rho\right\}. \tag{G.14}$$

The transversality conditions are given by

$$\lim_{t\to\infty}e^{-\rho t}\mu_K K=0, \qquad \lim_{t\to\infty}e^{-\rho t}\mu_A A=0. \tag{G.15}$$

From the necessary optimality conditions (G.11)–(G.14) it follows that

$$\mu_K=C^{-\theta}, \tag{G.16}$$

$$\frac{\dot{\mu}_K}{\mu_K}=-\theta\frac{\dot{C}}{C}, \tag{G.17}$$

$$\mu_A=\mu_K\left(1-\alpha\right)\lambda^{-1}\left[\frac{K}{A\left(L-L_A\right)}\right]^{\alpha}=\left(1-\alpha\right)\lambda^{-1}\left[\frac{K}{A\left(L-L_A\right)}\right]^{\alpha}C^{-\theta}, \tag{G.18}$$

$$\frac{\dot{\mu}_A}{\dot{\mu}_A}=\alpha\frac{\dot{K}}{K}-\alpha\frac{\dot{A}}{A}+\frac{\alpha}{L-L_A}\dot{L}_A-\theta\frac{\dot{C}}{C}. \tag{G.19}$$

Substituting (G.16)–(G.19) into (G.13)–(G.14) yields

$$\frac{\dot{C}}{C}=\frac{1}{\theta}\left\{\alpha\left[\frac{K}{A\left(L-L_A\right)}\right]^{-(1-\alpha)}-\rho\right\}, \tag{G.20}$$

$$\alpha\frac{\dot{K}}{K}-\alpha\frac{\dot{A}}{A}+\frac{\alpha}{L-L_A}\dot{L}_A-\theta\frac{\dot{C}}{C}=-\left(\lambda L-\rho\right). \tag{G.21}$$

From (G.4) and (G.10) it follows that

$$\frac{\dot{K}}{K}=\left[\frac{K}{A\left(L-L_A\right)}\right]^{-(1-\alpha)}-\frac{C}{K}, \tag{G.22}$$

$$\frac{\dot{A}}{A}=\lambda L_A. \tag{G.23}$$

Substituting (G.20), (G.22), and (G.23) into (G.21), we obtain the following differential equation for employment in the R&D sector:

$$\dot{L}_A = (L - L_A)\left(\lambda L_A - \frac{\lambda L}{\alpha} + \frac{C}{K}\right). \tag{G.24}$$

Using (G.16) and (G.18), the transversality conditions (G.15) can be expressed as follows:

$$\lim_{t\to\infty} e^{-\rho t} C^{-\theta} K = 0, \qquad \lim_{t\to\infty} e^{-\rho t} (1-\alpha)\lambda^{-1}\left[\frac{K}{A(L-L_A)}\right]^{\alpha} C^{-\theta} A = 0. \tag{G.25}$$

The considerations made above show that the dynamic evolution of C, L_A, K, and A in the socially planned economy is determined by the four differential equations (G.20), (G.24), (G.22), and (G.23), the initial conditions (G.8), and the transversality conditions (G.25).

This system exhibits the property that there exists a BGP characterized by the following features:

$$L_A = \text{constant}, \quad \frac{C}{K} = \text{constant}, \quad \frac{K}{A} = \text{constant}, \quad \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \text{constant}.$$

In the following we denote the steady-state values of an arbitrary variable x in the socially planned economy by x^P, where the superscript P stands for "Planner". The four differential equations (G.20), (G.24), (G.22), and (G.23) imply that

$$\begin{aligned}
g^P &= \frac{1}{\theta}\left\{\alpha\left[\left(\frac{K}{A}\right)^P \frac{1}{L-L_A^P}\right]^{-(1-\alpha)} - \rho\right\}, \\
0 &= \lambda L_A^P - \frac{\lambda L}{\alpha} + \left(\frac{C}{K}\right)^P, \\
g^P &= \left[\left(\frac{K}{A}\right)^P \frac{1}{L-L_A^P}\right]^{-(1-\alpha)} - \left(\frac{C}{K}\right)^P, \\
g^P &= \lambda L_A^P,
\end{aligned}$$

where g^P denotes the common growth rate of C, A, and K along the BGP. Solving for g^P, $(C/K)^P$, L_A^P, and $(K/A)^P$ we obtain

$$g^P = \frac{\lambda L - \rho}{\theta}, \tag{G.26}$$

$$\left(\frac{C}{K}\right)^P = \frac{(\theta-\alpha)\lambda L + \alpha\rho}{\alpha\theta}, \tag{G.27}$$

$$L_A^P = \frac{\lambda L - \rho}{\lambda\theta}, \tag{G.28}$$

$$\left(\frac{K}{A}\right)^P = \frac{(\theta-1)\lambda L + \rho}{\lambda\theta}\left(\frac{\lambda L}{\alpha}\right)^{-1/(1-\alpha)}. \tag{G.29}$$

From (G.5) and (G.9) it then follows that

$$L_Y^P = \frac{(\theta - 1)\,\lambda L + \rho}{\lambda\theta}, \tag{G.30}$$

$$\left(\frac{Y}{K}\right)^P = \left[\left(\frac{K}{A}\right)^P \frac{1}{L - L_A^P}\right]^{-(1-\alpha)} = \frac{\lambda L}{\alpha}. \tag{G.31}$$

The transversality conditions (G.25) require that the expressions

$$e^{-\rho t} C^{-\theta} K \qquad \text{and} \qquad e^{-\rho t}\,(1-\alpha)\,\lambda^{-1}\,\{K/\left[A\,(L - L_A)\right]\}^{\alpha}\, C^{-\theta} A$$

converge to zero for $t \to \infty$. Along the BGP the common constant rate of growth of these two expressions is given by

$$-\rho - \theta g^P + g^P = -\frac{(\theta - 1)\,\lambda L + \rho}{\theta} = -\lambda L_Y^P. \tag{G.32}$$

From (G.26), (G.27), (G.28), (G.30), and (G.32) it then follows that if ρ satisfies the condition that

$$(1 - \theta)\,\lambda L < \rho < \lambda L, \tag{G.33}$$

then

$$g^P > 0, \qquad \left(\frac{C}{K}\right)^P > 0, \qquad L_A^P > 0, \qquad \text{and} \qquad L_Y^P > 0$$

hold and both transversality conditions are satisfied.

Now we proceed with the socially optimal solution of an economy with *relative wealth preferences*. A sensible comparison with the status-free economy in which the instantaneous utility function is given by

$$u\,(C) = \frac{C^{1-\theta} - 1}{1 - \theta},$$

requires that we restrict our attention to relative wealth preferences that are obtained by setting $\xi = 1$ in (42):

$$u\,(C, S) = \frac{\left[C h\,(S)\right]^{1-\theta} - 1}{1 - \theta}, \qquad S\left(\Omega, \bar{\Omega}\right) = \varphi\left(\frac{\Omega}{\bar{\Omega}}\right), \qquad \varphi' > 0, \varphi'' \leq 0.$$

According to (44) and (45) these specifications of $u\,(C, S)$ and $S\left(\Omega, \bar{\Omega}\right)$ imply that

$$\chi = S\,(\Omega, \Omega) = \varphi\,(1)\,, \qquad \sigma^S = 1/\theta, \qquad \eta = \beta \equiv \frac{h'\left[\varphi\,(1)\right]\varphi'\,(1)}{h\left[\varphi\,(1)\right]}. \tag{G.34}$$

In contrast to the representative household of the decentralized economy, the benevolent social planner takes into account the externalities resulting from relative wealth preferences. It is optimal for the social planner to assign identical choices to households that are identical in every respect so that $\Omega_j = \bar{\Omega}$ holds for each household j at any time t. Taking into account that $S\,(\Omega, \Omega) = \varphi\,(1) \equiv \chi$ holds, the optimization problem of the social planner can be written as follows: Maximize lifetime utility of the representative consumer

$$\int_0^\infty e^{-\rho t}\left(\frac{[Ch(\chi)]^{1-\theta}-1}{1-\theta}\right)dt$$

by optimally choosing the time paths of C and L_A and taking into account the economy's resource constraint (G.10), the production function for blueprints (G.4), and the initial conditions (G.8). It is easily verified that the optimal time paths of C, L_A, K, and A (in contrast to the time paths of the costate variables) are independent of the constant factor $h(\chi)$. Hence, regardless of whether we consider standard preferences or introduce relative wealth preferences, the socially optimal time paths of C, L_A, K, and A are determined by the four differential equations (G.20), (G.24), (G.22), and (G.23), the initial conditions (G.8), and the transversality conditions (G.25). These results imply that the solutions for the steady-state values given by (G.26)–(G.31) are also valid for the model with relative wealth preferences provided that $\xi = 1$. In other words, the socially optimal solution of the economy with relative wealth preferences coincides with the socially optimal solution of the economy with standard preferences.

Proof of (ii):

If $\xi = 1$ then $\sigma^S = \theta^{-1}$ and $\eta = \beta$ [see (G.34)]. Setting $\sigma^S = \theta^{-1}$ in (50) and (51) yields

$$-\left(1-\alpha^2\theta^{-1}\right)g^* + \theta^{-1}\left\{\alpha^2 + \frac{\phi_K\eta}{\phi_K + \dfrac{\phi_Z(1-\alpha)}{\lambda L - g^*}\left[g^* + \left(\dfrac{C}{K}\right)^*\right]}\right\}\left(\frac{C}{K}\right)^* = \theta^{-1}\rho, \tag{G.35}$$

$$(1+\alpha)g^* + \frac{1}{\alpha}\left\{\alpha^2 + \frac{(\phi_K - \phi_Z)\eta}{\phi_K + \dfrac{\phi_Z(1-\alpha)}{\lambda L - g^*}\left[g^* + \left(\dfrac{C}{K}\right)^*\right]}\right\}\left(\frac{C}{K}\right)^* = \lambda L. \tag{G.36}$$

These two equations determine the steady-state values of g and (C/K) in the decentralized economy with relative wealth preferences. From (G.26) and (G.27) it follows that the conditions for social optimality are given by

$$g^* = g^P = \frac{\lambda L - \rho}{\theta}, \tag{G.37}$$

$$\left(\frac{C}{K}\right)^* = \left(\frac{C}{K}\right)^P = \frac{(\theta-\alpha)\lambda L + \alpha\rho}{\alpha\theta}. \tag{G.38}$$

Substituting these two conditions into (G.35) and (G.36) we obtain

$$-(1-\alpha)\alpha\theta\lambda L + \frac{\phi_K\eta[(\theta-\alpha)\lambda L + \alpha\rho]}{\phi_K + \phi_Z\dfrac{(1-\alpha)\theta\lambda L}{\alpha[(\theta-1)\lambda L + \rho]}} = 0, \tag{G.39}$$

$$\alpha^2(\lambda L - \rho) + \frac{(\phi_K - \phi_Z)\eta[(\theta-\alpha)\lambda L + \alpha\rho]}{\phi_K + \phi_Z\dfrac{(1-\alpha)\theta\lambda L}{\alpha[\lambda L\theta - \lambda L + \rho]}} = 0. \tag{G.40}$$

If $\phi_K = 0$, then condition (G.39) simplifies to

$$-(1-\alpha)\,\alpha\theta\lambda L = 0.$$

Obviously, this condition can never be satisfied. Hence, in the following we restrict our attention to $\phi_K > 0$. Introducing the definition

$$\varsigma \equiv \frac{\phi_Z}{\phi_K},$$

conditions (G.39) and (G.40) can be rewritten as

$$-(1-\alpha)\,\alpha\theta\lambda L + \frac{\eta\left((\theta-\alpha)\,\lambda L + \alpha\rho\right)}{1+\dfrac{\varsigma\,(1-\alpha)\,\theta\lambda L}{\alpha\left((\theta-1)\,\lambda L+\rho\right)}} = 0,$$

$$\alpha^2\left(\lambda L - \rho\right) + \frac{(1-\varsigma)\,\eta\left((\theta-\alpha)\,\lambda L + \alpha\rho\right)}{1+\dfrac{\varsigma\,(1-\alpha)\,\theta\lambda L}{\alpha\left(\lambda L\theta - \lambda L + \rho\right)}} = 0.$$

Solving these two equations for η and ς and denoting the solutions by $\tilde{\eta}$ and $\tilde{\varsigma}$, we obtain

$$\tilde{\varsigma} = 1 + \frac{\alpha\left(\lambda L - \rho\right)}{\theta\,(1-\alpha)\,\lambda L} > 1, \tag{G.41}$$

$$\tilde{\eta} = \frac{(1-\alpha)\,(\theta\lambda L)^2}{\left[(\theta-\alpha)\,\lambda L + \alpha\rho\right]\left[(\theta-1)\,\lambda L + \rho\right]}. \tag{G.42}$$

If $(\eta, \phi_Z) = (\tilde{\eta}, \tilde{\varsigma}\phi_K)$ happens to hold, then by definition both $g^* = g^P$ and $(C/K)^* = (C/K)^P$ hold. Using (G.37)–(G.38), (G.41)–(G.42) as well as (C.1)–(C.18), we finally obtain

$$\left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^* = \frac{\lambda L}{\alpha},$$

$$L_A^* = \frac{g^*}{\lambda} = \frac{\lambda L - \rho}{\theta\lambda},$$

$$L_Y^* = L - L_A^* = \frac{\lambda L - g^*}{\lambda} = \frac{(\theta-1)\,\lambda L + \rho}{\theta\lambda},$$

$$r^* = \alpha^2\left(\frac{Y}{K}\right)^* = \alpha\lambda L,$$

$$\left(\frac{\pi}{p_A}\right)^* = \alpha\lambda L_Y^* = \frac{\alpha\left[(\theta-1)\,\lambda L + \rho\right]}{\theta},$$

$$\left(\frac{C}{\Omega}\right)^* = \frac{\left(\dfrac{C}{K}\right)^*}{\phi_K + \tilde{\varsigma}\phi_K\dfrac{1-\alpha}{\lambda L - g^*}\left(\dfrac{Y}{K}\right)^*} = \frac{\left[(\theta-\alpha)\,\lambda L + \alpha\rho\right]\left[(\theta-1)\,\lambda L + \rho\right]}{\phi_K\theta^2\lambda L},$$

$$\left(\varepsilon^K\right)^* = \phi_K\tilde{\eta}\times\left(\frac{C}{\Omega}\right)^* = \frac{1}{\sigma^S}g^* - r^* + \rho = (1-\alpha)\,\lambda L,$$

$$\left(\varepsilon^Z\right)^* = \tilde{\varsigma}\phi_K\tilde{\eta}\times\left(\frac{C}{\Omega}\right)^* = \frac{(1-\alpha)\,\theta\lambda L + \alpha\left(\lambda L - \rho\right)}{\theta},$$

$$r^* + \left(\varepsilon^K\right)^* = \left(\frac{\pi}{p_A}\right)^* + \left(\varepsilon^Z\right)^* = \lambda L,$$

$$\left(\frac{C}{Y}\right)^* = \frac{(\theta-\alpha)\,\lambda L + \alpha\rho}{\theta\lambda L} = 1 - \frac{\alpha\,(\lambda L-\rho)}{\theta\lambda L},$$

$$\left(\frac{K}{K+p_A A}\right)^* = \frac{1}{1+\dfrac{1-\alpha}{\lambda L_Y^*}\left(\dfrac{Y}{K}\right)^*} = \alpha\left(1-\frac{(1-\alpha)\,(\lambda L-\rho)}{(\theta-\alpha)\,\lambda L+\alpha\rho}\right) < \alpha.$$

While the effective rates of return of physical capital and shares are identical,

$$r^* + \left(\varepsilon^K\right)^* = (\pi/p_A)^* + \left(\varepsilon^Z\right)^* = \lambda L,$$

the market rates of return and the status related extra returns differ due to $\phi_Z = \tilde{\varsigma}\phi_K > \phi_K$:

$$r^* = \alpha\lambda L > \frac{\alpha\,[(\theta-1)\,\lambda L+\rho]}{\theta} = (\pi/p_A)^*,$$

$$\left(\varepsilon^K\right)^* = (1-\alpha)\,\lambda L < \frac{(1-\alpha)\,\theta\lambda L + \alpha\,(\lambda L-\rho)}{\theta} = \left(\varepsilon^Z\right)^* \quad \blacksquare$$

Illustration:

Consider the values

$$\alpha = 1/3, \quad \xi = 1 \Rightarrow \sigma^S = 1/\theta = 1/2, \quad \rho = 0.02, \quad L = 300\times10^6, \quad \lambda = 5.0\times10^{-10}.$$

Then we have

$$\tilde{\varsigma} = 1 + \frac{\alpha\,(\lambda L-\rho)}{\theta\,(1-\alpha)\,\lambda L} \approx 1.22,$$

$$\tilde{\eta} = \frac{(1-\alpha)\,(\theta\lambda L)^2}{[(\theta-\alpha)\,\lambda L+\alpha\rho]\,[(\theta-1)\,\lambda L+\rho]} \approx 1.38.$$

If $\phi_Z = \tilde{\varsigma}\phi_K$ and $\eta = \tilde{\eta}$ happen to hold, then

$$g^* = g^P = 6.5\times10^{-2}, \qquad \left(\frac{C}{K}\right)^* = \left(\frac{C}{K}\right)^P = 0.385,$$

$$\left(\frac{Y}{K}\right)^* = 0.45, \qquad \left(\frac{C}{Y}\right)^* = 85.56\times10^{-2},$$

$$L_Y^* = 170000000.0, \qquad L_A^* = 130000000.0, \qquad \frac{L_A^*}{L} = 43.33\times10^{-2},$$

$$r^* = 5.0\times10^{-2}, \qquad \left(\frac{\pi}{p_A}\right)^* = 2.83\times10^{-2},$$

$$\left(\varepsilon^K\right)^* = 10.0\times10^{-2}, \qquad \left(\varepsilon^Z\right)^* = 12.17\times10^{-2},$$

$$r^* + \left(\varepsilon^K\right)^* = \left(\frac{\pi}{p_A}\right)^* + \left(\varepsilon^Z\right)^* = 15.0\times10^{-2},$$

$$\left(\frac{K}{K + p_A A}\right)^* = 22.08 \times 10^{-2},$$
$$z_1 = -0.08623, \qquad z_2 = 0.15703, \qquad z_3 = 0.45178.$$

For this parameter setting (that can only hold by pure coincidence), the growth rates of the socially planned economy and its decentralized counterpart are the same. Note that one eigenvalue of the Jacobian is negative and two are positive, which implies saddle-point stability of the BGP.

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