Essays on the History of Dynamic Economic Analysis

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To the memory of my mother, who sacrificed herself for her children.

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Contents

List of Figures

Preface

The subsequent three studies in the history of economic analysis - one monograph and two articles - constitute my doctoral dissertation. The analyses are not restricted to a specific field of study, but range over a wide area of subjects. The common ground between the self-contained contributions arises from my rather idiosyncratic approach to the history of economics. To begin with, I do not share the hostility towards the latest vintages of economic analysis, especially towards axiomatic general equilibrium analysis, that has become all too common among historians of economics. Rather, I perceive the history of 'mainstream' economics as a story of success, at least on balance. This does not suggest that I have a Whiggish view of history, since I do not believe in the inevitability of scientific progression; it just happened to take place. Nor does it imply that economics has reached 'the end of history', in the sense that I predict the survival of the standard economics toolset; our 'new' tools just provide superior frameworks for 'old' ideas, no matter what standards may follow next.

Secondly and closely related, my interests also extend to the more recent history of economics. In addition to more or less detailed discussions of usual suspects like Adam Smith, F. A. Hayek, Ludwig von Mises, or Lionel Robbins, the reader is invited to track my historical analysis of axiomatic reasoning. In fact, it is the confrontation of the theories of established figures in the history of economics with the various results of 'modern' general equilibrium analysis to which this work owes its specific flavor. What are the implications of the Sonnenschein-Mantel-Debreu results for 'Scottish Political Economy' as defined by David Hume and Adam Smith? What has Walras's law to do with Hume's vision of a 'civil' society? What is the relationship between 'myopic economic agents' as defined by Donald J. Brown and Lucinda M. Lewis (1981) and Hayek's capital theory? What does the "need for impatience" as proclaimed by Aloisio Araujo (1985) suggest with regard to the historians account of the heated debates on the proper role of time preference in economic analysis? Questions like these are raised and answered in the subsequent analyses.

The history of economics is a difficult field of study. Past authors often used dissimilar terms to define similar concepts, and equally often they used similar terms to define dissimilar concepts. They declared universal laws, yet could not escape the specifics of their time and social environment. They often derived positive results in the context of heated political and, thus, normative debates. This problem is accelerated by the normative biases and methodological convictions involved by the secondary literature (just think of the different 'schools of thought' that have contributed to the history of economics). Given this state of affairs, historical reconstructions are by necessity indeterminate, in the sense that 'true intentions' of past authors are forever out of reach. Thus, the history of economics by necessity relies on subjective interpretations that, at best, inform present theory choices by putting meat to the bones of mathematical reasoning. No more is attempted in the subsequent studies.

Part one, entitled '**The Past and Present of the Invisible-Hand Proposition: From 'Scottish Political Economy' to General Equilibrium Analysis**', raises the following questions: To what extent is axiomatic general equilibrium analysis a rational reconstruction of 'Scottish Political Economy' as defined by the writings of David Hume and Adam Smith? How much is gained and how much lost by the axiomatic transformation of the invisible-hand proposition? What are the implications of negative results like the Sonnenschein-Mantel-Debreu demonstrations for the Scottish point of view? Did it reach deadlock, or is there still hope for the dominant trajectory in the history of economics? In contrast to the rich historical literature on the invisible-hand proposition, the present study does not level any paradigmatic criticism at neo-Walrasian analysis. Rather, by focalizing the most important results against the backdrop of Scottish Political Economy, it may inform theory choice within the neo-Walrasian paradigm.

Bernard Mandeville is introduced as the originator of the invisible-hand proposition. It is shown that his statement of the proposition already involved a notion of general equilibrium, and that his vision of bottom-up coordination was generalized by exponents of Scottish Enlightenment, foremost by David Hume and Adam Smith. It is argued that predictions of Arrow-Debreu-McKenzie (ADM) models can be regarded as ideal types of Hume's idea of anonymous social coordination. Right because Smith's 'model' provides such a precise description of the then prevailing mercantile feudal system, it is introduced as an inferior host of Hume's vision. With respect to the existence and efficiency of general equilibrium, the transformation of Hume's invisible-hand proposition is a great success, a highlight in the history of economics. However, not only are ADM models prohibitively restrictive in requiring informational symmetry, they also fall victim to the Sonnenschein-Mantel-Debreu results: no matter how vigorously we restrict individual characteristics, by adding up individual excess demands we lose all structure beyond what is necessary to demonstrate existence and efficiency. It is therefore impossible to restrict

ADM economies to those which satisfy sufficiency conditions for global uniqueness and stability. While Walras's law introduces (some) Humeian structure via the rule of the quid pro quo condition, the ADM setting is in general too arid to determine aggregate outcomes, or to ensure convergence in case of arbitrary initial values. Rational Expectations Equilibria (REE) are considered as the last remaining resort of the invisible-hand proposition within the neo-Walrasian paradigm. Not only do models that predict REE involve price systems as communication devices, they also allow to translate the invisible-hand proposition for genuinely dynamic equilibrium models. However, REE models also imply the Grossman-Stiglitz paradox, which remains a major impediment for the neo-Walrasian transformation of the invisible-hand proposition.

Part two, entitled '**Myopia and Optimal Growth: A Rational Reconstruction of F. A. Hayek's Capital Theory**', translates F.A. Hayek's informal capital theory into a dynamic equilibrium model. The focus is restricted to Hayek's largely unrecognized contribution in Utility Analysis and Interest (UAI), published by *The Economic Journal* in 1936, being restated in *The Pure Theory of Capital* (PTC), first published in 1941. The underlying premise is that Hayek adopts infant versions of modern analytical tools during his time at the London School of Economics such that a rational reconstruction of his capital theory by established neoclassical tools is admissible. The major result is that UAI and PTC contain a generalization of the Ramsey-Cass-Koopmans model. In concrete, Hayek provides the solution to an infinite-horizon deterministic social planner optimization problem in a one-sector economy such that the rate of pure time preference encapsulated in the discount factor increases in prospective utility. The endogeneity of myopia is due to intertemporal complementarities and accounted for by the modified Uzawa aggregator (the Epstein-Hynes aggregator is introduced for expository reasons). It is shown that Hayek remains within the scope of modern capital theory as defined by Roland Becker (2008). The contribution of Fwu-Ranq Chang (1994) is identified as the most commensurable framework for Hayek's capital theory, because it closely relates the case of increasing myopia to the standard case with time-additive utility. Accordingly, the generalization of the Ramsey-Cass-Koopmans model is introduced as the Hayek-Chang model. From a purely theoretical point of view, the implications of 'increasing myopia' are significant: individual corner solution are exiled; on the aggregate level, convergence is ensured even in case of constant returns to per-capita accumulation. The latter result is of central importance to Hayek and establishes a partial alliance with Frank Knight.

Part three, entitled '**The Foundation of Ludwig von Mises's Business Cycle Theory: Real Analysis as a Chain of Tautologies**', addresses Ludwig von Mises's

business cycle theory at maturity, as advanced in his opus magnum *Human Action*. In this work, Mises embeds the business cycle theory which he initially developed in *Theorie des Geldes und der Umlaufmittel* into the broad context of his methodological convictions. Whereas the initial outline of his cycle theory strongly relies on Böhm-Bawerk's capital theory, its mature version is built upon a significantly altered framework of real analysis. The paper describes and evaluates the impact of Mises's praxeology on his conceptualization of real analysis; it provides a simple model to depict and clarify Mises's outline; it draws implications for his business cycle theory and its core prediction that 'any money-induced traverse by necessity reverses'; it argues that Mises's core prediction ultimately depends on his barren analytical device; it concludes that Mises's mature business cycle theory is a regression.

Part I

The Past and Present of the Invisible-Hand Proposition: From 'Scottish Political Economy' to General Equilibrium Analysis

Chapter 1

Introduction

1.1 General Equilibrium Analysis from the Scottish Point of View

Each day, billions of heterogeneous people make interdependent economic choices. Together they produce and consume billions of highly differentiated commodities, and find billion ways to serve one another. At each moment, the success of billions of individual plans depends on their simultaneous execution. Given such a complex coordination problem, surprisingly few of them are disappointed. As long as we are able and willing to pay its price, almost any of the billions of commodities is made available to us. Likewise, most of us are able to realize our stocks at given prices. 'Trade at false prices' seems to be an exception to the rule. In fact, the orchestration of trade plans is so regular and smooth that effective coordination is usually taken for granted. Also because of our genetic code, we get excited over irregular clusters of errors instead (e.g., the 'Great Depression', the 'Great Recession', currency crises, 'unnatural' rates of unemployment, etc.). What accounts for such an amazingly high degree of social coherence? And what mechanism decides for each firm what to produce, where and when to produce, how to produce, how much to produce, and for whom to produce?[1] Why do we not observe economic disorder or outright chaos, even though suppliers of valuable resources follow independent laws of motion?

Although we prosper most rapidly by peaceful cooperation, mankind is an aggressive and greedy species. By now, our social fabric spans widely across the globe, and even tends to overwhelm it. Our self-love and faculty of imagination stand in no advantageous relationship to our natural endowments, including our sharpest tool, the faculty of reason. Human insatiability stems from the imbalance between our profuse and multifarious

[1]According to the young Paul A. Samuelson, economics is defined by the answers to these questions (Samuelson 1947). See Backhouse and Medema 2009: 229.

imaginations of betterment, and the meager individual means to bring them about. This 'scarcity problem' not only motivates human conduct, but at the same time constitutes our dependence on other people's skills and resources (the conflict of interest). However, mankind is also a "company of strangers" (Seabright 2004): even though evolution taught us love and compassion, only few can rely on us. And even though we just learn to sympathize universally, that is, independent of origin or phenotype, our altruism is bounded so that we do not care reliably enough. Yet, if we cannot "expect our dinner" from "the benevolence of the butcher, the brewer, or the baker", to invoke Adam Smith's famous phrase (see 2.2), how then is the natural conflict over scarce resources resolved? If not by appeal to altruism or reason, how then is social coherence sustained?

'Predominantly by appeal to the self-interest of strangers!' is how the representative economist answers this question, as generations of economists have done before. What else remains? In doing so, he continues the oldest surviving tradition in economics. As early as in 1714, Bernard Mandeville argued that the pursuit of individual happiness - deemed vicious at that time - aggregates to a beneficial social order. Even though individuals pursue independent ends, their plans attune "as in Musick harmony" (2.1). Only a few decades later, Mandeville's vision of bottom-up coordination was seized by exponents of Scottish Enlightenment, foremost by David Hume and Adam Smith (2.2-2.3). They identified the 'quid pro quo condition' (2.2) as *the* fundamental ingredient to decentralized social organization: "*Give me that which I want, and you shall have this which you want* [...] it is in this manner that we obtain from one another the far greater part of those good offices which we stand in need of" (Smith 2003: 23). Given social and legal norms that facilitate the exchange nexus by harnessing greed (especially property rights), binding budget constraints submit *tamed* self-interested conduct to the service of strangers: '*do ut des*' implies that individual improvement is subject to the material needs of others. It ensures that *individuals unintentionally serve one another as eager as they serve themselves.*

Today, of course, maximization of utility under budget constraints (optimization) is common currency in economics. Adding up binding budget constraints (each of which states the quid-pro-quo condition) over all individuals gives Walras's law; a corollary establishes anonymous market interdependence. In this sense, Arrow-Debreu-McKenzie models restate one of the most important postulates of 'Scottish Political Economy' (2.2): impersonal exchange relations are the predominant putty of large commercial societies, in contrast to the discretion of supreme leaders, or to 'godly' virtues being enforced on their subjects. Ironically, by denying the necessity of feudal hierarchies (including slavery) for maintaining social order is "how the Dismal Science got its name" (Levy 2002). Furthermore, economics owes its focus on the scarcity problem to Mandeville's *Fable of the Bees*, Hume's *Treatise*, and Smith's *Wealth of Nations*. Although axiomatic general equilibrium models accommodate free goods, we naturally invoke the Scottish emphasis on the 'natural poverty of man' (the individual imbalance introduced above). We are 'Scots', whenever we teach students that, in general, 'there ain't no such thing as a free lunch'; that whenever there is a choice, there is a next best alternative foregone.[2] The Scottish inheritance has become so natural to us that we call it 'thinking like an economist'.

In fact, Lionel Robbins, who persistently stressed continuity in the history of economics (e.g. Robbins 1970), explicitly attributed his 'scarcity definition of economics'[3] to Mandeville (Robbins 1998: 119-22; see section 2.1 for textual evidence supporting Robbins's interpretation of Mandeville's *Fable*). As Denis O'Brien has pointed out in his intellectual biography of Robbins, he

> "was particularly impressed with the work of the eighteenth-century Scottish philosophers Hume, Smith, Ferguson and Millar, with their view of the evolution and development of society. [...] David Hume attracted Robbins's particular admiration [for his welfare and monetary analysis]." (O'Brien 1988: 46; see also page 9)

Yet, due to his focus on the unfortunate influence of apriorism on Robbins, O'Brien introduced "*English* Classical economics, Jevons [to a limited extent], and Wicksteed, and the Austrians [minus Hayek]" as the major sources from which Robbins predominantly drew (ibid: 24; my emphasis): "From Senior, Mill and Cairnes, Robbins took an emphasis on *a priorism*" (ibid: 25; with respect to the "Austrian connection", see also Howson 2004; for the relation between Philip Wicksteed and the Austrians, see Kirzner 1999). The Scottish influence he almost exclusively restricts to Robbins's normative analysis (ibid: 24-5).

However, O'Brien's interpretation underestimates the impact of Allyn A. Young and Friedrich A. Hayek on Robbins's perception of Scottish Political Economy and, therefore, understates its significance for Robbins's positive analysis, even though he himself indicates their influence (ibid.: 45-6). Most importantly, it was Young, who drew Robbins's attention to the strong congruency between Scottish Political Economy and Walras-Pareto

[2]The notion of opportunity costs, of course, is only implicit in classical reasoning. Prominent examples are Smith's beaver-and-deer example, and David Ricardo's international trade theory (constant objective opportunity costs). See Hollander 1987.

[3]"Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses" Robbins 1932: 15.

general equilibrium models! He himself attempted to integrate into general equilibrium analysis Adam Smith's "famous theorem that the division of labor depends on the extent of the market", which he considered to be "one of the most illuminating and fruitful generalizations which can be found anywhere in the whole literature of economics" (Young 1928: 529). Even though Young did not succeed, he involved a social indifference map, a construction "which owes much to Pareto" (ibid: 540), and a transformation curve. In doing so, he brought Smith's reasoning in line with the notions of objective and subjective opportunity costs; he tried to reformulate Smith's theorem for the case where notions of (value) productivity and scarcity have "the same meaning" (ibid: 532; see also section 3.2.2).

Under the influence of Young (and later Hayek), Robbins came to understand that Walras-Pareto general equilibrium models are superior hosts of the Scottish vision of decentralized economic coordination, at least relative to Smith's own 'model' (see 2.3), and to nineteenth-century English classicism in general (Robbins 1930, 1970, 1998). Right because classical models provide excellent stylizations of the then prevailing mercantile feudal system, they fail as hosts of Hume's alternative draft to such privilege-based societies. In a fully meritorious society, so Hume, access to resources is not granted by privilege, but earned on markets. Each individual is weighted by the 'exchange value' of its productive net contributions to society, and is therefore ranked in a 'civil' manner, that is, according to market consensus (2.2). Whereas the separation of income distribution and the allocation of resources across industries is characteristic for classical analysis (including Smith's), Walrasian equilibrium prices decentralize such 'civil' allocations, where each agent is ranked (via its budget) according to the relative scarcity of its net supply.

Taking Hume's distinction between 'is' and 'ought' seriously, Robbins attempted to minimize the impact of normative biases on positive analysis. In fact, he is most known for his argument that inter-subjective utility comparisons have no scientific (i.e., positive) foundation (Robbins 1932, 1938). He sympathized with Pareto's brilliant concept of social efficiency, which incapacitates the theorist's normative judgment, and empowers his hypothetical agents instead, that is, those who inhabit our models, and who must actually bear the consequences of their conjoint choices (3.2.2). Equipped with Pareto efficiency, it becomes possible to reformulate the invisible-hand proposition without Scottish or any other biases, and to discuss decentralized coordination on neutral ground (see also Scitovsky 1951).

After succeeding Young in the Chair of the London School of Economics (LSE) in 1929 (at the age of thirty!), Robbins began to establish a research program that dominated mainline economics for half a century. At its heart was the transformed invisible-hand proposition, which hereby returned to its birthplace (2.1), and which was supposed to provide the link between rational choice and 'macro phenomena'. Under Robbins's benign reign, the LSE soon became the general-equilibrium based antipode to Cambridge, UK (for the "Marshall-Walras Divide", which underlay the conflict between Cambridge and London, see DeVroey 2004). In few years, he gathered together the so-called 'Robbins Circle'[4], a cluster of young guns like Roy R. D. Allen, F. A. Hayek, John R. Hicks, Abba Lerner, Nicolas Kaldor, Tibor Scitovsky, and George Shackle (Hicks 1982), and advised them to read Cassel, Walras, Pareto, Edgeworth, Taussig, Wicksell, Knight, Mises, and the Wieser-Austrians (O'Brien 1988: 42).[5] As Hicks later recalled,

> "[w]e seemed, at the start, to share a common viewpoint, or even, one might say, a common faith. Some of us, especially Hayek, have in later years maintained that faith; [...] The faith in question was a belief in the free market, or 'price-mechanism' - that a competitive system, free of all 'interferences' by government or by monopolistic combinations, of capital or of labour, would easily find into 'equilibrium'." (ibid.: 3)

Thus, not only did the Robbins Circle focus on general equilibrium analysis, initially it did so to analyze the invisible hand as a real-world phenomenon. And even though normative beliefs became increasingly heterogeneous (also because of the Great Depression), Walrasian equilibrium reasoning remained dominant over a much longer period (an instance where Pareto efficiency ensured scientific 'hygiene'; see 3.2.2). Indeed, the idea of beneficial decentralized coordination has become so natural a part of positive analysis, i.e., free of Scottish biases, that Lange and Lerner (as an undergraduate!) could use the Walrasian apparatus to introduce the notion of 'market socialism' (Lange 1936, 1937, Lerner 1934b, 1937, 1944). Soon, general equilibrium analysis was generalized to the case of monopolies (e.g., Lerner 1934a) and externalities (e.g., Scitovsky 1954).[6] Only later did Kaldor (1972) and Shackle (e.g. 1992) completely depart from equilibrium reasoning.[7]

The most important contribution of the LSE circle, at least with respect to the present study, is that it successfully launched the 'Ordinalist Revolution', by which the psychological root of utility analysis was removed and, therefore, the stage prepared for axiomatic general equilibrium analysis (Allen and Hicks 1934; for the Ordinalist Revolution, see Mandler 1999, ch. 4). Still today, introductory courses in microeconomics, where students are for the first time confronted with the first principles of economics, heavily rely on the LSE toolset (e.g., the Hicks decomposition of individual demand). The LSE approach

[4]As to my knowledge, Hicks invented the term.
[5]Robbins himself read German fluently.
[6]On the LSE tradition, see also Shackle (1967), Buchanan (1981), and Ingrao and Israel (1990): 230-5.
[7]Hayek did not! See Part II.

to general equilibrium analysis culminated in Hicks's *Value and Capital* (1939), which, together with Paul A. Samuelson's *Foundations of Economic Analysis* (1947), inspired the craft activities associated with the Cowles Commission (especially Hicks's concept of the Pure Futures Economy, which is a deterministic forerunner of Arrow-Debreu-McKenzie models).

Such is the link between the Scottish vision of decentralized coordination and 'modern' general equilibrium reasoning. Via Robbins and his Circle at the LSE, Smith's flowery metaphor came under the scrutiny of axiomatic reasoning as associated with the Cowles Commission. In fact,

> "[i]n the early 1950s, the only unambiguous endorsements of the Robbins defnition in the journal literature were by members of the Cowles Commission, the center of work on general equilibrium modeling. But by the end of the 1960s, when mathematical methods — including the axiomatic approach — were much more pervasive, this definition had become widely accepted." (Backhouse and Medema 2009: 227)

Even Gérard Debreu, who is well-known for his reluctant use of economic interpretation (see Weintraub and Düppe 2011, and Düppe 2010), did involve the Scottish point of view:

> "The complexity of an economy stands in sharp contrast to the simplicity of a question that must be raised about its operation. Many agents compose the economy, and they have to deal with a large number of commodities. Each one of those agents makes decisions about the quantity of each one of those commodities that he will produce or consume: the number of variables involved is the product of the number of agents and the number of commodities. Moreover, in this decision-making process the agents act independently of each other, and they are guided by self-interest. Why is high disorder not the result? The agents of an economy are counted in millions, if not billions. The number of commodities is similarly large. The self-interests of the independent decision-makers are sometimes in agreement, sometimes in conflict. Why does one not observe for every commodity a large excess of demand, evidenced for instance, by lengthy waiting times for orders to be filled, or a large excess of supply over demand, evidenced, for instance, by massive inventories? Agents no longer make independent decisions, and they interact with each other, if there are markets for commodities. Their interaction then reduces the difference between demand and supply." (Debreu 1998: 10)

Today, a representative economics student learns that the precise manifestation of Smith's metaphor is the First Fundamental Theorem of Welfare Economics: for no more assumptions than necessary and sufficient to ensure existence, competitive equilibria decentralize Pareto-efficient allocations. Eventually, he is told, economists have succeeded in finding "a mathematical statement of Adam Smith's notion of an invisible hand" (Starr 1997: 146; see also Arrow and Hahn 1971: 3). Since the theorem is the first and also fundamental, the student might think, the invisible hand must be really important. To him, the invocation of the metaphor seems to be a convenient way to introduce a voluminous idea prior to the tools that are necessary to command the more demanding levels of inquiry. He easily gets the impression that there is some precise and uncontroversial idea of what the invisible hand is about. However, our student is also warned that the operation of the invisible hand is conditional. Market power and externalities are introduced as major impediments to the working of the price system. It may occur to him early that some economists consider the assumptions behind the invisible-hand proposition to be more restrictive than others. On average, however, economists still reckon market failures as exceptions to the rule, even though they hold strategic behavior and missing markets liable for serious welfare problems.

1.2 Plan of the Study

The present study raises the following questions: To what extent is axiomatic general equilibrium analysis a rational reconstruction of 'Scottish Political Economy' as defined by the writings of David Hume and Adam Smith? How much is gained and how much lost by the axiomatic transformation of the invisible-hand proposition? What are the implications of negative results like the Sonnenschein-Mantel-Debreu demonstrations for the Scottish point of view? Did it reach deadlock, or is there still hope for the dominant trajectory in the history of economics? In contrast to the rich historical literature on the invisible-hand proposition, the present study does not level any paradigmatic criticism at neo-Walrasian analysis. Rather, by focalizing the most important results against the backdrop of Scottish Political Economy, it provides some flesh to the bones of axiomatic economics and, insofar, may inform theory choice within the neo-Walrasian paradigm. Naturally, the answers to the questions raised are complex and do not fit into an abstract. Instead, the reader is referred to the final section, which lists, interrelates, and discusses the major results of the study.

Section 2 outlines the Scottish invisible-hand proposition, and places it into historical context. Section 2.1 introduces the contribution of Bernard Mandeville. Section 2.2 delineates 'Scottish Political Economy' by focusing on the writings of David Hume.

To show that the classical models are inferior hosts of the invisible-hand proposition, section 2.3 provides a simple Smithian model. Section 3 discusses the axiomatic transformation of the invisible-hand proposition against the backdrop of Scottish Political Economy. To structure and clarify the subsequent analysis, section 3.1 introduces and extends Roy Weintraub's portrayal of the 'neo-Walrasian program' according to Lakatosian standards. Section 3.2 discusses the axiomatic invisible-hand proposition and, thus, the class of Arrow-Debreu-McKenzie models. Section 3.3 focuses on aggregation problems as indicated by the Sonnenschein-Mantel-Debreu results. Rational expectations equilibria are discussed with respect to the Grossman-Stiglitz results. Section 4 provides the concluding discussion.

Chapter 2

The Invisible-Hand Proposition: Origin and Context

To escape the narrow and, insofar, sterile reading of the invisible-hand proposition that has become common in doctrinal histories of general equilibrium analysis, I restate Smith's argument within the broader context of the Scottish Enlightenment. The major premiss in what follows is that the broadest and most ambitious interpretation of the metaphor is also the most productive, independent of Smith's own use of the term. Consequently, my history of the invisible-hand proposition starts before Smith. It begins with Bernard Mandeville, the Dutch philosopher-physician who migrated to London in 1693, and whose writings later became - in one way or another - a major influence on Scottish moral philosophy and political economy.

2.1 The Heresies of Bernard Mandeville

In 1705, Bernard Mandeville bootstrapped a then disconcerting perspective on social organization in *The Grumbling Hive: Or, Knaves Turn'D Honest*, a biting poem which he anonymously republished in 1714 under the title *The Fable of the Bees: Or, Private Vices, Publick Benefits*, complemented by a commentary section and a series of remarks (Mandeville 1988). The title indicates his heretic proposition: a buoyant, or 'vibrant', society is neither sustained by the volition and prudence of kings, nor by godly virtues being enforced on their subjects, but by commercial activity, motivated by instinctive self-love and worldly needs. Even though the details of Mandeville's outline (including comments and remarks) significantly differ from our present interpretation of the invisible-hand proposition, and naturally so, the poem gives expression to Mandeville's intuition of bottom-up coordination. It adumbrates the self-organization of individuals which, because of their limited capacity for sympathy and reason, are subject to rather independent laws of motion. And it suggests that it is the exchange-nexus that ultimately ties large societies.

The diffusion of Mandeville's 'vision' (as defined by Schumpeter 1954) of decentralized coordination took a while. But eventually in 1723, the publication of the *Fable*'s second edition triggered expectable reactions and evoked a public scandal (for an overview, see Hayek 1991b). To the extent that invisible-hand studies constitute the dominant trajectory in the history of economics, at least until general equilibrium analysis ran into negative results and diminishing returns, the publication of the *Fable* must be regarded as the big-bang event for that very tradition. One way to distill the basic characteristics of Mandeville's vision and, as a corollary, to evaluate its historical significance for mainline economics, is to confront his glimpse of decentralized coordination with a stylized reconstruction of the then prevailing 'Zeitgeist' (as will become evident, the short historical outline on the following pages is mostly inspired by David Hume's *The History of England* (1826). See also Streminger 1994).

With the exception of the Highlands in the north of Scotland (dominated by a clan system and collective property rights), the Kingdom of Great Britain is a feudal system of continental origin (a game-changing inheritance from the Norman conquest in 1066) and, as such, characterized by extractive political and economic institutions. The king, of course, is the focal point of the feudal society. Before the Glorious Revolution in 1688-89, the 'divine-right theory of kingship' as propagated by the clergy and enforced by the worrier nobility, granted the exclusive ownership of economic resources to the king. According to the feudal maxim "No man without a lord! No land without a lord!", subsistence in serfdom was accepted as the natural lot of the 'unprivileged many'. The feudal state was primarily a system of taxation, a vertical hierarchy purposefully designed to extract as much surplus as possible (surplus defined as a residual according to the Classical School). And it prevailed long enough to incubate a common believe in the legitimacy of such exquisite privileges.

The doctrine of divine rights could gain acceptance beyond the minority of beneficiaries, that is, beyond the 'extraordinary' members of society, also because it was derived from the much older doctrine of Original Sin (or rather from varieties thereof). Catholics and Protestants alike believed that the 'fall of man' - Adam and Eve's violation of the original contract between God and mankind and, consequently, their expulsion from Paradise - constitutes a collective guilt of humankind and its everlasting depravity. According to the English standard version of the bible, "They have *all* turned aside; *together* they have become corrupt; *there is none who does good, not even one*" (Psalm 14.3). In the eyes of his Lord, man is spoiled and tainted. Divine acts of mercy, he can only expect for afterlife. To earn it, he must submit himself to comprehensive rules of conduct, which were designed to curtail instincts and selfish emotions. The individual's pursuit of happiness was neither a legitimate end, nor a feasible mean to secure divine reward. Rather, the submission of individual laws of motion to rigorous self-discipline is a necessary condition to escape eternal damnation (divine mercy - the sufficiency condition - cannot be enforced by a 'good' record only; human behavior cannot lay claim to such a reward). Consequently, the moral codex expelled from the definition of 'virtue' those activities that were seen to be motivated by mundane self-interest. Man is able to rise above the animal to the image of God by self-denial only. His self-interest is merely a potential source of sin. Even though commercial activity, especially consumption, was not generically deemed vicious (for evident reasons), it was regarded as a quite risky path to divine reward. After all, economic ambition - especially the desire to rise above each other - is a slippery slope to 'usury' and 'greed'. The threat of being accused as such imposed more or less narrow constraints on economic choice, depending on the respective Zeitgeist.

To all this, the divine-right doctrine adds that, by the unfathomable will of God, a few supreme bloodlines (a sequence of dynasties) are appointed or predestined to rule over their fellow specimen (since Henry VIII, of course, such appointments were made independent of Rome). Such legitimate holders of absolute power, immunities, and exclusive privileges are commissioned to adapt the rules of 'good' conduct in light of external circumstances, and to enforce them upon the inferior members of society ("by the grace of God" as Henry I. laid down in the Charter of Liberties in 1100). Any constraint on the Sovereign's discretion is *ipso facto* a constraint on God's will. His faculty of reason is equally of divine origin and able to identify eternal and universal truth and regularities in worldly phenomena. Accordingly, it is neither warranted, nor legitimate to submit the monarch to the same laws and conventions that he drafts for his subjects. The king stands above the law. Social organization is the result of his purposeful design, the manifestation of his deliberate volitions. Most importantly, given that human nature is depraved, the common good is served best, if such societal designs disregard or even suspend individual laws of motion. Like sheep, individuals were supposed to execute the will of their noble shepherds, to fully submit to the control of the authority. From their occupation to the color of their clothes, the ordinary many made predetermined choices. Unsurprisingly, the 'good society' was broadly considered to be rather static, coordinated top-down by the quite conspicuous hand of the Sovereign.

The feudal system proved robust to changing beliefs, foremost to the continuation of religious doctrines and consequent patterns of conduct. For the late feudal system in the seventeenth century, the English philosopher Thomas Hobbes was even able to derive the wretchedness of man (*homo homini lupus est*), and the legitimacy of the feudal system (the social contract) from a purely rationalist perspective (Hobbes 2010). The grim vision of the Leviathan as a mean to enforce the unconstrained discretion of the monarch became so natural under the prolonged reign of absolutism that Adam Smith, seven decades after the Glorious Revolution, still felt compelled to retort with one of his most famous statements:

> "The man of system [like Hobbes, as opposed to the someone "whose public spirit is prompted altogether by humanity and benevolence" and who "will respect the established powers and privileges even of individuals"], is apt to be very wise in his own conceit, and is often so enamoured with the supposed beauty of his own ideal plan of government, that he cannot suffer the smallest deviation from any part of it. He goes on to establish it completely and in all its parts [...]: he seems to imagine that he can arrange the different members of a great society with as much ease as the hand arranges the different pieces upon a chess-board; he does not consider that the pieces upon the chess-board have no other principle of motion besides that which the hand impresses upon them; but that, in the great chess-board of human society, every single piece has a principle of motion of its own, altogether different from that which the legislature might choose to impress upon it. If those two principles coincide and act in the same direction, the game of human society will go on easily and harmoniously, and is very likely to be happy and successful. If they are opposite or different, the game will go on miserably, and the society must be at all times in the highest degree of disorder." (Smith 2000: 342-3)

Of course, the control over the pieces upon the chess-board of human society is always incomplete. In real life, power is never absolute. To sustain a smooth tax revenue, the 'man of system' had to rely on those, who actually enforced the extraction of economic surplus. In fact, the Parliament of England, the birthplace of individual liberty *through* constitutional law (Hayek 2006), was initially designed to coordinate taxation. It was a council composed of tenants-in-chief, those who sustained an army on behalf of the king and generated additional revenue in return for privileges such as land tenure (which explains the name). Murray Rothbard would call it a 'gang of thieves'. The feudal system, the collusion of a small set of individuals to implement extractive economic and political institutions upon their complement, was based on the perception of such mutual benefits. On one side the monarch, who exclusively owns and legitimizes, but who cannot make full use of his property and is in need of an executive council (a government); on the other side of the bargain the worrier nobility that could indeed use labor and land to extract revenues, but neither owns any of it, nor can it invoke much legitimacy (which was crucial to sustain the unequal distribution of political power and economic benefits at relatively low costs).

In this trade, the most saleable service of the monarch was his grant of privilege. As monopolist supplier of privilege, and in full control of its distribution, he dominated the reward system and, thus, the distribution of income. Centered on the monarch, a rather complex rent-seeking system emerged - the court - as the result of a discontinuous bargaining process that was driven by Machiavellian-type strategies and 'court intrigue'. Naturally, the most refined privilege-seeking society was located in Versailles, the focal point of the purest absolutist system in medieval history. The first major discontinuities in this prolonged struggle between monarchy and nobles (much later involving non-noble merchants) occurred with the Charter of Liberties in 1100 and, of course, with the Magna Charta in 1215, which granted to the council of nobles the monopoly as supplier of extracted surplus, to be the king's exclusive tax collector (the monarchy, however, could win back ground in subsequent 'bargains', e.g., the First Barons' War).

Whatever it is that makes the Charter of Liberties or the Magna Charta so special to the 'Western society', it played no role for any party in this trade. Principled reasoning that led to the (re-)discovery of percepts like the 'rule of law' and 'government of law', and to the emergence of institutional arrangements like the constitutional monarchy with the (two houses of) English Parliament as legislature, was primarily motivated by the prospect of higher rents (and status). The occasion of the Magna Charta as a mean to withdraw the monarch's outside options exemplifies the general practice to generate rents by arrangements that lever out competition. More often, however, such bargains imposed most of the costs on third parties, on the army of unprivileged subjects who perceived the bargaining process as an endless series of negative externalities. Only with the benefit of hindsight was it possible for a generation of Scottish thinkers to identify the joint products of this process, e.g., the Bill of Rights in 1689 or the system of 'checks and balances' in general, as necessary ingredients to what they referred to as the 'Great Society'.

This, then, was the dominant worldview of Mandeville's contemporaries in 1714 (and 1723), still more than three decades before the Scottish Enlightenment, just at the beginning of five decades of 'Whig supremacy': there exists a natural order with a strictly vertical hierarchy by which individual laws of motion are tamed for the common good, subordinated to a single end, and buried under tax obligations; the division of labor is fixed by a master plan, the distribution of income controlled by the distribution of rank and authority. According to this worldview, the undesigned interplay of erring individuals whose self-serving discretion is only constrained by their respective budgets, and who are free to choose their occupation, implies disorder, or even outright social chaos.

Mandeville proclaimed the polar opposite. Perhaps because he stemmed from the Dutch Republic at the end of its golden age, where incomes were not extracted but *predominantly* earned in international trade, his *Fable of the Bees* makes the case for bottom-up coordination and commerce as the predominant social kit of large societies. By analogy of a hive, which in terms of humor and wit easily keeps up with the writings of David Hume and Adam Smith, Mandeville makes accessible the apparently paradoxical (conditional) proposition that the non-coercive interplay of 'individual principles of motion' finds expression in ordered and even benign aggregate outcomes (in terms of "Luxury and Ease"). It is the first statement of the invisible-hand proposition.

Long before French Physiocrats gathered around Quesnay's *Tableau économique*, which is a crude exemplification of the circular flow (published in 1758), Mandeville used the analogy of the bee-society to bootstrap a proto-type model of an exchange economy, still the nucleus of axiomatic general equilibrium reasoning (in the sense that exchange economies already contain the gist of ADM-models so that production is "inessential'). He confronts the long established faith in the predominantly hierarchically organized society with the idea of a *heterarchy*: the agents of his model, the bees, are driven by instinctive self-love, motivated by lower needs, and guided by a minimum of reason. Nevertheless, the relation of the bees to one another is unranked, or at least admits multiple potential orders. A thriving hive, so Mandeville's thesis, does not depend on exceptional individual capabilities, neither on those of the ordinary many, nor on those of the 'chosen few' (note that his contemporaries identified hives with hierarchy, not with swarm intelligence as we do).

Social interaction is introduced by exchange relations only, in the following referred to as the 'exchange nexus'. In short, he introduces *The Grumbling Hive* as a 'back-scratching economy' (Mandeville 1988: 65-75):

> Vast Numbers throng'd the fruitful Hive;
> Yet those vast Numbers made 'em thrive;
> Millions endeavouring to supply
> Each other's Lust and Vanity. (ibid.: 66)

It is remarkable that already the first incidence of the invisible-hand proposition involved the notion of largeness, and prominently so. Today, of course, we know that a strictly 'unranked' society is composed of a continuum of agents. Also note that Mandeville assumes a causal relation between largeness and prosperity, as did Smith more than a half a century later. In fact, it was Mandeville, not Smith, who introduces the term "division of labour" as a characteristic property of economic growth in commercial societies. Individual rationality translates into collective rationality via an ever increasing partitioning of economic activity (ibid.: 141-3).

In arguing that social interdependence is closely intertwined with the individual "Business of Self-Preservation", Mandeville even anticipated Lionel Robbins's scarcity definition of economics:

> "[T]he Sociableness of Man arises only from these Two things, viz. the multiplicity of his Desires, and the continual Opposition he meets with in his Endeavours to gratify them." (ibid.: 218)

Add to this statement, and to the preceding remarks on Mandeville's back-scratching economy, the notions of choice and opportunity costs, and we arrive at:

> "The ends are various. The time and the means for achieving these ends are at once limited and capable of alternative application. Here we are, sentient creatures with bundles of desires and aspirations, with masses of instinctive tendencies all urging us in different ways to action. But the time in which these tendencies can be expressed is limited. The external world does not offer full opportunities for their complete achievement. Life is short. Nature is niggardly. Our fellows have other objectives." (Robbins 1932: 13)

Note that the above citation from Mandeville is not taken from the poem (i.e., the model), but from his subsequent remarks. His assertion that the "Sociableness of Man" *only* arises from trade incentives, implied by the concurrence of multiple ends and limited means (with alternative uses), is no simplifying restriction devised to isolate exchange relations in his back-scratching bee-economy. Rather, it was intended as a statement about the world he lived in, about what he saw 'by looking out of the window' (a second phrase borrowed from the GMU oral tradition).

In hindsight, and in due consideration of the context in which Mandeville formulates the term 'vice', we can easily identify *The Grumbling Hive* as an embryonic statement of the invisible-hand proposition:

> Thus every Part was full of Vice,
> Yet the whole Mass a Paradise;
> Flatter'd in Peace, and fear'd in Wars,
> They were th' Esteem of Foreigners,
> And lavish of their Wealth and Lives,
> The Balance of all other Hives.

Such were the Blessings of that State;
Their Crimes conspir'd to make them a Great:
And Virtue, who from Politicks
Had learn'd a Thousand Cunning Tricks,
Was, by their happy Influence,
Made Friends with Vice: And ever since,
The worst of all the Multitude
Did something for the Common Good.
This was the States Craft, that maintain'd
The Whole of which each Part complain'd:
This, as in Musick Harmony,
Made Jarrings in the main agree. (ibid.: 69)

Thus Vice nurs'd Ingenuity,
Which join'd with Time and Industry,
Had carry'd Life's Conveniencies
It's real Pleasures, Comforts, Ease,
To such a Height, the very Poor
Liv'd better than the Rich before,
And nothing could be added more. (ibid.: 70)

Thus, "strange ridic'lous Vice, was made the very Wheel that turn'd the Trade" (ibid.: 69). It is beyond doubt that this first formulation of the invisible-hand theme is also the most witty and beautiful. And it is packed with content. Most importantly, already in Mandeville's *Fable* we find a notion of general equilibrium that specifies the meaning of the "Common Good". The conflict of interest that is implied by the scarcity problem, intimated by the agents' "jarrings", is peacefully dissolved by the exchange nexus such that individual laws of motion are "mainly" attuned "as in Musick Harmony". Instead of the metaphor of an invisible hand, Mandeville invokes that of an orchestra: individual jarrings add up to a symphony 'as if' they were orchestrated.

Further, Mandeville makes a conditional proposition: private vices aggregate to public benefits, *only if* "Virtue", by which he insinuates the societal rule book, makes "Friends with Vice". Rephrased according to Smith's terminology: for the multiple individual principles of motion to attune 'as if' they were directed by an invisible hand like figures on a chess board, the rules of conduct impressed upon them by the legislature must concord. For institutional configurations to concord, they must respect the individual's impulse. It is a necessary condition: the effective operation of the invisible hand implies a 'proper' institutional configurations. The 'rule of law' and 'government of law' are introduced explicitly:

> No Bees had better Government,
> More Fickleness, or less Content:
> They were not Slaves to Tyranny,
> Nor rul'd by wild Democracy;
> But Kings, that could not wrong, because
> Their Power was circumscrib'd by Laws. (ibid.: 66)

Clearly, abstract rules that constrain the bee queen's discretion are introduced as beneficial. Rosenberg argues that for Mandeville the proper role of government is "to establish the rules of the game by the creation of a framework of wise laws" (Rosenberg 1963: 190) such that "arbitrary exertions of government power would be minimized" (ibid.: 193).

Finally, just before the Scottish Enlightenment, Mandeville suggests that the wisdom expressed by such laws is nowhere pooled and beyond any individual's reach. Instead, he hints at an adaptive and anonymous selection process: "Virtue", that is, the comprehensive rule book, "*learn'd* a Thousand Cunning Tricks" in the business of "Politicks". The system's intelligence is *impersonal* (This explains Hayek's deep admiration for Mandeville).

Mandeville not only established 'proper' institutions as a necessary condition for individual impulses to attune 'as if' they were orchestrated, his 'model' also attempts to 'prove' sufficiency ('proper institutions exist' implies 'the invisible-hand proposition applies') by contraposition ('proper institutions do not exist' implies 'the invisible-hand proposition does not apply'). To do so, Mandeville introduces "Jove", the Lord of the bees, whom he makes listen to some bee-equivalents of John Knox, Scotland's most fervent supplicant (known as the 'killer of joy' as Maria Stuart could tell), who impudently pray for a virtuous society:

> [...] *Jove* with Indignation mov'd,
> At last in Anger swore, *He'd rid*
> *The bawling Hive of Fraud;* and did.
> The very Moment it departs,
> And Honesty fills all their Hearts. (ibid.: 70)

The results are devastating:

But, Oh ye Gods! What Consternation,
How vast and sudden was th' Alteration!
In half an Hour, the Nation round,
Meat fell a Peny in the Pound.
The Mask Hypocrisy's flung down,
From the great Statesman to the Clown:
And some in borrow'd Looks well known,
Appear'd like Strangers in their own.
The Bar was silent from that Day;
For now the willing Debtors pay. [...]

Now mind the glorious Hive, and see
How Honesty and Trade agree.
The Shew is gone, it thins apace;
And looks with quite another Face.
For 'twas not only that They went,
By whom vast Sums were Yearly spent;
But Multitudes that liv'd on them,
Were daily forc'd to do the same.
In vain to other Trades they'd fly;
All were o'er-stock'd accordingly. [...]

As Pride and Luxury decrease,
So by degrees they leave the Seas.
Not Merchants now, but Companies
Remove whole Manufactories.
All Arts and Crafts neglected lie;
Content, the Bane of Industry,
Makes 'em admire their homely Store,
And neither seek nor covet more.
So few in the vast Hive remain,
The hundredth Part they can't maintain
Against th' Insults of numerous Foes;
Whom yet they valiantly oppose. (ibid.: 71)

A commercial society cannot be sustained, if individuals do not primarily act on self-interest, but follow virtuous motives instead. An individual unmotivated by self-interest is indifferent to commercial or 'material' reward and, thus, indifferent to the 'material' needs of his fellow specimen. Mandeville effectively argues that the strict and universal enforcement of the then prevailing moral standards implies economic regression, described by cumulative income effects that dissolve the exchange-nexus and curtail the division of labor. Hence, the moral:

> Then leave Complaints: Fools only strive
> To make a Great an Honest Hive
> T' enjoy the World's Conveniencies,
> Be fam'd in War, yet live in Ease,
> Without great Vices, is a vain
> Eutopia seated in the Brain.
> Fraud, Luxury and Pride must live,
> While we the Benefits receive:
> Hunger's a dreadful Plague, no doubt,
> Yet who digests or thrives without?
> Do we not owe the Growth of Wine
> To the dry shabby crooked Vine? [...]
>
> So Vice is beneficial found,
> When it's by Justice lopt and bound;
> Nay, where the People would be great,
> As necessary to the State,
> As Hunger is to make 'em eat.
> Bare Virtue can't make Nations live
> In Splendor; they, that would revive
> A Golden Age, must be as free,
> For Acorns, as for Honesty. (ibid.: 74-5)

Mandeville's embryonic invisible-hand proposition can now be stated in full: trade plans of predominantly self-interested individuals reconcile 'as if' they were attuned by an invisible hand, *if and only if* institutional configurations concord with individual laws of motion.

Given the prevailing Zeitgeist, it comes as no surprise that Mandeville saw himself confronted by "philosophical hysterics" (Kaye in Mandeville 1988: 21), being treated like a "moral monster" (Hayek 1991b: 82). Also not surprisingly, John Maynard Keynes quotes the *Fable* approvingly as an example of mercantilist economic thought (Keynes 1964: 359-62). Just think of the 'silent bar' due to the austerity of debtors. Mandeville indeed classifies the postponement of consumption, that is, the act of saving, as virtuous, as an instance of the triumph of godly reason over passions. He makes no exception to his rule and argues that such prudence aggregates to economic decline.

2.2 David Hume and Scottish Political Economy

The towering figure of the Scottish Enlightenment is David Hume. According to Hayek, "it is only in Hume's work that the significance of Mandeville's efforts becomes wholly clear, and it was through Hume that he exercised his most lasting influence" (Hayek 1991b: 96). Hume's *Treatise of Human Nature - An ATTEMPT to introduce the experimental Method of Reasoning into Moral Subjects* is one of the most significant and influential contributions to Western Philosophy, despite its disappointing short-run performance. The first two volumes, *Of the Understanding* and *Of the Passions*, were published in 1739; the third, *Of Morals*, followed in 1740. His favorite work, however, was the *Enquiry concerning the Principles of Morals*, published in 1751, in which he consolidated the third book of the *Treatise* to a smoother and more compact reading. The subsequent elaborations are largely built on these two works, with few additional references to Hume's *Political Discourses* (1752).[1]

The individual scarcity problem and the consequent social conflict of interest - as introduced in the previous section on Mandeville - also lies at the heart of Scottish Political Economy. Hume declined the Calvinist dictum that human 'depravity is always total' such that 'there is no such thing as a half rotten man'; he refused to evaluate human nature from the perspective of some speculative higher being. Instead, he relied on the bare observation that

> "Of all the animals, with which this globe is peopled, there is none towards whom nature seems, at first sight, to have exercised more cruelty than towards man, in the numberless wants and necessities, with which she has loaded him, and in the slender means, which she affords to the relieving these necessities. In other creatures these two particulars generally compensate each other. [...] In man alone, this unnatural conjunction of infirmity, and of necessity, may be observed in its greatest perfection. Not only the food, which is required for his sustenance, flies his search and approach, or at least requires his labour to be produced, but he must be possessed of cloaths and lodging, to defend him against the injuries of the weather; though to consider him only in himself, he is provided neither with arms, nor force, nor other natural abilities, which are in any degree answerable to so many necessities." (Hume 2000: 311-2)

[1] See Wennerlind and Schabas 2008 for further aspects of Hume's Political Economy.

The human mind is constantly generating more needs than we have means to gratify them; more images of improvement than solutions of how to implement them. In modern parlance, individuals are (at least locally) *insatiable*. It is the imbalance between excessive needs and meager endowments that constitutes the individual's law of motion, the impulse being potentially increasing in the degree of individual disequilibrium.

However, where the Christian tradition saw collective guilt, Hume just saw a problem to be solved, free of normative or even metaphysical speculation (the latter in contrast to Hobbes). He realized that while individuals in natural balance (man or other animals) can prevail on their own, individuals in inherent disequilibrium can only survive by the support of others, that is, by means of resources which they either 'earn' in cooperation, or extract by credible threats of coercion, in addition to what they can obtain as gifts, or by fraud. His focus, of course, is on cooperation:

> "It is by society alone he is able to supply his defects, and raise himself up to an equality with his fellow-creatures, and even acquire a superiority above them. By society all his infirmities are compensated; and though in that situation his wants multiply every moment upon him, yet his abilities are still more augmented, and leave him in every respect more satisfied and happy, than it is possible for him, in his savage and solitary condition, ever to become." (ibid.: 312)

Against Hobbes, who claimed that the faculty of reason enables man to submit (by means of a social contract) his 'brutish' interests and 'evil' passions to the coercive force of a Leviathan, Hume argued that the faculty of reason is sufficient to reveal the necessity of social interaction, and to ensure that individuals find self-interest in submitting their instincts and passions to general rules of conduct. Like John Locke, he claims that humans are social animals, and that society precedes the state. Instead of the Leviathan as safeguard to the orchestration of individual conduct according to the 'Grande Design' of a supreme leader (supremacy in terms of reason), Hume envisions voluntary adaptions of moral standards, customs, and other conventions, predominantly motivated by selfish emotions, and guided by rational insight in the inescapable dependence on others.

While reason is introduced as a necessary condition for beneficial social and legal norms to arise, it is dismissed as sufficiency condition. The individual's impulse, so Hume, is by necessity "an impulse of passions" which cannot be effectively countered by the faculty of reason. Rather, reason is introduced as a capability to identify means to satisfy quite unreasonable ends, and as systematically 'practical'. Hobbes's dark utopia is infeasible right because self-interest and reason tend to align. "Reason is, and ought only to be the slave of the passions, and can never pretend to any other office than to serve and obey them." Once in perfect balance, man's faculty of reason becomes an idle tool that by itself "can never produce any action, or give rise to volition". It *therefore* fails as a "contrary impulse" to the individual's law of motion (all citations taken from Hume 2000: 312).

Hume's notion of self-interest, however, is much broader than Mandeville's, including beyond selfishness also the passions *of* others. In fact, he introduced 'sympathy' - defined as the inherent ability to resonate with each other on a purely affective basis - as an essential glue to the social fabric:

> "No quality of human nature is more remarkable, both in itself *and in its consequences*, than that propensity we have to sympathize with others, and to receive by communication their inclinations and sentiments, however different from, or even contrary to our own." (Hume 2000: 206; emphasis mine)

Most importantly, sympathy as "communication of sentiments" is introduced as prerequisite for a *social reward system* and, hence, as a necessary condition for social and legal norms to prevail (ibid: 210-1). Individuals (as social animals) seek for benevolence, praise, and accolade; and they eschew disapproval and social expulsion. To prevail in the eyes of others (or to evade the costs of insubordination), they *tend* to conform and, thereby, to stabilize the societal rule book.

Altruism as the passion *for* others is introduced as an effective survival strategy: "by mutual succour we are less expos'd to fortune and accidents" (ibid: 312). Hume even argues that altruism *typically* dominates selfishness as driving force of human conduct:

> "I am sensible, that generally speaking, the representations of [selfishness] have been carried much too far; and that the descriptions, which certain philosophers delight so much to form of mankind in this particular, are as wide of nature as any accounts of monsters, which we meet with in fables and romances. So far from thinking, that men have no affection for any thing beyond themselves, I am of opinion, that though it be rare to meet with one, who loves any single person better than himself; yet it is as rare to meet with one, in whom all the kind affections, taken together, do not overbalance all the selfish. Consult common experience: Do you not see, that though the whole expence of the family be generally under the direction of the master of it, yet there are few that do not bestow the largest part of their fortunes on the pleasures of their wives, and the education of their children, reserving the smallest portion for their own proper use and

entertainment This is what we may observe concerning such as have those endearing ties; and may presume, that the case would be the same with others, were they placed in a like situation." (2000: 312-3)

Note, however, that Hume suggests an important qualification. As it is rare to find someone, who is not concerned for the welfare of his family (or tribe), it is rare to find someone, who extends such generosity to the multitude of strangers. Altruism, so Hume, is locally bounded. Due to such "limited generosity", the "contrariety of passions" cannot be resolved by the appeal to our "softer affections" (ibid: 313). If it were so, social and legal norms would indeed be completely redundant:

> "[Human conventions] are intended as a remedy to some inconveniences, which proceed from the concurrence of certain qualities of the human mind with the situation of external objects. The qualities of the mind are selfishness and limited generosity: And the situation of external objects is their easy change, joined to their scarcity in comparison of the wants and desires of men. [...] [I]f every man had a tender regard for another, or if nature supplied abundantly all our wants and desires, that the jealousy of interest, which justice supposes, could no longer have place; nor would there be any occasion for those distinctions and limits of property and possession, which at present are in use among mankind. Encrease to a sufficient degree the benevolence of men, or the bounty of nature, and you render justice useless, by supplying its place with much nobler virtues, and more valuable blessings. The selfishness of men is animated by the few possessions we have, in proportion to our wants; and it is to restrain this selfishness, that men have been obliged to separate themselves from the community, and to distinguish betwixt their own goods and those of others." (Hume 2000: 317)

Thus, although Hume considered altruistic behavior and, in consequence, individual "safety" to be necessary for social progress to materialize in "large societies",[2] he explicitly sides with Mandeville and forcefully argues that altruism is not sufficient, and that social progress must predominantly be based on selfishness:

> "It is certain, that no affection of the human mind has both a sufficient force, and a proper direction to counterbalance the love of gain, and render men fit members of society, by making them abstain from the possessions of

[2] Such safety, so Hume, is important to cushion the impact of unemployment in case of highly specialized individuals who are dismissed because of economic change.

others. Benevolence to strangers is too weak for this purpose; and as to the other passions, they rather inflame this avidity, when we observe, that the larger our possessions are, the more ability we have of gratifying all our appetites. There is no passion, therefore, capable of controlling the interested affection, but the very affection itself, by an alteration of its direction. Now this alteration must necessarily take place upon the least reflection; since it is evident, that the passion is much better satisfied by its restraint, than by its liberty, and that in preserving society, we make much greater advances in the acquiring possessions, than in the solitary and forlorn condition, which must follow upon violence and an universal licence. The question, therefore, concerning the wickedness or goodness of human nature, enters not in the least into that other question concerning the origin of society; nor is there any thing to be considered but the degrees of men's sagacity or folly. For whether the passion of self-interest be esteemed vicious or virtuous, it is all a case; since itself alone restrains it: So that if it be virtuous, men become social by their virtue; if vicious, their vice has the same effect." (ibid. 316)

By means of a thought experiment, he exemplifies the *ultima ratio* of social conventions like 'justice' and 'property' as (more or less effective) partial solutions to the generic problem of how to resolve the conflict of interest (as implied by individual imbalances) *at once* with the opposition of passions (as implied by constrained altruism):

"Let us suppose, that nature has bestowed on the human race such profuse abundance of all external conveniences, that, without any uncertainty in the event, without any care or industry on our part, every individual finds himself fully provided with whatever his most voracious appetites can want, or luxurious imagination wish or desire. His natural beauty, we shall suppose, surpasses all acquired ornaments: The perpetual clemency of the seasons renders useless all cloaths or covering: The raw herbage affords him the most delicious fare; the clear fountain, the richest beverage. No laborious occupation required: No tillage: No navigation. Music, poetry, and contemplation form his sole business: Conversation, mirth, and friendship his sole amusement.

It seems evident, that, in such a happy state, every other social virtue would flourish, and receive tenfold encrease; but the cautious, jealous virtue of justice would never once have been dreamed of. For what purpose make a partition of goods, where every one has already more than enough? Why give rise to property, where there cannot possibly be any injury? Why call this object mine, when, upon the seizing of it by another, I need but stretch out my hand to possess myself of what is equally valuable? Justice, in that case, being totally USELESS, would be an idle ceremonial, and could never possibly have place in the catalogue of virtues." (Hume 2006: 33-4; his emphasis)

It is therefore Hume, who introduces property rights as an effective tool to internalize the conflict of interest, and to appease the opposition of selfish emotions. It is in the self-interest of each individual that the property he accumulates (to ease his imbalance) is shielded by society against coercive invasion, that his *exclusive* entitlement to income or wealth is either accepted or enforced. *In return* for such protection, he has to constrain his discretion by submitting himself to the same rules of the game; he has to tolerate that his set of *admissible* wealth-seeking methods is equally restricted to the subset that is subject to the *quid pro quo condition* (or the *do ut des* principle), by which the exchange nexus is prominently introduced in Hume's analysis. Common rules of conduct arise and prevail due to such individual cost-benefit analyses. Of course, the most prominent formulation of 'quid pro quo' as a necessary condition for potentially conflicting interests to attune is not Hume's, but that of Adam Smith:

> "[M]an has almost constant occasion for the help of his brethren, and it is in vain for him to expect it from their benevolence only. He will be more likely to prevail if he can interest their self-love in his favour, and show them that it is for their own advantage to do for him what he requires of them. Whoever offers to another a bargain of any kind, proposes to do this. *Give me that which I want, and you shall have this which you want*, is the meaning of every such offer; and it is in this manner that we obtain from one another the far greater part of those good offices which we stand in need of. It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our own necessities but of their advantages." (Smith 2003: 23-4; emphasis mine)

Against John Locke and the teleological natural right tradition (including the ancient tradition), Hume emphasized that social and legal norms are human "*artifacts*", that is, the result of purposeful problem-solving activity:

> "Those rules, by which properties, rights, and obligations are determined, have in them no marks of a natural origin but many of artifice and contrivance. They are too numerous to have proceeded from nature: They are changeable by human laws: And have all of them a *direct and evident* tendency to public good, and the support, of civil society. This last circumstance is remarkable upon two accounts. First, because, though the cause of the establishment of these laws had been a regard for the public good, as much as the public good is their natural tendency, they would still have been artificial, as being purposely contrived and directed to a certain end. Secondly, because, if men had been endowed with such a strong regard for public good, they would never have restrained themselves by these rules; so that the laws of justice arise from natural principles in a manner still more oblique and artificial." (Hume 2000: 339)

Fortunately, Hume did not stop at this point, although his analysis so far deserves highest credits already. Recall that he introduced conventions as partial solutions to the myriads of problems associated with the conflict of interest. So what problems does the convention of property rights actually solve? As part of the Scottish Enlightenment, he refused to derive the purpose of social and legal norms from some holy book, or from metaphysical reasoning. As an empiricist, he looked out of the window instead, systematically searching for impediments to *social* progress, for 'frictions' that account for social dysfunctionalities. Once a set of problems was identified, social and legal norms were evaluated according to their ability to (partially) neutralize such frictions. Evidently, Hume is the pioneer of 'new' institutional economics, most visibly in the works of Harold Demsetz (e.g. Demsetz 1967) and Douglass North (e.g. North 1990).

With this 'research program' in mind, Hume introduced property rights as a social technology that governs the distribution of power, control, and privilege in violent contrast to the feudal distribution 'mechanisms'. Given that the quid pro quo condition generally applies - due to the acceptance or effective enforcement of property rights -, wealth constraints ensure that power and control over scarce resources is only obtained by the consent of other members of society, and that privilege is always temporary. A decentralized and, thus, impersonal system regulates that those who rise, cannot systematically do so at the costs of others, but only by continually supplying their needs. Thus, property rights are (inter alia) a rational response to the problem of 'Why only the worst rise to top?' (inherent to the feudal system).

The general acceptance of property rights, even though sustained by individual long-run interests and sheer habit alone, suggests beneficial, yet unintended social consequences: a system of 'checks and balances' - enforced by budget constraints - puts the individual's law of motion to the service of strangers, and therewith unleashes the "partition of employments" by which "our ability increases" and "society becomes advantageous" (Hume 2000: 312). The quid pro quo condition is introduced as a necessary condition for an *anonymous intelligence* to become effective, and to exempt the few visible, yet altogether inefficient hands of feudal masters from the control over the division of labor. It is a *benevolent intelligence*, and in this sense of superior efficiency, in that it sets free dynamic productivity gains. And it is a *decentralized, yet indivisible intelligence*, in the sense that social outcomes are more than the sum over individual activities:

> "[S]ocial virtues of justice [including property rights] and fidelity [...] are highly useful, or indeed absolutely necessary to the well-being of mankind: *But the benefit, resulting from them, is not the consequence of every individual single act; but arises from the whole scheme or system, concurred in by the whole, or the greater part of the society.*" (Hume 2006: 214-5; emphasis added)

Bearing the insufficiencies of reason and generosity in mind, the following citation encapsulates Hume's invisible-hand proposition:

> "[A]s the self-love of one person is naturally contrary to that of another, these several interested passions are obliged to adjust themselves after such a manner as to concur in some system of conduct and behaviour. *This system, therefore, comprehending the interest of each individual, is of course advantageous to the public; though it be not intended for that purpose by the inventors.*" (Hume 2000: 339)

Note that Hume's 'system' is not just complicated, but also complex. In fact, his invisible hand solves a higher-order problem: it does not only orchestrate the partitioning of employments, but simultaneously improves upon the institutional design that frames and directs the individual laws of motion. Furthermore, economic activity is not only informed by social and legal norms as in the case of property rights and the implied quid pro quo condition, it also generates feedbacks, and gives direction to institutional change. Hume describes how the division of labor confronts individuals with an ever increasing number of milieus and life plans, and how such experiences of diversity and pluralism 'teaches' them the lessons of tolerance and even benevolence. A more tolerant and benevolent social environment, in turn, reduces the costs of (civil) disobedience and, therewith, increases the diversity of problem-solving strategies ('trials') - including economic decision making -, and the pace of *impersonal learning* (from 'errors'). In this sense, economic progress and the more general process of civilization are mutually reinforcing (note that Hume neither excludes the government as such, nor egalitarian traditions as emergent achievements of this civilization process). Information always runs both ways such that non-market interaction matters for the system's cumulative performance. His economic system is open and non-teleological.

The *maxim of Scottish Political Economy* immediately follows from the above analysis, and is lucidly summarized in Hume's *Political Discourses* (*Of Commerce*):

> "Sovereigns must take mankind as they find them, and cannot pretend to introduce any violent change in their principles and ways of thinking. A long course of time, with a variety of accidents and circumstances, are requisite to produce those great revolutions, which so much diversify the face of human affairs. And the less natural any set of principles are, which support a particular society, the more difficulty will a legislator meet with in raising and cultivating them. It is his best policy to comply with the common bent of mankind, and give it all the improvements of which it is susceptible. Now, according to the most natural course of things, industry and arts and trade encrease the power of the sovereign as well as the happiness of the subjects; and that policy is violent, which aggrandizes the public by the poverty of individuals." (Hume 1752: 10-11)

2.3 Adam Smith and the Mercantile Feudal System

David Hume's difficult, yet light-footed analysis constitutes the core of Scottish Political Economy, and accordingly dominates its aims and scope. Hume's close friend Adam Smith, who was introduced to the *Treatise* already at the age of sixteen (by his admired teacher Francis Hutcheson), found himself in deep agreement with his twelve years older countryman. *The Theory of Moral Sentiments* and *The Wealth of Nations* must both be understood as contributions to, and extensions of Hume's research program. Gerhard Streminger illustrates the relationship between the two celebrated Scots as pioneers of *British* political economy, and later economics, with great wit and appeal to intuition: David Hume he views as the 'Adam' of political economy; and Adam Smith as its 'David' (Streminger 1994: 389).

Smith's most important contribution in the *Wealth of Nations*, at least with respect to the purpose of this study, is that he applied Hume's new tools for social analysis to the "system of commerce" in greater isolation (books I-III), and that he specified the maxim of Scottish Political Economy for predominantly economic considerations (books IV-V). He isolated the economic subsystem by assuming an "obvious and simple system of natural liberty" (Smith 2003: 873), defined as a gradually 'moving equilibrium' in social and legal norms. Evidently, institutions as designed by Hume's anonymous intelligence are neither obvious in their concerted functions, nor simple in configuration. Smith had in mind a beneficial and relatively stable limit point of Hume's higher-order learning process (stable relative to economic structures). Such a general institutional equilibrium ensures that individual laws of motion are sufficiently tamed and aligned to admit the aggregation of self-interested *economic* conduct to *economic* order. "Natural liberty" means that individuals are 'free within natural limits', that is, free within (and due to) efficient institutional constraints (the Scottish ideal of individual liberty *through* the law). With the assumption of a benign and stable general institutional equilibrium, Smith simplifies his 'model' of the world relative to Hume's by eliminating all feedbacks between the economic system and its institutional framework.

Not efficient institutional solutions, but the task of government is obvious and simple, if framed by such optima. Thus, Smith's economic specification of the maxim of Scottish Political Economy:

> "All systems either of preference or of restraint, therefore, being thus completely taken away, the obvious and simple system of natural liberty establishes itself of its own accord. Every man, as long as he does not violate the laws of justice, is left perfectly free to pursue his own interest his own way, and to bring both his industry and capital into competition with those of any other man, or order of men. *The sovereign is completely discharged from a duty, in the attempting to perform which he must always be exposed to innumerable delusions, and for the proper performance of which no human wisdom or knowledge could ever be sufficient; the duty of superintending the industry of private people, and of directing it towards the employments most suitable to the interest of the society.* According to the system of natural liberty, the sovereign has only three duties to attend to; three duties of great importance, indeed, but plain and intelligible to common understandings: first, the duty of protecting the society from the violence and invasion of other independent societies; secondly, the duty of protecting, as far as possible, every member of the society from the injustice or oppression of every other member of it, or the duty of establishing an exact administration of justice; and, thirdly, the duty of erecting and

> maintaining certain public works and certain public institutions." (Smith 2003: 873-4; emphasis mine)

Hereby, Smith introduced the knowledge problem to economics (later taken up and popularized by Hayek). No central intelligence can substitute for economic self-organization. The allocation of resources is best left to the interplay of individual laws of motion, while the sovereign shall focus on the rules of the game (and the provision of public goods). He was very well aware of the fact that such institutional convergence from the mercantile feudal system to the system of natural liberty is risky and path dependent in that the success of Hume's impersonal approximation routine depends on the chronological order of re- and deregulations:

> "Such are the unfortunate effects of all the regulations of the mercantile system! They not only introduce very dangerous disorders into the state of the body politic, but disorders which it is often difficult to remedy, without occasioning for a time at least, still greater disorders. In what manner, therefore, the colony trade ought gradually to be opened; what are the restraints which ought first, and what are those which ought last to be taken away; or in what manner the natural system of perfect liberty and justice ought gradually to be restored, we must leave to the wisdom of future statesmen and legislators to determine." (Smith 2003: 770)

Evidently, Smith was no adherent of 'big bang' approaches to institutional transformation. In contrast to the French Enlightenment, especially to Maximilien de Robespierres and his Jacobines, the Scots preferred gradualism.[3]

A crucial aspect of such beneficial institutional transformation, as Smith saw it, was the deregulation of international trade, especially the repeal of import restrictions designed to protect domestic industries. He argued that, while the mercantile system - with its inefficient objective to maximize net inflows of gold - would systematically invite merchants and landlords to hijack the legislature with the purpose to shield themselves against foreign competition, domestic protection could be equally accomplished by free trade (as part of the system of natural liberty), yet without invoking the social costs of privilege. Free trade, so Smith, provides a free lunch.

His proposition is stated explicitly:

> "*No regulation of commerce can increase the quantity of industry in any society beyond what its capital can maintain.* It can only divert a part of it into

[3] Charles-Louis de Montesquieu and Charles Alexis de Tocqueville are the prominent exceptions that prove the rule.

a direction into which it might not otherwise have gone; and it is by no means certain that this artificial direction is likely to be more advantageous to the society than that into which it would have gone of its own accord." (Smith 2003: 539; my italics)

The proposition is 'proved' by the following theorem: given the home bias in international investment, that is, given the empirical fact that risk averse investors prefer relatively save domestic investments to foreign investments of equal profitability, their self-interest suffices to implement domestic protection, that is, to aggregate to public interest. Given the social costs of privilege, it follows that a central intelligence cannot outperform the decentralized intelligence of the economic system in the provision of domestic protection (which 'proves' the proposition). It is the only context in the *Wealth of Nations*, for which Smith invokes the invisible-hand metaphor:

> "As every individual, therefore, endeavours as much as he can both to employ his capital in the support of domestic industry, and so to direct that industry that its produce may be of the greatest value; every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it. *By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this,* AS IN MANY OTHER CASES*, led by an* INVISIBLE HAND *to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it.* I have never known much good done by those who affected to trade for the public good. It is an affectation, indeed, not very common among merchants, and very few words need be employed in dissuading them from it." (Smith 2003: 572; emphases via capital letters by AMV)

Evidently, the operation of the invisible hand is not restricted to the case of domestic protection, as it applies "in many other cases", and since there is "never known much good" that stems from the smoke-blowing rhetorics of merchants, who sell their self-interest as public interest. The case of domestic protection just exemplifies a general principle (the other case where Smith exemplifies the operation of the invisible hand by explicitly invoking the metaphor can be found in *The Theory of Moral Sentiments*, where he argues that workers are unintentionally sustained by luxury spendings of the owning classes (Smith 2000: 264-5)).

Given that the attribution of social progress to the operation of a decentralized and, thus, impersonal intelligence is the most significant analytical innovation of Scottish Enlightenment, and given Smith's own words, William D. Grampp's conclusion that "the invisible hand is not a power that makes the good of one the good of all, and it is not any of a number of other things it is said to be," but merely "a part of an argument for free trade" must be rejected (Grampp 2000: 441-2). Likewise, Emma Rothschild's suggestion that "Smith did not particularly esteem the invisible hand, and thought of it as an ironic but useful joke" (Rothschild 1994), may apply to the invisible-hand *metaphor* as a semantic stopgap and short-cut device, yet not to the invisible-hand *proposition.* The confusion of the metaphor with the proposition also underlies Gavin Kennedy's claim that "Smith had no 'theory' of invisible hands and that he showed no inclination to treat it as anything more than an isolated, though well-known, 18th-century literary metaphor" (Kennedy 2009: 240).

What is true, however, is that the modern perception of Smith elevates the scope and information-processing capabilities of his anonymous economic system. Most importantly, *the price mechanism plays only a subordinate role in his analysis* (this claim constitutes a partial alliance with Grampp 2000: 446). Instead, Smith provides an impressive account of the mercantile feudal system, yet of one to which the system of natural liberty applies. The motion of this system is predominantly governed by quite obvious feudal mechanisms and, in consequence, to a lesser extent by price competition. In what follows next, I substantiate the claim that the operation of decentralized intelligence in Smith's system is subordinate to mercantile mechanisms. For the sake of clarity, and to economize on space, his complex and capacious deliberations are summarized by a stylized model. It is a rational reconstruction and as such a consistency check (which the model fails). The model focuses on the following chapters in the *Wealth of Nations*: Book I, ch. I-III, VI-IX,XI; Book II, ch. III.

Subjects and objects of the model: The index set of agents $I \subseteq \mathbb{N}$ is partitioned into three denumerable equivalence classes of agents $\{A, B, C\}$ with empty intersection. These three classes are exclusively characterized by the type of individual endowment. Each $i \in A$ (i_A) is exclusively endowed with common labor L_i (a worker), $i \in B$ (i_B) is exclusively endowed with homogeneous stock of capital K_i (a capitalist), and $i \in C$ (i_C) is exclusively equipped with homogeneous land ownership H_i (a 'Juncker'). Within each class, aggregate endowments are uniformly distributed.

Agents in set A may be active ('alive'), denoted by i_A^+, or inactive ('dead'), denoted by i_A^-. Define $\alpha = \#i_A^+ \in \mathbb{N}$ as the total number of active workers. Workers inelastically supply the same quantity of labor, $L_i = \bar{L} > 0, \forall i \in A$. Then the aggregate supply of labor is given by $L^+ = \alpha\bar{L}$. Likewise, agents in class B may active ('in the market'), i_B^+, or inactive ('out of the market'), i_B^-. Again, define $\beta = \#i_B^+ \in \mathbb{N}$ as the total number of active capitalist competing for labor. Capitalists inelastically supply the same quantity of capital, $K_i = \bar{K} > 0, \forall i \in B$. Then the aggregate supply of capital can be expressed by $K^+ = \beta\bar{K}$. Any increase in the aggregate stock of capital is displayed as an intensification of competition (the competition process). We normalize $\bar{L} = \bar{K} = \bar{H} = 1$. Note, active subsets of agents may be large, but are always finite, while their respective complements are infinite. Perfect competition is approximated as $\alpha, \beta \to \infty$.

Further, there exists just one produced commodity, say corn, which serves as consumption and capital good (capital and consumption goods as perfect substitutes). All prices are expressed in terms of this commodity. The representative capitalist is constrained by a Leontieff technology. Given the aggregate production function $\ell : I_A \times I_B \times I_C \mapsto \mathbb{N}$, the scale-constrained efficiency frontier is given by $\min_{\bar{Y}}(\alpha, \beta, \delta)$, with $Y = \sum_B y_i$ as aggregate net output. The indivisibility of agents implies the indivisibility of stocks which, in turn, suggests the overall production possibility set to be a lattice. Nonconvexity will allow for the introduction of 'increasing returns', as the average productivity of labor depends on the scale of activity. This scale is determined by profit maximization subject to ℓ. Profit is

$$\Pi = \min_{\bar{Y}}(L^+, K^+, H^+) - wL^+ - RK^+,$$

with $H^+ = \delta\bar{H}$, where $\delta = \#i_C^+ \in \mathbb{N}$ gives the number of homogeneous land units supplied. It is assumed that $\delta \geq \delta_{min}$, where δ_{min} ensures that land supply is always in excess supply such that Ricardo's later notion of differential rent does not apply. Given that land is homogeneous and a free good, landowners earn zero rent by just relying on markets forces. R is the rental price of capital.

Given constant returns to scale, profits are zero in equilibrium. Thus, as an equilibrium condition,

$$wa_L + Ra_K = 1 \Leftrightarrow w = \frac{1}{a_L} - k \cdot R \Leftrightarrow R = \frac{1}{a_k} - \frac{w}{k},$$

where, as usual, a_K, a_L denote the input coefficients with respect to capital and labor, and $k = K/L$. The wage rate (the rental rate of capital) is increasing (decreasing) in average capital intensity. For given technology, profit maximization gives the famous inverse linear relationship between the average wage rate and the average rental rate of capital. This simplistic statement of production - summarizing the economy's efficiency frontier -

suits best in a rational reconstruction of Smith's reasoning.

Advanced capital and some national accounting: A modern theorist would distinguish between consumption and net additions to the stock of capital as constituents of net "annual produce". Smith, however, argues that $Y \equiv C$, with C as total consumption (Smith 2003: I.vi; Hollander 1987: 148-52). Equating net output with overall consumption is perfectly legitimate as a stationary equilibrium condition, describing the "dull" state in which no additions are made to the stock of capital. It is however illegitimate as an accounting identity. His notion of income cannot account for changes in the stock of capital. As Samuelson points out, Smith's "exposition is 1776, not 1876 or 1976, in its vagueness" (Samuelson 1977: 43).

To reproduce a given stock of capital, the capitalist has to maintain workers active by supplying them with corn over the reproduction period ('food and shelter'). A fraction of the available stock of corn must be foregone, i.e., saved, to sustain net available output. Savings beyond what is necessary to sustain a stationary state, i.e., net savings, add to the next period stock of capital. Gross saving is given by

$$S(t) = K^+(t+1) - (1-\rho)K^+(t) = I_n(t) + \rho K^+(t),$$

where ρ gives the rate at which the stock of corn depreciates, and $I_n(t) = K^+(t{+}1) - K^+(t)$ denotes net investment. Smith's primitive society is defined by $S = 0$. Further, given a closed economy,

$$Y(t) = C(t) + I_n(t) \Leftrightarrow S_n(t) = I_n(t).$$

Note that $C = \sum_{(I_B^+ \cup I_C^+)} C_i$ denotes non-labor consumption. The consumption of labor is already accounted for by S, which is *capital advanced* (in this sense, a subsistence fund). Say's Law applies due to the capitalist as saver-investor (the credit market and, thus, the relation between the rate of interest and the rate of profit play a minor, subordinate role in Smith's analysis). $S_n > 0$ defines a progressive commercial economy. The excess of gross savings over the stock of corn necessary to sustain labor over the reproduction period is used to draw upon labor for the production of new capital, increasing next-period output. Accordingly, a retrogressive commercial economy is characterized by negative net savings, that is, the capitalist consumes his 'working' capital; he turns workers inactive. In steady-state, $Y(t) = C(t)$.

Power relations and the distribution of income: Aggregate net available output, Y, is exhausted by active workers, active capitalists, and land owners (for convenience, the time index is skipped):

$$Y \equiv W + P + T \equiv \sum_A W_{i^+} + \sum_B P_{i^+} + \sum_C T_i \equiv w \sum_A L_i^+ + (R - \rho - \tau) \sum_B K_i^+ + \tau \sum_C H_i^+,$$

where τ the average rate of rent. Evidently, rent income T is introduced as a transfer payment, drawn from the capitalist class. Define $r = R - \rho - \tau$ as the average rate of profit. As usual, the wage sum is given by W, total profits by P, the aggregate stock of capital by K^+. H^+ is total land in use. The steady-state value of each, denoted by a star, is determined by extra-economic, sociological forces. At this point, Smith introduces a modern feudal system, that is, the mercantile system in 1776, into his analysis:

(a) *The iron law of wages* (Smith 2003: I.viii): As argued in section 2, the feudal system is characterized by extractive political and economic institutions. The unprivileged masses that relied on their labor alone, were condemned to scrape a living at subsistence level. According to Smith, the system of natural liberty and economic progress cannot improve their long-run *per capita* living standard. In steady state, and on average, wage rates are just sufficiently high for workers to maintain themselves on the behalf of the two more privileged classes of society. They are free through the law (instead of being tenants), yet use each increase in income to 'activate' perfect substitutes as additional competitors. "Wages are not high in a stationary country, however rich." (Smith 2003: 101)

The time motion of labor supply is determined by the 'iron law of wages', which is introduced as a crude, linear feudal mechanism:

$$\alpha(t+1) - \alpha(t) = \lambda[w(t) - w^*], \text{ with } \lambda > 0,$$

being the elasticity of reproduction, i.e., measuring the impact of excess wage rates on the number of workers 'alive', and with w^* as the subsistence wage rate (potentially determined by cultural factors as well). Given an income W^* per period that just maintains the representative worker over that period, we have $w^* = L_i/W^*$. If the average wage rate exceeds the subsistence rate, the fraction of active workers increases. It is an iron law, since the opposite holds as well. The initial fraction of active workers, $\alpha(1)$, is given. Evidently, $|w(t) - w^*| \to 0$ (or $w(t) \to w^*$) implies $|\alpha(t+1) - \alpha(t)| \to 0$ (or $\alpha(t) \to \alpha^*$) for $t \to \infty$.

Ignoring the distinction between productive and unproductive labor, the aggregate demand for labor, L^D, is determined by the wage rate and the level of gross saving, i.e., $S = wL^D \Leftrightarrow L^D = S/w$. Since w^* is predetermined, the steady-state level of aggregate labor demand, L^*, is *exclusively* controlled by gross saving. In general, labor market short-run equilibrium is given by

$$L^+ = L^D \Leftrightarrow \alpha(t) = \frac{S(t)}{w(t)} \Leftrightarrow w(t) = \frac{S(t)}{\alpha(t)},$$

such that the market-clearing average wage rate is increasing in gross saving, and decreasing in the fraction of workers alive. Thus, the 'short-run' or market rate of wages is determined by supply and demand, yet is dominated by the choices on capital markets (a non-Walrasian feature) and, as will be shown, controlled by feudal laws of reproduction. Further, given a steady-state level of gross saving, $\alpha^* = S^*/w^*$, that is, the long-run equilibrium fraction of active workers is increasing in the subsistence fund, and decreasing in the subsistence wage rate (given the capitalists' choices, cultural norms and other social conventions that increase individual welfare levels reduce equilibrium population).

Given $\alpha(1)$ as well as the capitalists choice of initial gross saving, $S(1)$, we get the initial wage rate $w(1)$. Assuming that the initial choice of gross saving is maintained such that $S(1) = S(t) = S^* < L, \forall t$, the change in equilibrium wages is governed by the following difference equation:

$$w(t+1) - w(t) = -S^* \cdot \frac{[\alpha(t+1) - \alpha(t)]}{\alpha(t)\alpha(t+1)},$$

such that $|\alpha(t+1) - \alpha(t)| \to 0$ (or $\alpha(t) \to \alpha^*$) implies $|w(t+1) - w(t)| \to 0$ (or $w(t) \to w^*$) for $t \to \infty$. Thus, deviations of the market wage rate from its subsistence level measures changes in the scarcity of labor. In this sense, α^*, w^* figure as centers of gravitation, as limits to the labor market process. Long-run equilibrium is almost exclusively determined by feudal considerations. Also during convergence, a feudal mechanism dominates.

Further, Smith assumed the convergence process to be highly sluggish. Thus, he plants most of his empirical observations into the convergence process, like all of his successors, who equally lacked concepts of dynamic equilibria (which explains the persistence myth that disequilibrium analysis is in some sense more 'real'). He concludes:

> "It deserves to be remarked, perhaps, that it is the progressive state, while the society is advancing to the further acquisition, rather than when it has acquired its full complement of riches, that the condition of the labouring poor, of the great body of people, seems to be the happiest and the most comfortable. It is hard in the stationary, and miserable in the declining state. The progressive state is in reality the cheerful and the hearty state to all the different orders of the society. The stationary is dull; the declining melancholy." (Smith 2003: 114)

Smith is primarily a protagonist for the "labouring poor", since the progressive situation (i.e., $w > w^*, \alpha < \alpha^*$) ranks as the highest valued possible state of society. His defense of the Whigs is derived from his primary target, the prosperity of the unprivileged class.

(b) *Class struggle over C - Whigs vs. Tories* (Smith 2003: I.xi): Also the average rent of land is determined by the mechanisms of the mercantile system (The rental rate of capital R is discussed in the next subsection and, thus, taken as given in this subsection). As emphasized in section 2.1, all variations of feudal systems are characterized by bargaining processes over extracted income that involve the privileged subjects of society. The aristocratic feudal system was described by the dominance of landowners, who struggled with the monarch. The mercantile feudal system, however, witnessed the rise of a producing (and internationally trading) class, and its increasing involvement in the privilege-seeking society. In this arena, the capitalists were represented by the Whigs, and the landowners by the Tories. After the implementation of constitutional monarchy, the feudal bargaining process was dominated by the hassles of these two political parties. Over time, the Whigs gained upper hand in this process. Smith celebrated the Whigs whenever they succeeded to repress Tory privileges. And he arraigned them, whenever they sought for their own grants of privilege, and manipulated the law for their own advantage (outrageously, whenever capitalists blew smoke in the sense of selling particular interest as public interest). Such discussions constitute the second part of the *Wealth of Nations*, where he applies the economic specification of Hume's maxim of Scottish Political Economy.

Smith was an astute, yet not impartial observer of the aforementioned bargaining process between Whigs and Tories. Even the reign of natural liberty does not change the nature (*das Wesen*) of rent income: it is predominately an undeserved extraction of other people's productive contributions: Rent, so Smith, is "*naturally* a monopoly price", "not at all proportioned to what the landlord may have laid out upon the improvement of the land" (Smith 2003: 199-200; my emphasis). Rent is to by and large a transfer payment: landowners reap what they have not sown. The "authority of riches" (ibid.: 903) is to some degree unavoidable in Smith's competitive system. The class of landowners can behave strategically, and exerts power. Thus, average rent is introduce as a tax rate τ on capital income. Accordingly, the rate of profit on capital advanced is given by $r = R - \tau - \rho$.

The Dynamics of the Mercantile System (Smith 2003: I.i-iii,I.ix; II.iii): Capitalists, if tamed by the system of natural liberty and sufficiently many, are the heroes of Smith's plot. Their saving decisions bottleneck economic progress, since Smith assumed a super-classical (Kaldorian) saving function, that is

$$\frac{S}{Y} = \frac{S_A}{Y} + \frac{S_B}{Y} + \frac{S_C}{Y} = s_A w a_L + s_B(R(t) - \tau)a_K + s_C \frac{\tau H}{Y},$$

with

$$s_B > 0 \text{ and } s_A = s_C = 0,$$

such that

$$S(t) = s_B(R(t) - \tau)\beta(t) = \beta(t+1) - \beta(t) + \rho\beta(t),$$

and

$$S_n = s_B r(t)\beta(t) = \beta(t+1) - \beta(t) \text{ with } r(t) = R(t) - \tau - \rho.$$

Given $\beta(1) > 0$, $S_n \geq 0$ (< 0), iff $r \geq 0$ $(r < 0)$ or, equivalently, $R \geq \tau + \rho$ $(R < \tau + \rho)$. From profit maximization we have

$$R = \frac{1 - w a_L}{a_k}.$$

We assume with Smith that the division of labor and, thus, average labor productivity is limited by the market, that is, production choice in each period is constrained by the aggregate net output of that period. Such output-constrained efficiency is due to indivisibilities, i.e., the impossibility to make use of the most efficient technique known independent of the level of aggregate income. Thus, $a_L(t) = f(Y(t)), f' < 0, f'' = 0$ and

$$a_L(t) - a_L(t+1) = g, \forall t,$$

where g is the exogenous growth rate of labor productivity. Denoting the efficiency frontier of global technology by $a_{L,\min}$, the exogenous productivity process is convergent, that is, $a_L(t) \downarrow a_L^* = a_{L,\min}$, for $t \to \infty$ (and $k(t) \uparrow k^*$). Thus, the productivity process is constructed as a sequence that starts from the interior of the technology set and monotonically gropes via output growth to the the efficiency frontier. It follows that $g = 0$ is an equilibrium condition.

The equilibrium dynamics of the rental rate of capital is governed by

$$R(t+1) - R(t) = -\frac{w(t+1)a_L(t+1) - w(t)a_L(t)}{a_K},$$

with

$$R^{**} = \frac{1 - w^* a_L^*}{a_k}$$

as the steady-state rental rate. Note that $R \to R^{**}$, if and only if both driving processes order converge, i.e., $|w(t) - w^*| \to 0$ *as well as* $|a_L(t) - a_L^*| \to 0$. Over time, the average rental rate of capital is monotonically decreasing in wage rates, and monotonically increasing labor productivity. To isolate the two effects on the motion of R, assume that $a_L(t) = \bar{a}_L < a_L^*, \forall t$. Then, $w(t) - w^* \geq 0$ implies $R(t) - R^{**} \leq 0$. Or, assume $w(t) = \bar{w}, \forall t$. Then, we can isolate 'increasing returns' due to the implementation of an increasingly efficient technique:

$$R(t+1) - R(t) = \frac{\bar{w}}{a_K} \cdot g.$$

For the general law of motion, note that labor productivity converges to a_L^* only if $g > 0$ does not induce an increase in the average wage rate such as to overcompensate the positive effect of g on ΔR. Most importantly, labor supply must be sufficiently elastic to avoid a bottleneck on the labor market, pushing up wage rates such that rental rates actually fall despite the potential for next-period productivity gains. In this case, Smith's system may display capital reversal. In the following, it is assumed that λ is sufficiently high to prohibit capital reversals.

Next, we introduce a competition process guided by excess rental rates. Given the structure of the model, this process is the process of capital accumulation (the representative capitalists net saving plan). Each capitalist inside the market maximizes steady-state capital income $P_i^* = r^*(t)\bar{K}_i, i \in B$. This is due to the fact that Smith does not allow intertemporal preferences to play a role (typical for classical long-run theory). Thus, we have to forego pure time preference (Böhm's second cause of positive interest) as well as the smoothing preference (Böhm's first cause). Without myopic preferences, however, the infinite steady-state tails of the average consumption program dominates the capitalists identical choices, and 'early' costs are completely ignored in light of such 'late' benefits.

During convergence, aggregate profit is $P(t)$. 'Short-run' excess profits are given by $EP = (R(t) - R^*)\beta(t)$, where

$$R^*(t) = \frac{1 - w(t)a_L^*}{a_k},$$

is the long-run *market* equilibrium rental rate of capital (in contrast to R^{**}, which asks for the convergence of the β- as well as the α-process). R^* is determined by the rate of return on investment that just covers fixed management costs, say $\bar{m}$, i.e., $r^* = R^* - \tau - \rho = \bar{m}$. Smith's theory of steady-state profit is perhaps his weakest analytical contribution, given that management cost is a bootstrap notion, and alien to his general outline.[4] Yet, R^*

[4]Smith did not argue that increases of "stock" increase average wages due to diminishing returns in agriculture (as David Ricardo did). He did not involve convex technology (marginal productivity). He did not think of profit as a reward for abstinence (as did Senior and Rae).

will prove to be a helpful device in separating the market long-run equilibrium from the feudal steady-state.

Since the capital stock is uniformly distributed on B, capital accumulation is displayed as subsequent market entry of more and more capital owners, that is, as an increase in β over time. The fraction of competing capitalists, so Smith, is increasing in excess rental rates. Let

$$\beta(t+1) - \beta(t) = \chi[R(t) - R^*]$$

be the competition process, where $\chi > 0$ measures the intensity of the competition process. The more entry barriers there are, the smaller is χ. Given the natural system of liberty, all judicial and legislative obstacles are removed and norms accept the rules of the game. What remains are genuine transaction costs as market entry depends on time and resources. Relaxing the auxiliary assumption of a constant level of gross saving made in (a), such that

$$S(t+1) - S(t) = \beta(t+2) - 2\beta(t+1) + \beta(t) + \rho(\beta(t+1) - \beta(t)),$$

the dynamics of average wage rates generalize to

$$w(t+1) - w(t) = -\frac{[S(t)\alpha(t+1) - S(t+1)\alpha(t)]}{\alpha(t)\alpha(t+1)},$$

which links up the β-process with the α-process, allowing for non-linearities in wage rates and, thus, in the rental rate of capital. Labor demand is driven by a market process; labor supply by a feudal mechanism. Note that wage rates converge, if and only if the β-process (capital accumulation due to intensifying competition, i.e., via market entries) *and* the α-process (the feudal mechanism via endogenous fertility rate) converge. Note also that the two processes are of opposite sign. The competition process tends to increase wages over time, while the iron law (the activation of hitherto passive labor supply) tends to depress the absolute change of the wage rate. As will be shown immediately, the convergence properties of Smith's dynamic system hinge on the relative strength of the α- and β-process (involving the relation of λ and χ).

Define the short run of the feudal system (the market long run) by $t = 1, \dots T$ such that $a_L(t) > a_L^* \geq a_{L\min}$ and, thus, $g > 0$. Assume $w(1) = w^*$ and $\alpha(1), \beta(1) > 0$. From $g > 0$ it follows that $R(2) - R(1) > 0$. Thus, $S_n(1) = \beta(2) - \beta(1) > 0, \forall t \in (1, \dots, T-1)$, proportional to χ. Given the dynamics of gross saving, $w(2) > w^*$ such that $\alpha(3) > \alpha(2) = \alpha(1)$. The increase in the wage rate is a counteracting force, dampening the impact of g on ΔR. Yet, it provokes the activation of labor supply which counteracts the counteracting force. Thus, the feudal system framing the market process induces second order effects. The convergence properties of the system crucially depend on $\phi = \chi/\lambda$.

If $\phi > 1$, the law of motion of the average wage rate is strictly dominated by the β-process and $R(t+1) - R(t) > 0, \forall t \in (1, \ldots, T-1)$. It follows that $\beta(t+1) - \beta(t) > 0, \forall t \in (1, \ldots, T-1)$ and, given $\chi > \lambda$, $w(t) > w^*, \forall t \in (2, \ldots, T)$. Thus, $a_L^* = a_{L\,\min}$ and $\beta(t) \uparrow \beta_g$, where $\beta_g \cdot K$ is the golden rule capital stock. The β-process is strictly monotonic and stops since $g(T) = 0$. However, even if β_g is the limit of the competition/accumulation process and, thus, a long-run market equilibrium, the overall system is still in motion. The feudal counterpart of the market process still operates: given $w(T) > w^*$, $\alpha_{T+1} - \alpha_T > 0$. Eventually, the α-process converges, i.e., $\alpha(t) \to \alpha^*$ as $t > T \to \infty$, as $w(T) \to w^*$. Yet, as the wage rate converges to its limit,

$$R(t) = \frac{1 - w(t)a_L^*}{a_k} \uparrow R^{**} \text{ as } t > T \to \infty.$$

As the tail of the physical history of Smith's hybrid system is dominated by a feudal mechanism, the owning classes are able to extract an increasing fraction of real aggregate income. Thus, R^{**} is a *surplus*, extracted from the working class via the iron law governing the labor market. In fact, the time path of R us U-shaped, with R^* as global minimum!

This immediately raises the question of why the β-process is not reinforced? Given that the competition/accumulation process is stronger, why should we not expect an eternal struggle between the market and feudal process in form of a cyclical equilibrium? Starting in T, the feudal process induces an extraction period, which provokes strong accumulation (now in form of capital widening); more competition, in turn, accounts for wage increases, counteracting accumulation, and so on and so forth. In this case, a steady-state does not exists. This inconsistency is due to Smith's insistence on the subsistence wages rate as the limit of his system, the 'dull state'. Given linear technology, this prediction suggests a surplus theory of the rental rate of capital and, thus, of the profit rate, which is conflicting with his independent determination of steady-state profit via management costs. As David Ricardo later showed, either you fix the equilibrium rate of profit, or the equilibrium wage rate, yet not both. If the profit rate is exogenous, then the steady-state wage rate is determined by average labor productivity, and the feudal α-process is dispensed with. Capital accumulation would ensure 'happiness' and 'comfort' for the working class in long-run equilibrium. All this makes clear that English political economy is analytically superior, which explains its subsequent dominance (the canonical model). With the rise of English dominance, Scottish moral philosophy was increasingly repressed by utilitarian considerations.

Equilibrium Price Systems and the Scope of the Invisible Hand (Smith 2003: I.vii, II.v): Smith's famous seventh chapter (book I) contains his price theory. Ignoring the inconsistencies mentioned above, assume with Smith that the mercantile system is stationary and that $\{w^*, R^{**}\}$ prevails at $S_g = \rho\beta_g K$. Given that his price theory explains the sectoral allocation of capital and (therefore) labor, define n sectors with divergent capital intensities producing n commodities traded at the price p_n. Prices are normalized with respect to p_1. For each sector, profit maximization now implies

$$w^* a_L^n + R^{**} a_K^n = P_n^*,$$

where P_n^* is the steady-state or "natural" price of commodity n and, accordingly, $\mathbf{p}^* = (1, p_2, \ldots, p_n)$ is the long-run equilibrium price system. In contrast to modern general equilibrium models, equilibrium price systems are not the major prediction (theorem) of Smith's model, but derived as a corollary. Note that *relative* equilibrium prices (Smith's "natural" prices) are exclusively determined by technological primitives, that is, by industry-specific differences in input coefficients. Since demand relations play no role in the determination of the equilibrium price system, it is no list of trade-offs that signal relative scarcities. This, of course, is due to the fact that the steady-state supply of labor and the steady-state stock of reproducibles are both variable, endogenously determined for given steady-state values of wage and profit rates. Smith himself was well aware of divergent risk premia (and the so-called home bias in international investment) as well as of the role of heterogeneous human capital and divergent hardships of labor in the determination of industry-specific wage rates. In contrast to Samuel Hollander (1973, 1987), however, I do not believe that such considerations fit Smith's core model, for they suggest increasing long-run supply curves and, thus, a role for demand relations in the determination of equilibrium prices. Given the iron law of wages, it is hard to believe that workers would prefer 'deactivation' to any degree of industry-specific disutility of employment; or to wage rates that do not pay for their talents.

Given that scarcity considerations play no role in Smith's notion of long-run general equilibrium and, thus, the exchange nexus as introduced by Mandeville and Hume is absent, the invisible-hand proposition does not apply. Foremost, commodity price relations do not inform the convergence process as introduced above. The drift of Smith's economy is not controlled by an impersonal intelligence, but by crude and rather evident feudal mechanisms. *Indeed, the invisible hand is inoperative during the convergence to steady state, right because Smith's model is such an excellent description of the mercantile feudal system.* Moreover, not only is Smith's long-run equilibrium not the outcome of a decentralized intelligence, it is *inefficient* as well. Recall that Smith measures efficiency in terms of average per capita income. His model, however, predicts that individual wage sums of the vast majority of 'labouring poor' remain unaffected by the competitive accumulation process.

Most modern authors, of course, do not associate Smith's invisible hand with the long-run performance of his economy, but with short-run disequilibrium processes on commodity markets, that is, with inter-industry competition among capitalists. However, given that commodity price relations do not matter for the competition process, it follows that Smith's notion of efficiency is unrelated to commodity prices in general, **p**, including disequilibrium prices (Smith's 'market' prices). Short-run convergence is defined as

$$p_n \to p_n^*, \ \textit{independently} \text{ for all } n,$$

such that

$$\mathbf{p} \to \mathbf{p}^*.$$

Since $\mathbf{p}^*$ is data to this process, a mercantile feudal system in long-run equilibrium frames the commodity-market processes, revealing Smith's assumption that the long-run structure of the economy, including technology and feudal power relations, is far more persistent than the inter-industry (or sectoral) allocation of resources. Intersectoral competition only regulates the convergence of market prices, that is, it reflects the systems response to accidental and temporary influences, but is unrelated to the systematic and persistent influences determining natural values. (Kurz and Salvadori 1995).

Given that such a long-run equilibrium suggests fixed aggregate supplies of labor and capital, sectoral supply curves are increasing, $x_n(p_n), \partial x_n/\partial p_n > 0$, and, therefore, demand relations matter. The individual demand for the n-th commodity is denoted by $d_n^i(p_n)$. Smith assumes the 'law of demand', that is, $\partial d_n^i/\partial p_n < 0, \forall i, n$. Sectoral demand is simply given by $d_n = \sum_i d_n^i$. Of course, $\partial d_n/\partial p_n < 0, \forall n$. For the sake of the argument, let sectoral short-run behavior be summarized by excess demand functions,

$$z_n(p_n) = d_n(p_n) - x_n(p_n), \text{ with } \frac{\partial z_n}{\partial p_n} < 0,$$

which, together with the 'law of supply and demand', i.e., $p_n = p(z_n), \partial p_n/\partial z_n > 0, \forall n$, is a necessary and sufficient condition for sectoral convergence, i.e.,

$$z_n \to 0 \Leftrightarrow p_n \to p_n^*.$$

This, according to Smith, is sufficient for $\mathbf{p} \to \mathbf{p}^*$. With regard to the system's short-run convergence, that's it!

He, however, preferred to restate short-run convergence as the equalization of sectoral profit rates, r_n, i.e.,.

$$r_n \to r_n^*, \ \textit{independently} \text{ for all } n,$$

such that

$$\mathbf{r} \to \mathbf{r}^*,$$

where $\mathbf{r} = (r_1, ..., r_N)$. Note that the average of $\mathbf{r}^*$ is r^*; $\mathbf{r}^*$ may provide additional information about sectoral risk premia. Note also that the level of gross saving is given by S^*. The link to commodity prices is given by

$$e_n = r_n - r_n^* = p_n - p_n^*, \forall n,$$

where e_n denotes sectoral excess returns (windfall profits if positive, and losses otherwise). Evidently, if $\mathbf{p} \to \mathbf{p}^*$, then $\mathbf{r} \to \mathbf{r}^*$ by $e_n \to 0, \forall n$ (and vice versa). The entire exercise is redundant. Yet, if risk premia are neglected, the *equalization of sectoral profit rates boils down to the 'law of one price'*, since S is homogeneous.

Thus, Smith's invisible hand coordinates the sectoral allocation of capital and labor, while aggregate levels are determined by his hybrid mercantile system. In arguing against monopoly in colony trade, Smith states the invisible-hand proposition:

> "It is thus that the private interests and passions of individuals naturally dispose them to turn their stock towards the employments which in ordinary cases are the most advantageous to the society. But if from this natural preference they should turn too much of it towards those employments, the fall of profit in them and the rise of it in all others immediately dispose them to alter this faulty distribution. Without any intervention of law, therefore, the private interest and passions of men naturally lead them to divide and distribute the stock of every society, among all the different employments carried on in it, as nearly as possible in the proportion which is most agreeable to the interest of the whole society." (Smith 2003: 799)

Thus, for given quantity constraints that prevail in the short-run, the invisible hand allocates capital most efficiently.

So what is the nature of the invisible hand in Smith's short-run analysis? First of all, the *short-run system is summative* and, thus, the convergence process *non-complex.* Alfred Marshall would later adhere to the same procedure: firstly, partitioning the economic system into independent industries, with tools specialized for such analysis; secondly, putting all separately conceptualized pieces together to gain insight into the working of the complete system.

> “Man’s powers are limited. Almost every one of nature’s riddle is complex. [...] Breaking up a complex system, studying one bit at a time, and at last combining his partial solutions with a supreme effort of his whole small strength into some sort of an attempt at a solution of the whole riddle.” (Marshall 1920: 366)

In assigning the analysis of income distribution exclusively to his long-run system, Smith had to ignore that - given the quid pro quo condition (budget constraints) - sectors are also interrelated by income effects, and that overall price convergence cannot be grasped by the logic of partial equilibrium analysis. In consequence of the summative approach, the minimum requirements imposed on the system’s intelligence are rather small. It is, I dare to say, not at all invisible how single markets converge. This does not imply, however, that impersonal intelligence is strictly inoperative on single markets, or that its operation is restricted to multi-market coordination. As Dhananjay K. Gode and Shyam Sunder have shown by computational (experimental) analysis, budget constraints suffice for partial convergence even in case of zero-intelligence agents (Gode and Sunder 1997).

Furthermore, the knowledge problem that Smith himself involved in economic analysis plays a minor role in his short-run system. Given that the long-run system is a stable framework for the operation of the ‘law of supply and demand’ - such that technology as well as the steady-state values of wage, profit, and rent rates persist over time -, equilibrium prices are ‘stylized facts’ and constitute the history of the short-run system. However, if equilibrium prices could be detected by means of statistical time-series analysis, they must not be discovered by an anonymous intelligence (and there is no need for a decentralized memory). Smith’s ‘center of gravitation’ (Smith 2003: 82) are quite visible attractors.

Chapter 3

The Modern Habitat of the Invisible Hand

As a result of the craft activities associated with the Cowles Commission, Smith's flowery metaphor came under the scrutiny of axiomatic reasoning. This section dissects the Neo-Walrasian program, as defined by E. Roy Weintraub (1985a, 1985b), and investigates its relationship to Scottish Political Economy as defined in section 2. The development of axiomatic general equilibrium analysis in the history of economics is confined to the citadel of the profession, associated with the most distinguished theorists like Kenneth J. Arrow, Robert J. Aumann, Gérard Debreu, Werner Hildenbrand, Roy Radner, or Hugo Sonnenschein. Given the scope and depth of the relevant literature, no single article can provide a complete list of the fundamental aspects involved. Tough choices have to be made: for each contribution introduced, dozens of others are omitted in the hope to isolate the major aspects that are relevant to appraise the place of the invisible-hand proposition in modern analysis.

3.1 The Neo-Walrasian Program

3.1.1 Neo-Walrasian Analysis as a Scientific Research Program

To handle the complex material subsumed under the heading of 'general equilibrium analysis', I resort to Weintraub's rational reconstruction of Walrasian economics by means of Imre Lakatos's Methodology of Scientific Research Programs (MSRP) (Weintraub 1985a, Lakatos 1978a, 1978b). Since this is not a contribution to the philosophy of science, such a procedure should not be understood as a statement on the relative merits of Lakatos's approach, which he introduced as "sophisticated methodological falsificationism" (Popper$_3$), although I have great sympathies for his intermediate stance between Karl Popper's normative falsificationism (Popper$_2$) and Thomas Kuhn's descriptive account of how science actually proceeds (for a neat discussion, see Blaug 1993 and Weintraub 1985b).[1] The resort to the MSRP rather serves to *structure* and *clarify* my own arguments. The ultimate goal is to introduce a general framework that allows to compare and, thus, to appraise two very distinct approaches to invisible-hand studies, i.e., Scottish Political Economy and the neo-Walrasian paradigm.

In a nutshell, the ingredients of a Scientific Research Program (SRP) are (1) internally consistent *hard-core propositions* on a conventional basis that demarcate a specific paradigm, (2a) a set of *positive heuristics*, i.e., metarules of how to derive models from hard-core propositions, (2b) a set of *negative heuristics* that inform how not to proceed, and (3) a *protective belt* composed of specific testable theories that result from the specification of hard-core proposition in conjunction with auxiliary assumptions such that each conjunction is guided and confined by the SRP's heuristics. Together, (1) and (2) comprise the SFP's *hard core*. The negative heuristics serve to shield or immunize the hard core from potentially conflicting empirical results. As long as the SRP remains progressive, only auxiliary assumptions are adjusted in case of falsified predictions. A SRP is progressive, if it generates a series of theories such that any single theory is superseded by one that (a) generates the same set of corroborated predictions, (b) provides excess content, i.e., allows for additional predictions which (c) are at least partly corroborated (Weintraub 1985b: 32). Otherwise, the SRP is degenerating. Only in this case is it admissible to scrutinize the program's hard-core proposition and, to the extent that alternative SRP's exist and are relatively progressive, to switch to another program.

By means of the MSRP it is possible to rationally reconstruct the history of science and, thereby, to dispel the actual historical process, which is the result of complex craft activities, and driven by all kinds of sociological factors. Weintraub (1985b) provides such an outline for the history of general equilibrium analysis. He defines the *neo-Walrasian program*, that is, the specific SRP suited to rationally reconstruct the history of general equilibrium analysis, by specifying its hard core (ibid.: 109; quotation marks are skipped for all items; emphasis mine):

(1) The neo-Walrasian hard-core propositions:

HC1. There exist economic agents.

HC2. Agents have preferences over outcomes.

HC3. Agents independently optimize subject to constraints.

[1]According to Lakatos, Popper_1 refers to the straw man constructed by Popper's critiques, usually introduced as 'naive' falsificationism (Lakatos 1970). Note that Popper_2 and Popper_3 are variations on critical rationalism.

HC4. Choices are made in interrelated markets.

HC5. Agents have full *relevant* knowledge.

HC6. Observable economic outcomes are coordinated, so they must be discussed with reference to equilibrium states.

To regulate the dynamics of (1), one may add the following metarule (*strict methodological individualism*):

MR. The properties of any economic system are completely reducible to the individual characteristics of its population.

Most importantly, this rule precludes restrictions on the distribution of individual characteristics. If the metarule is regarded as innate to neo-Walrasian reasoning, the applicability of general equilibrium analysis is severely narrowed in light of the Sonnenschein-Mantel-Debreu results (discussed in section 3.3.2).

(2) An incomplete list of positive and negative heuristics:

PH1. Go forth and construct theories in which economic agents optimize!

PH2. Construct theories that make predictions about changes in equilibrium states!

To Weintraub's list, one may add:

PH3. Construct models that account for non-summative wholeness!

...

NH1. Do not construct theories in which irrational behavior plays any role!

NH2. Do not construct theories in which equilibrium has no meaning!

NH3. Do not test hard core propositions!

Again, to Weintraub's list, one may add:

NH4. Do not directly relate out-of-equilibrium behavior and convergence properties to real-world dynamics!

...

In contrast to the common treatment of a SRP's hard core as a rather sluggishly progressing framework for the problem-solving activities in the protective belt, Weintraub is also concerned with the discontinuous dynamics of the hard core, taking into account that propositions HC1-6 did not fall from heaven. Also our understanding of notions like 'agent' or 'equilibrium' did change over time, following the introduction of new mathematical concepts and the increasing precision of deductive reasoning. Accordingly, Weintraub describes as "hardening of the hard-core" the process initiated by the craft activities associated with Karl Schlesinger's generalization of Gustav Cassel's system of equations in 1933, and Abraham Wald's existence proofs in 1933/4 (in the context of Karl Menger's Vienna colloquium), a process that eventually culminated in the final generalizations by Arrow, Debreu and (independently) McKenzie in 1954 (Weintraub 1983, 1985b).

Note, however, that hard-core propositions do not specify economic content. Only models in the protective belt provide the mathematical concepts that associate with economic notions. Most importantly, different models suggest different specifications.

> "In the neo-Walrasian program [...] the analyst [is led] to construct theories [i.e., models] based on optimizing choice and equilibrium outcomes and to explore the effect of giving different interpretations [i.e., different one-to-one correspondences between mathematical and economic concepts], in different models, to undefined terms like 'agent', 'outcome', 'knowledge', or 'market'." (Weintraub 1985b: 113)

This fact exacerbates the task to evaluate the proximity of the neo-Walrasian program and the Scottish Political Economy. The major difficulty will be to isolate an appropriate subset of models, preferably non-empty, according to the criteria carved out of the works of Mandeville, Hume, and Smith in section 2. Some generic statements are nevertheless admissible and are discussed in the following two subsections.

3.1.2 Generic Properties of Neo-Walrasian Models

HC1-3 and HC 5 are propositions on individual characteristics, while HC 5 restricts the class of economic systems. HC 6 is a restriction on aggregate outcomes. For each model in the protective belt, the existence proof checks for the consistency of the specific individual properties that are taken as given, and the specific propositions on equilibrium outcomes. HC1-6 are consistent, if individual properties add up to market interdependency, and do not rule out mutually consistent set of choices (Weintraub 1985b: 113-4). It follows that only propositions on individual characteristics are independent and form the axiomatic basis of the economic system, whereas the equilibrium notion depends on the specifications of individual properties. This dependency manifests itself in the long-familiar tradeoff between the cognitive and information-processing requirements imposed on agents, and the requirements on the market system's scope and structure (discussed in the subsequent section).

HC1 gives arbitrary many hypothetical agents. They are not introduced as realistic images of human beings, but as potentially convergent choice processes or, to give a constructivist definition, as (often continuous) algorithms of unspecified cognitive and information-processing capabilities that start from an initial guess and may generate a sequence of approximate solutions to a choice problem. Consistent individual restrictions (axioms given by HC2) imply conditionally convergent choice processes: if provided with sufficient data, i.e., 'relevant information', an individual approximation routine is able to detect (small neighborhoods of) 'best elements' (or bliss points) *independent of its initial position*. Evidently, such choice processes are the neo-Walrasian specification of 'individual laws of motion'. If a choice process conditionally converges, it is called rational. Thus, rationality involves the cognitive capabilities of hypothetical agents. Evidently, rationality varies over the different classes of general equilibrium models in the protective belt of the neo-Walrasian program, and *should in no case be understood as intended to describe actual cognitive processes of real human beings* (as the myth goes).

HC2 specifies the notion of self-interest. Binary preference relations impose structure on individual opportunity sets on which the choice algorithms are applied respectively. The partitioning of choice sets into equivalence classes renders meaningful the direction of the individual laws of motion. As will be shown immediately, HC2 in conjunction with HC3 and HC4 implies 'local nonsatiation' as a generic minimum requirement on preferences, that is, order and preference relations generically concur within an arbitrary small neighborhood of each element in the individual's choice set. Thus, self-interest at least suggests that, locally, more is preferred to less, which resembles Hume's individual imbalance. Only additional individual restrictions on preferences specify rationality for each class of model in the protective belt.

Inter alia, HC3 introduces Hume's quid pro quo condition (and thus implies a non-empty set of price systems): if an agent draws commodities from the system (or scarce resources that would otherwise contribute to the provision of such commodities), it is obliged to contribute to the system other commodities (or resources) of equal value. An equilibrium price system is more than a list of tradeoffs that controls individual substitution decisions. It also attaches weight to each agent according to the relative scarcity of its potential contribution, i.e., its initial endowments. In private ownership economies, such endowments are those inputs to transformation processes that are not also outputs of other production activities. Thus, HC3 corresponds to Hume's ideal of a just society, where each individual can only rise above the others (in terms of purchasing power) by contributing more or more valuable benefits than others. Note that HC3 also restricts the interplay of individual laws of motion to impersonal market interactions.

Local nonsatiation ensures that the individual wealth constraints are binding. Thus, HC4 can be easily derived from HC2 in conjunction with HC3. For a finite exchange economy, let $\mathbf{x}_i(\mathbf{p})$ be a bliss point of individual $i \in \{1, ...I\}$ for an arbitrary price vector $\mathbf{p} >> 0$, and let $\mathbf{e}_i$ be its initial endowment vector. Given local nonsatiation, HC3 implies $\mathbf{px}_i = \mathbf{pe}_i \Leftrightarrow \mathbf{p}(\mathbf{x}_i - \mathbf{e}_i) = 0 \Leftrightarrow \mathbf{pz}_i = 0, \forall i$, where $\mathbf{z}_i$ is the i-th agent's excess demand. It follows that $\mathbf{p}\sum_i \mathbf{z}_i = 0 \Leftrightarrow \mathbf{pz} = 0$, which, of course, is Walras' Law. Given that $\mathbf{z} = \sum_i \sum_n z_i^n$, where $n \in \{1, ...N\}$ is a *marketed* commodity, HC4 can be identified with the famous corollary of Walras' Law: $\mathbf{pz} = 0 \Leftrightarrow \sum_1^{N-1} p_n z_n = -p_N z_N$. Given interior price systems, $z_N = 0$, if and only if $\sum_1^{N-1} z_n = 0$, which establishes market interdependence and, given that HC3 isolates market relations, the *full* interdependence of individual laws of motion.

Naturally, since HC1-3 are generic properties of neo-Walrasian economies (and do not only apply to small-square exchange economies), so is Walras' Law and, thus, HC4. Evidently, local nonsatiation is a necessary condition for HC1-3 to aggregate to HC4. Note that HC4 takes to the extreme Mandeville's and Hume's proposition that the exchange nexus provides the predominant social kit of commercial societies. This accounts for the fact that the 'logic of exchange' dominates even in private ownership models (ownership of firms), and that the scarcity problem is generic to neo-Walrasian models. This, of course, is a major difference to Smith's model (and to the 'canonical classical model' as such), where production relations dominate general equilibrium outcomes. Hume's vision, however, is successfully isolated: the inevitable conflict of interest is appeased by a price system that provides a 'civil' ranking of agents, and that thereby gives expression to a consensus view with regard to the relative scarcity of individual contributions.

Most importantly, however, *HC4 constitutes 'non-summative wholeness' as a generic property of neo-Walrasian economies.* Given that price systems weight agents via their respective wealth constraints, and given that relative weights matter in the orchestration of individual laws of motion (in general equilibrium), a summative approach that would decompose the neo-Walrasian economy (always summarized by an aggregate excess demand function) into partial markets, and analyzed them independently, would lose essential information about the system's logic. Given such irreducibility, it follows that the simplest neo-Walrasian model must contain two markets and two agents (Smith's beaver-and-deer exchange economy). In this smallest-square-economy, however, a partial equilibrium on any of the two market suggests the existence of general equilibrium (already by Walras'

Law!). For non-summative wholeness to suggest complexity, at least three markets and three agents must be given. Herbert Scarf's (non-)convergence results revealed that the system's intelligence may already misbehave in such an economy, and that convergence is far from trivial (Scarf 1960).

Note, however, that the degree of complexity is not increasing in $N, I > 2$! According to John H. Miller and Scott E. Page, a distinction must be made between complex and complicated systems:

> "Complexity is a deep property of a system, whereas complication is not. A complex system dies when an element is removed, but complicated one continue to live on, albeit slightly compromised. [...] Complicated worlds are reducible, complex ones are not." (Miller and Page 2007: 9)

Given this distinction, one may say that while the step from $N, I = 2$ to $N, I > 2$ involves complexity, any further increase in the number of markets or agents just complicates the system. By removing the n-th market, the n-th price is removed as well, and neither the notion of general equilibrium, nor its approximation is altered in kind. With regard to complexity, the system's intelligence counts 'one, two, many'.

Evidently, if notions of general equilibrium are *per se* regarded as at variance with Scottish Political Economy (e.g. Blaug 1996: 60-1, Blaug 2008; see also section 3.2.2), it is legitimate to reject neo-Walrasian models *ab initio* as feasible reconstructions of the invisible-hand proposition. The problem however is that neither original sources, nor the pre-WWII literature can provide admissible arguments to substantiate such a grim perspective. This is because of the much more diversified portfolio of modern equilibrium analysis. It is one thing to discount the potential uses of equilibrium reasoning, if the only concept at hand is a deterministic and stationary long-run notion of general equilibrium. But it is quite another thing to apply such arguments to contemporary theory without a detailed study of why these arguments also pertain to more powerful notions of choice-consistency, like dynamic stochastic general equilibria. By its nature, the axiomatic equilibrium method challenges its opponents to generalize their critique.

Due to the higher diversity of equilibrium concepts, the classical (and pre-WWII neoclassical) identification of disequilibrium adjustments with the 'short run', and of equilibrium states with the 'long run' collapses (yet not the important distinction between trend and deviations therefrom). The prediction of a deterministic intertemporal equilibrium model, for instance, is a market-clearing price system that coordinates the entire physical history of the economy, including the very first instance of delivery, and potentially extending to an infinite horizon. Stability analysis concerned with the convergence to such price systems is incompatible with classical short-run or market-process analysis. To identify the classical short-run notion with the 'initial period' in intertemporal analysis, i.e., a period of unspecified length preceding the beginning of the physical history of the economic system during which all consumption and production plans are pre-reconciled via recontracting, is to overload with economic content a very artificial device against the intention of its innovators. Given that the neo-Walrasian program restricts admissible predictions to equilibrium outcomes, convergence processes are not intended to relate directly to empirical data. If the goal is to reduplicate actual time series, then the program dictates to choose between notions of dynamic equilibria and, by HC6, discriminates against the use of out-of-equilibrium processes.

A common charge against general equilibrium reasoning in invisible-hand studies is its lack of realism (e.g. Boettke 1997). According to this view, HC6 constitutes a generic inferiority of the neo-Walrasian program relative to competing programs that predominantly rely on disequilibrium analysis and, thus, on the distinction between the short and the long run. However, such a charge is fundamentally unfounded. '*Dis*equilibrium' is the antagonist notion of 'equilibrium', and the antagonism of a hypothetical notion is not real, but hypothetical as well. *A priori*, the relative inferiority of equilibrium analysis cannot be substantiated. This is also true *a posteriori*, as long as the (pattern) predictions of general equilibrium models are able to reproduce empirical results (e.g., 'if prices are fixed according to extra-economic considerations, the economy degenerates').

If, instead, the efficiency of theory choice is measured by its productivity (as defined above) *as well as by a program's ability to yield generic negative results*, general equilibrium analysis clearly dominates its alternatives. In fact, the most serious problems raised against general equilibrium analysis were discovered by economists who argued from within the neo-Walrasian paradigm. Such negative results, like the aforementioned Sonnenschein-Mantel-Debreu results (section 3.3.2), are important to delimit the *potential* scope of modeling strategies, and to inform and reappraise standards of applied analysis in the protective belt. The health and hygiene of a research program crucially depend on its ability to confute itself (i.e., dominant modeling strategies derived from its hard core) on a purely analytical basis.

If, by HC6 and NH4, disequilibrium processes do not relate to real-world dynamics, what role do they play in neo-Walrasian analyses? The task of axiomatic stability analysis is to augment the list of individual restrictions developed in static analysis (existence and uniqueness studies) by the minimum number of items necessary and sufficient to ensure the isolation of convergence mechanisms that are generically reliable (zero additional items is the theorist's Nirvana). *Convergence is of no independent significance, but serves to strengthen the respective equilibrium notion* (or fails to do so). Most importantly, given the generic global non-uniqueness of Walrasian equilibrium, 'appropriate' convergence mechanism are necessary to select from the set of market-clearing price systems (Duffie and Sonnenschein 1989: 575).

3.1.3 The Generic Tradeoff between Individual and Impersonal Intelligence

As mentioned above, HC1 gives potentially convergent choice processes, and HC1-2 (i.e., potentially convergent algorithms in conjunction with individual axioms) give individual convergence conditional on the availability of 'relevant information'. In this context, 'relevant information' means 'sufficient individual information for at least one general equilibrium to exist', and, thus, suggests minimum requirements for individual information-processing capabilities *as a (discontinuous) function of general equilibrium specifications*: different notions of general equilibrium - i.e., different equivalence classes of models in the (discrete) protective belt of the neo-Walrasian program - imply different minimum information-processing requirements. HC5 predetermines that agents are generically endowed with sufficient information, that is, irrespective of equilibrium specifications. Again, *in no case should HC5 be understood as related to actual information-processing skills of real individuals*. Rather, it is a technical convention that reveals the interrelation between information-processing skills of hypothetical agents, and the processing capabilities of a hypothetical system. In concrete, HC5 reveals an inverse relationship between minimum individual processing capabilities and minimum system requirements. To highlight this important relationship, the class of Arrow-Debreu-McKenzie (ADM) models - discussed in greater detail in subsequent sections - is introduced. ADM models are fundamental for the neo-Walrasian program, and of particular importance in invisible-hand studies.

A characteristic property of ADM models is that they allow for a potentially unlimited scope of the market system. They rest on the insight that commodities are the minimum set of objects necessary to formalize marketed consumption and production choices (Geanakoplos 2004: 116). A commodity is characterized by its physical properties, by the location and date of its delivery, and by the state of nature on which delivery is conditional (Debreu 1959). Each commodity is perfectly homogeneous, that is, even the slightest variations in physical characteristics as well as in locations, dates, and states of delivery are sufficient to differentiate commodities. Evidently, this modeling strategy exploits the facts that an increase in the list of commodities (beyond three) implies additional complications, yet not an increase in complexity. *Intra*-sectoral trade is meaningless in ADM models.

Agents choose between commodity bundles, each specified as an element in vector (sub-)space (over the field of real numbers) denoted by, say, L; they trade state-contingent claims, introduced as the exclusive property rights corresponding to a particular vector. In early small-square ADM models, characterized by finitely many agents and commodities, L is naturally specified as a Euclidean space (i.e., the textbook case of $\mathbb{R}^N$). In contemporary, infinite dimensional ADM-models, which can cope with continuous state spaces and non-terminal time horizons, L is specified as a sequence subspace (i.e., $L \subset \mathbb{R}^\infty$) or, as is more often the case, as a Lebesgue subspace (i.e., $L \subset \mathcal{L}_p, 1 \leq p < \infty$). It is therefore possible to differentiate marketed choices in infinitely many ways (on infinite dimensional spaces in economics, see Mas-Colell and Zame 1991, and Aliprantis and Border 2006). In the case of sequence subspaces, each vector is characterized by countably infinitely many quantitative attributes. In case of $\mathcal{L}_p$-subspaces, where any choice is a continuous function, it is even possible to differentiate in uncountably infinitely many ways!

The market space, say M, is spanned by feasible and linearly independent marketed choices. A characteristic property of ADM models is that they presume all linearly independent choices to be feasible. Thus, the basis of the market space is identical to the Hamel basis of L, say A, such that $M = \text{span}(A) = \text{L}$ (complete markets). The finite linear combination of any two pairwise orthogonal market choices forms a feasible third choice. Since $A \subset \text{L} \subset \mathbb{R}^\infty$or $\mathcal{L}_p$ is infinite, there are infinitely many finite linear combinations possible and, *therefore*, admissible. If $M \subset \mathcal{L}_p$, the market space is not only infinite, but continuous, and allows agents to fully insure themselves even in case of continuous state spaces (potentially defined along a continuous timeline). Evidently, the ADM-property of completeness gives the maximum scope of the neo-Walrasian market system, or, equivalently, its maximum effectiveness.

In consequence, the system's intelligence operates at full information-processing capacity. As is generally the case in neo-Walrasian models, the system's law of motion is given by a feedback relation between a central algorithm (the super auctioneer) subject to an approximation method (e.g. the 'law of supply and demand'), which takes excess demands as input to generate price systems, and an mean excess demand correspondence, which takes prices as input, and which summarizes the primitives of the economy (market by market). In continuous ADM-models, excess demand information is provided by an average excess demand relation, given as a Lebesgue integral over the continuous market space. Thus, the super auctioneer has to process uncountably infinitely many inputs to generate a continuous prices system (given linear pricing, such that the market value of the sum of any two choices equals the sum of their market values, the quid pro quo condition remains well defined in infinite dimensional analysis).

Such a sumptuous market system - and, thus, the operation of impersonal intelligence at maximum capacity - has immediate consequences for individual minimum requirements. As is well known, the definition of state-contingent commodities in conjunction with the completeness presumption ensure that the "Arrow-Debreu model collapses the future into the present and almost all uncertainty into certainty" (Hahn 1992: 184). Most importantly, agents are able to 'price in' all their local information and private beliefs at once. All data that could ever affect individual choice in a non-terminal economic system is aggregated simultaneously, during an initial period preceding the entire physical history of the economy. Hence, *the market system and the physical operation of the economy, i.e., the actual execution of the agreed upon set of deliveries, are disjunct* (markets never reopen).

Accordingly, the notion of 'plan revision' is never associated with imperfect foresight, but collapses to the notion of 'recontracting', which characterizes individual behavior during the radically hypothetical convergence process in the initial period. Once (an ϵ-neighborhood of) an equilibrium price system is detected, a (continuous) list of tradeoffs inform individual substitution decisions symmetrically across time, space, and states of the world. Such a *comprehensive list of prices turns redundant individual price-expectation operators*. This, of course, significantly reduces individual minimum requirements. Note that a continuous equilibrium price system does not suggest that individuals have to process uncountably infinitely many inputs to be able to generate optimal choices. If the model is competitive, as is generically the case in ADM models, existence does not rest on the (weak) convexity of preference relations (as will be discussed in subsequent sections). Thus, agents are not biased towards interior solutions. In infinite dimensional analysis, a corner solution suggests finitely many non-zero entries in individually optimal consumption choices, so that agents can easily ignore infinitely many tradeoffs without affecting the system's effectiveness (this statement is conditional on a 'realistic' endowment profile across agents).

Further, a complete insurance market implies that individuals are subjects to a single lifetime wealth constraint of which they have certain knowledge (e.g. Radner 1982). In making optimal choices, no agent has to tinker with broken promises or bankrupt trading partners. Accordingly, there is no role for an institutional framework to harness individual laws of motion such as to ensure "the stability of possession, [...] its transference by consent, and [...] the performance of promises" (Hume 2000: 337). Recall that the acceptance (and otherwise enforcement) of property rights was essential in Hume's analysis, in that they safeguarded *over time* the ranking of individuals according to the quid pro quo condition. Given an ADM-type price systems, however, also the ranking of agents via wealth constraints collapses into the initial period. Whereas convergence processes are severely stricken by wealth effects, ADM (equilibrium) predictions simply extend the logic of atemporal exchange economies: once an equilibrium price system is approximated (sufficiently close), *all agents are ranked simultaneously* according to the relative scarcity of their certain lifetime contribution to the system, and are allowed to draw from the system scarce benefits of equal value.

In summary, it can be stated that the definition of state-contingent commodities in conjunction with the presumption of completeness, both inherent to ADM models, imply a maximum scope of the market system and, *thereby*, drive to a lower limit the minimum requirements on individual information-processing skills (specified by HC5), and on individual cognitive capabilities (specified by HC2). Hence, *ADM models maximize the wedge between individual and impersonal intelligence*. This explains why the class of ADM models became the modern habitat of the invisible-hand proposition. Most importantly, if the proposition does not hold for ADM economies, that is, if an equilibrium price system does not exist, or does not translate individual optimization into *efficient* social outcomes even in case of maximum effectiveness of competitive markets, then it holds for no other class of models in the protective belt of the neo-Walrasian program. In this sense, the class of ADM models is *primus inter pares*; it exclusively provides other model classes with a normative benchmark, and thereby transcends the invisible-hand proposition to all of them.

For instance, the class of sequence models (including models that predict Radner equilibria or, in case of asymmetric information, rational expectations equilibria) predict efficient outcomes, if they implement ADM allocations (meaning that an ADM model fed with identical primitives would predict equivalent equilibria). Money as a unit of account facilitates the introduction of security markets and, insofar, is a mean to economize on the number of markets that is necessary to implement ADM allocations. Instead of individuals who all know with certainty the (bounded) present value of their lifetime wealth constraint, agents are described as being subject to flow budget constraints: they choose consumption and trade plans on a sequence of open spot markets. In contrast to the ADM system, markets are intertemporally separated and connected only via the interdependency of budget constraints or, equivalently, by economic choices (Radner 1972: 293). Thus, money as a unit of account is 'qualitatively non-neutral' in that it allows to switch to a market setting with weaker requirements on the system's information-processing capability. But money is still 'inessential' in the sense that dynamically complete sequence economies generically implement ADM allocations (Hahn 1973, Arrow and Hahn 1999).

Given a numeraire-commodity, it is possible to define a complete set of 'Arrow securities', each being a claim to future payoff (Arrow 1965). More precisely, each Arrow securities pays one unit of account conditional on a single mutually observable event at a subsequent date, and pays nothing otherwise. Evidently, a full set of Arrow securities (each for one possible state of the world) spans a dynamically complete security space such that any individual portfolio can be compiled by finitely many linear combinations of Arrow securities (for an excellent survey, see Duffie 1988). At any date, each agent can observe a collection of spot prices. At each further date, there exists a price function measurable with respect to a σ-field of elementary events. Thus, a price system is a collection of current prices as well as the mathematical expectation of all future spot prices. For strictly positive prices, the behavior of each agents is described by a set-valued function from prices to admissible consumption and trade plans that maximize expected lifetime utility.

The sequence economy equipped with the common information property predicts *equilibria of plans, prices,and price expectations* or, in short, Radner equilibria (Radner 1972). The system solves for a collection of observable current prices and a sequence of expected prices common to all agents. More precisely, it solves for a common price expectation function that takes value on a common information base (ibid.: 289, 293). Given free disposal and 'well-behaved' agents, Radner proves existence of RE by means of Kakutani's fixpoint theorem (ibid.: 293-4). Hence, despite the fact that coordination requirements are immensely weakened by the assumption that agents receive identical signals, the 'full relevant knowledge' for at least one Radner equilibrium to exist is perfect foresight : for a solution to exist, all agents must forecast the solution. In concrete, each agent must somehow be able to assign an equilibrium price to each event at each future date. As Radner himself points out,

> "the perfect foresight approach is contrary to the spirit of much of competitive market theory in that it postulates that individual traders must be able to forecast, in some sense, the equilibrium prices that will prevail in the future under all alternative states of the environment. [...] *[T]his approach still seems to require of the traders a capacity for imagination and computation far beyond what is realistic. An equilibrium of plans, prices and price expectations might be appropriate as a conceptualization of the ideal goal of indicative planning, or long run steady state toward which the economy might tend in a stationary stochastic environment.*" (Radner 1982: 942; emphasis mine)

Security markets, *even if dynamically complete*, accelerate the minimum requirements on individual cognitive and information processing skills. This reveals the inverse relationship between individual and impersonal intelligence.

3.2 Primus Inter Pares: The Class of ADM Economies

The years 1951 and 1954 are milestones in the history of economics. For the present study, the year 1951 is of particular importance, since it witnessed the appearance of the two fundamental theorems of welfare economics, of which the first is assumed to encapsulate Smith's invisible-hand proposition. The substitution of the powerful tools of convex set theory for the local techniques of differential calculus (Arrow 1951 and Debreu 1951) not only clarified the relations between Pareto-efficient and competitive allocations but, in doing so, also expanded the definition of competitive equilibrium to include individual corner maxima and linear transformation activities as introduced by Koopmans (1951), a framework that allows to determine a commodity's status as a 'free good' by the system's primitives (for a quick introduction, see Arrow 1974, Debreu 1984, and Debreu 1991; fore more extensive literature, see Ingrao and Israel 1990, Weintraub 1985b, 2002).

In 1954, the long quest for a general and neat existence proof eventually culminated in the celebrated paper by Arrow and Debreu (1954; see also Debreu 1952), who employed Kakutani's generalization of Brouwer's fixpoint theorem (Kakutani 1941). For the case that also entrepreneurial skills are marketed such that firms earn zero profits (and are thus redundant), existence is proved by McKenzie (1954). Although of particular importance for the success of the neo-Walrasian program, existence proofs play a relatively minor role in this study. Further, the first fundamental theorem is not reconstructed according the Arrow's original demonstration (in addition to Arrow 1951, see Arrow and Hahn 1971, Arrow 1974, and Duffie and Sonnenschein 1989). For the sake of clarity, and to economize on space, the theorem is rather introduced as a corollary of Aumann-type equivalence results.

Darrell Duffie and Hugo Sonnenschein provide an informal, yet precise statement of the invisible-hand proposition for the ADM setting:

> "For externality-free economies in which all agents are infinitesimal, there is theoretical support for the assertion that individual agents, free to bargain, exchange, and make deals that are best for themselves, will be forced to act as if they are price takers. Furthermore, in the presence of mild technical assumptions, such economies have at least one Walrasian (price-taking) equilibrium and each of these is Pareto optimal. Finally, in this equilibrium the action of agents will be decentralized in the sense that they will depend only on their own characteristics and the common prices." (Duffie and Sonnenschein 1989: 582-3)

The absence of externalities is implied by the completeness presumption introduced in the previous section. It simply mirrors the fact that the provision of a potentially infinite market space must be costless (and, thus, the economy frictionless), for otherwise the bounded stocks of resources would be completely exhausted without affecting individual utilities. Given zero transaction costs, the Coase theorem rules out externalities (private and social costs concur; Coase 1960, 1988). Recall that Hume's higher-order invisible-hand proposition was also concerned with the evolution of friction-minimizing institutional solutions. Recall further that Smith assumed this institutional groping process to have reached some efficient equilibrium, where all such frictions are minimized, with the purpose to isolate purely economic mechanisms. State-contingent commodities in conjunction with the completeness presumption takes this approach to the extreme, and suggest a *perfect* system of natural liberty. A market system at maximum capacity (in conjunction with 'infinitesimal' individuals) fully eliminates the rationale for bilateral social interaction, and is *perfectly* anonymous. Hume's 'contrariety of passions' never erupts, and must not be harnessed by social or legal norms.

3.2.1 On Competitiveness as a Static Property of ADM Economies

At least since Cournot (1838) and Edgeworth (1881), perfect competition is associated with the insignificance of individuals relative to the number of identical peers that can provide outside options to potential trading partners. Individual insignificance extinguishes the incentive to behave strategically and, therefore, guarantees that individuals behave *as if* prices were primitives of their choice problem. For the case of non-cooperative firms in a homogeneous-good market, where each firm chooses among quantity strategies conditional on the rivals' decisions, Cournot establishes that

> "[t]he effects of competition have reached their limit when each of the partial producers is *inappreciable*, not only with reference to total production but also with respect to the derivative [of the market demand function], so that the partial production could be subtracted from the total production without any appreciable reduction in the price of the commodity." (quoted from Khan 2010: 4; Cournot's italics)

Here, perfect competition is only a limiting case, indicating convex production opportunity sets. Cournot competition is a more general concept, also applying to the case of indivisible production activities such that, for each firm, minimum average costs are bounded away from zero (i.e., the efficient scale is non-infinitesimal), and such that their is a limit on the number of firms which are active in equilibrium. Thus, for nonconvex technologies the number of firms is endogenously determined, given that entry is free (no frictions). Strategic behavior prevails, if the efficient scale is non-infinitesimal and, thus, excess profits are positive.

For Edgeworth (1881: 17-9), a "perfect field of competition" with reference to a given allocation depends, inter alia, on a set of agents such that each item can improve upon a given allocation by recontracting "with any out of an *indefinite number*" of other agents (my emphasis). Further,

> "we see that in general for any number short of the practically infinite (if such a term is allowed) there is a finite length of the contract curve [...] at any point of which the system is placed, it cannot by contract or recontract be displaced; that there are an indefinite number of final settlements, a quantity continually diminishing as we approach a perfect market." (ibid.: 38-9)

Like Cournot's concept of competition, Edgeworth competition also converges in case of non-infinitesimal agents. Again, individual insignificance is a limiting case. Unlike Cournot competition, competition in the sense of Edgeworth is described by the formation of independent 'coalitions', that is, by *cooperative* agents that cut tentative (or non-binding) deals to the advantage of all counterparties involved. A purposed allocation is rejected or, equivalently, 'blocked', if a coalition of agents identifies a preferable allocation that is also affordable once coalition members pool their budgets. The primitive solution concept is the 'core', i.e., the set of allocations no coalition can improve upon (Edgeworth's contract curve; see Shubik 1959). A 'core economy' is defined as a mapping from the space of agents into the space of their characteristics, where each agent is described by a preference ordering over its choice set (a subset of the market space), by its location according to its initial endowment, and by its knowledge of how to transform commodities by means of production activities.

Even though Novshek and Sonnenschein, among others, extended Cournot competition to the case of general equilibrium and showed that, given the 'law of demand', a Cournot equilibrium approximates Walrasian solutions (Novshek and Sonnenschein 1978, Theorem 2; see also Novshek and Sonnenschein 1987), core competition has proven to be the far more influential notion. This is because neo-Walrasian and core analysis involve the same set of mathematical tools, share the same set of primitives, and because sufficiency conditions for core equivalence results rest on little more than neo-Walrasian existence requirements (see Scarf 1962, Debreu 1962, Debreu and Scarf 1963 in contrast to the transferable utility approach by Gillies 1953 and Shapley 1953).

Further, core analysis is of higher generality. For instance, whereas Cournot competitors are restricted to choose between quantity strategies, no such restriction is imposed on the bargaining activities associated with the formation of blocking coalitions. Most importantly, inferences are drawn from institutionally unspecified models that are, thus, predestinated to supply benchmark allocations for the more specific models in ADM- or Cournot-type analysis (Mas-Colell 1982). As usual, such an advantage comes at costs: core allocations are essentially non-operational. On its own, analysis of large core economies has no resilient implications. Rather, it exclusively serves to support more operational frameworks. As Werner Hildenbrand argues,

> "the theoretical (non-operational) concept of the core of an economy is [...] used to justify concentrating the attention on the more operational concept of Walrasian equilibrium. Hence, our main goal is to show that for large competitive economies every allocation in the core can be 'approximately' decentralized by a suitably chosen price system [...]." (Hildenbrand 1982: 836)

A more serious problem is that right because models that predict core allocations do not specify trading technologies, the only sensible feasibility constraint for a coalition to form is that the arbitrage opportunities associated with a purposed reallocation are attainable independent of any other trader for all commodities simultaneously considered. Whereas a price system weighs and, insofar, interrelates individual budget sets, institution-free core competition as collusive action "works by setting of entirely separated subeconomies"(Mas-Colell 1982: 16). Further - and unsurprisingly - informational requirements are prohibitively high, suggesting public information of tastes, technologies, and trade sets of active agents (Hildenbrand 1982: 832, Mas-Colell 1982).

Robert Aumann's improvement on our understanding of perfect competition in ADM economies by means of core analysis is a preeminent example of its supportive role for neo-Walrasian analysis (Aumann 1964). There, he proves that Debreu's *Theory of Value* (1959) is not the final world on general equilibrium, since a finite number of agents cannot account for individual insignificance and, thus, perfect competition. By extending neo-Walrasian analysis to include continuous ADM models, he reinforces the intuition of Cournot and Edgeworth:

> "Though writers on economic equilibrium have traditionally assumed perfect competition, they have, paradoxically, adopted a mathematical model that

> does not fit the assumption. Indeed, the influence of an individual participant on the economy cannot be mathematically negligible, as long as there are only finitely many participants. Thus *a mathematical model appropriate to the intuitive notion of perfect competition must contain infinitely many participants*." (ibid.: 39; Aumann's emphasis)

In concrete, the number of agents must be uncountably infinite and, thus, the agent space continuous: the index set of agents is a closed unit interval with aggregate Lebesgue measure of unity such that each set of positive weight contains a subset of smaller positive measure (Lyapunov's Theorem); the precise meaning of 'individual insignificance' is 'to be of zero measure', or to be 'being non-atomic'. Like a real number whose removal does not alter the length of the unit interval, the economic weight of a single agent is negligible relative to a continuum of peers. To gain significance, sufficiently many agents must pool their resources and, thereby, form an atomic coalition.

Thus, whereas individual insignificance is a defining characteristic of ADM allocations (a *static* property), it is a limiting case in core analysis. Aumann exploits this relationship and proves for a continuum of traders that "the core coincides with the set of equilibrium [or competitive] allocations" (ibid.: 43). Whenever agents are atomless, they act *as if* prices are given. This, then, is the meaning of perfect competition as a generic property of ADM economies. Moreover, since core allocations are both, Pareto efficient and individually rational, the first fundamental welfare theorem - the *modern invisible-hand proposition* - can be inferred as a corollary of Aumann's equivalence result: *equilibrium price systems invariably decentralize efficient allocations.* Note that in contrast to Arrow's original proof for small-square ADM economies, convexity of preference relations is no necessary condition (Largeness in case of heterogeneous agents already guarantees convex aggregate excess demand correspondences).

Before Aumann, Shubik (1959) has proven such equivalence for two types of agents and two commodities, where a 'type of agent' is an atomic equivalence class with respect to tastes, technologies, and endowments. In accordance with Edgeworth, each of the two types of agents is replicated until the core shrinks to the set of competitive equilibria. Given monotonic, transitive, and strongly convex preferences, the core shrinks because replication overcomes individual indivisibility such that there are more and more blocking opportunities (see also Hildenbrand and Kirman 1988, Starr 1997). In the n-fold replica (with $n > 1$), n agents of type one can afford a convex combination of their initial endowment and the least preferred core allocation of the $(n-1)$-th replica by colluding with $n-1$ agents of type two (and vice versa). As n increases to infinity, all but competitive core allocations survive, that is, those at which type-specific indifference curves are separated by the set of all such convex combinations.

Subsequently, Scarf (1962) and Debreu and Scarf (1963) generalized Shubik's result to the case of k types of agents and l commodities. Thus, a core allocation associated with the n-th replica economy lies in the Euclidean space of dimension $n \cdot k \cdot l$. Equal treatment of equal agents by core allocations is, again, ensured by strongly convex preferences: whenever a purposed redistribution assigns different bundles to agents of the same type, the agent with the smallest bundle can afford a type-specific average by pooling its resources with the 'worst off'-members of other types, who thereby move to type-specific averages as well. Since each replication uniformly adds to each type such that the distributions of agents' characteristics remain fixed, it is sufficient to focus on $k \cdot l$ dimensional allocations, that is, on mappings between arbitrary large but finte dimensional spaces.

Hence, equivalence theorems that rely on replica economies, i.e., on a sequence of copies of an initial core economy that is increasing in each of the k basic types of agents, and which approximates an ϵ-neighborhood of a set of Walrasian equilibria, evade an explicit statement of the limit economy. Although convenient, the downside is that the replica approach assumes an arbitrary large number of *identical* agents. Aumann's immediate focus on a continuous model avoids such a prohibitive restriction:

> "The continuous model allows *all* traders to have different initial bundles and different preferences. There is no problem of working in spaces of varying dimension, because we start with a space of *infinite* dimension and remain in this same space throughout the investigation." (Aumann 1964: 48-9; his emphases)

Instead of relying on identical agents, Aumann introduces a homeomorphism between the agent space and the convex and preference-partitioned positive orthant of a finite dimensional commodity space, i.e., a (pointwise) continuous mapping of agents onto characteristics which has a continuous inverse, such that the inverse image of any open choice set is an open set of agents, and vice versa. Each such open set is contained in a neighborhood of 'close' or 'similar' points so that the topological structure of the commodity space carries over to the space of agents. Aumann defines a set of agents to be 'full', if it includes 'almost all' agents, that is, if it includes all but the sets of zero measure (singletons, finite sets, or countable sets) such that the full set is of measure one. Thus, the set of agents is partitioned into a large number of *atomic* coalitions of similar agents (ibid.: 45).

The "key to the proof" is a lemma (Lemma 4.1; ibid.: 45, 49). For each agent, Aumann defines a set of net trades which give an allocation strictly preferred to a core allocation.

The lemma proves by contradiction that the convex hull of the union of all agents' preferred net trades and the interior of the negative orthant have an empty intersection for an allocation to be in the core for a full set of agents with monotonic preferences. Otherwise, it would be feasible for some (disjoint) atomic sets of agents to move to a preferred gross trade by simply giving away initial endowments to insatiable fellows such that the allocation could not be in the core. Given this lemma, Minkowski's Theorem guarantees that there exists a supporting hyperplane to the convex hull of preferred net trades such that its normal, i.e., the price system, decentralizes the core allocation. This proves Aumann's theorem (ibid.: 46-7).

The supportive role of core analysis for neo-Walrasian reasoning is evident: independent of any concrete specification of the allocation mechanism, the universal absence of the incentive to behave strategically - or to 'game the system' - is shown to be exactly true for a mass economy only. The strength of Aumann's equivalence result roots in its generality. Besides the economically insignificant assumption of measurablity, monotonicity and continuity suffice to prove the theorem. Note what Aumann does not assume: preferences are neither transitive, nor irreflexive, nor convex. In contrast to Debreu and Scarf, different agents might hold different commodities. Even though each commodity is initially held by some agents, no single agent is endowed with a positive amount of all commodities (ibid.: 44). By relaxing monotonicity to local insatiability, Hildenbrand (1969, 1974) has shown subsequently that a core allocation is a quasi-equilibrium, that is, a price system such that 'almost all' agents have 'little' incentive to deviate from the core allocation.

Of course, the resort to uncountably infinite agents is 'unrealistic' by itself. It seems as if individual assumptions are weakened by imposing a prohibitively high restriction on the economic system. Aumann, however, defends his theorem:

> "The idea of a continuum of traders may seem outlandish to the reader. Actually, it is no stranger than a continuum of prices or of strategies or a continuum of 'particles' in fluid mechanics. In all these cases, the continuum can be considered an approximation to the 'true' situation in which a large but finite number of particles (or traders or strategies or possible prices). The purpose of adopting the continuous approximation is to make available the powerful and elegant methods of the branch of mathematics called 'analysis', in a situation where treatment by finite methods would be much more difficult or even hopeless [...]." (ibid.: 41)

In fact, it can be shown that for economies with a 'sufficiently large' but finite number of agents every core allocation is a quasi-equilibrium (Hildenbrand 1982). The proof rests on the Shapley-Folkman-Starr Theorem. Thus, 'sufficiently large' suggests that the number of agents exceeds the dimensionality of commodity space.

3.2.2 Static Efficiency versus Dynamic Efficiency

Are the modern concepts of perfect competition and static efficiency consistent with Scottish Political Economy? The answer to this question divides historians of economics and economic practitioners. The answer of the latter is affirmative. With respect to perfect competition, Smith is explicit that largeness is a prerequisite for his (short-run) system to converge. Already in the *Lectures on Jurisprudence*, he refers to a "plentifull market" (Smith 1982: 364). There, he explicitly argues against the strategic collusion of the few against the many:

> "All such companys [Smith's monopolies] prevent a free concurrence, which brings down the price of every thing to its naturall height. A few persons can never make this concurrence, and these few can easily lay their heads together and join in bringing in a small quantity and selling it very high, which would be prevented by a free concurrence." (ibid.: 363)

Smith understood very well that collusions imply a reduction of outside options, and that largeness reduces the "chance" of collusion. In the *Wealth of Nations*, he argues:

> "The quantity of grocery goods, for example, which can be sold in a particular town, is limited by the demand of that town and its neighborhood. The capital, therefore, which can be employed in the grocery trade cannot exceed what is sufficient to purchase that quantity. If this capital is divided between two different grocers, their competition will tend to make both of them sell cheaper, than if it were in the hands of one only; *and if it were divided among twenty, their competition would be just so much the greater, and the chance of combining together, in order to raise the price, just so much the less.*" (Smith 2003: 460; my emphasis)

Thus, the intensity of competition is a function of the number of homogeneous firms among which a given level of capital is distributed. Free competition is a limiting case where the sheer number of competitors prohibits any collusive activity. Indeed, it seems natural to esteem Smith's formulation as the nucleus of the modern understanding of perfect competition.

Nevertheless, a phalanx of renowned historians of economics (and Austrian economists) insist that Smith's notion of competition is misrepresented by general equilibrium analysis (e.g. Blaug 1996: 594-595, 1997, 2008, 2001, Hayek 1980, 2002, Hutchison 2000). Mark Blaug's critique is representative: he argues that general equilibrium analysis reduces the notion of competition to describe "an end-state in the rivalry between buyers and sellers" such that "the contest between market traders is finally resolved", whereas Smith's concept suggests "a process of rivalry that may or may not terminate in an end-state" (Blaug 1996: 593). Against the evidence quoted above, Blaug even maintains that for Adam Smith "[n]either competition nor monopoly was a matter of the number of sellers in a market [...]" (Blaug 2001: 153)! Consequently, he argues that

> "[p]erfect competition is an utterly misleading concept, not least because it directs attention to what equilibrium looks like when we get to it, whereas the real problem is that of the process of converging on it. In short, competition is a disequilibrium phenomenon and yet general equilibrium à la Arrow and Debreu, the existence problem, the fundamental theorems of welfare economics, perfect competition and the like, all foster concern with the end-state of competition at the expense of a consideration of competition as a dynamic process." (Blaug 1997: 4)

However, it should be clear by now that perfect competition as an "end-state", that is, as an equilibrium condition, is a misrepresentation. Rather, perfect competition is a defining characteristic of the economic system. Largeness is a static property that excludes strategic behavior in equilibrium *as well as in disequilibrium*: in equilibrium it ensures that (almost) no one has the incentive to deviate from the social optimum; in disequilibrium it ensures that (almost) no agent misrepresents itself to the super-auctioneer. Both, Smith's 'free competition' and the contemporary notion of perfect competition prevent agents from 'cheating' during the convergence process.

Note also that the fixed number of firms in ADM models is not in opposition to Smith's emphasis on market entry and exit. If inactivity or 'free disposal' is allowed for, it is possible to regard market entry as activity of a formerly inactive firm; accordingly, exit is a switch from active to passive; see Arrow and Hahn (1971). In fact, Smith's model as introduced in section 2.3 exploits this 'trick', which is fundamental in explaining his long-run convergence process. Further, the "Cournot-Marshallian-ADM synthesis" by Novshek and Sonnenschein (1987: 1282; 1978) provides a class of general equilibrium models that do explicitly account for free entry and exit. Most importantly, the modern concept of perfect competition extends to this class of models as well:

> "We use the term *perfect competition* to describe a situation in which firms are arbitrary small relative to their markets. [...] As firms become small relative to the market, we observe in accordance with Cournot that their influence on prices disappears and it is the limit of this that we call *perfect competition*." (Novshek and Sonnenschein 1987: 1282)

Hence, Blaug's reasoning does not suffice to reject generally the consistency between Smith's free competition and the contemporary notion of perfect competition as a static property of neo-Walrasian models.

Against the identification of the Scottish invisible-hand proposition with the first fundamental welfare theorem, Blaug argues that the Scots' notion of efficiency is intimately related to an out-of-equilibrium competition process and, thus, ultimately dynamic. Thus, "to imagine that the 'dynamic efficiency' which they clearly ascribed to the competitive process is *exactly the same thing* as the 'static efficiency' of Pareto and Arrow-Debreu is to pile travesty on travesty" (Blaug 2001: 153; italics mine). Recall from section 2.3 that Smith measured efficiency in terms of average per capita income, and that such efficiency is highest during the convergence of his long-run system. By contrast, Smith's general equilibrium - the 'dull' state of society - is inefficient in terms of average per capita income. Recall further that Smith assumed the convergence of the long-run system to be highly sluggish, due to counteracting productivity gains induced by the (intra-firm) division of labor, and that he therefore approached actual economic processes with his disequilibrium apparatus. In this sense, Smith's notion of efficiency exclusively applies to disequilibrium dynamics, and Blaug's argument is insofar valid. The concept of 'dynamic efficiency' is indeed a characteristic property of Scottish Political Economy. Recall from section 2.2 that Hume associated efficiency with civilization processes in real time, i.e., with non-teleological dynamics driven by a cumulative feedback loop between the 'partitioning of employments' (and, thus, economic growth), and decentralized innovations in social and legal norms.

However, the predominance of dynamic efficiency in Scottish Political Economy does not per se delegitimize the attempt to rationally reconstruct the invisible-hand proposition by means of the ADM apparatus. Being well-versed in the history of economics, theorists like Kenneth Arrow or Paul Samuelson of course never maintained that the "static efficienty of Pareto and Arrow-Debreu" and the Scottish notion of dynamic efficiency meant "exactly the same thing". ADM models are rather designed to give *new* expression to an old idea. Or, in the words of Terence Hutchison (2000: 314), the invisible-hand proposition is not 'translated', but '*transformed*': ADM models isolate the role of market prices in shaping the orchestration of locally informed and cognitively limited individuals that seek for their own interest to the benefit of society. They free the exchange nexus as implied by the scarcity problem from the classical long-run system and, therefore, from feudal artifacts like subsistence wages, profit as extracted (residual) surplus, or the struggle between Whigs and Tories over net income. Hence, ADM models predict stylized images of Hume's vision of impersonal coordination in a civil society ('ideal types' in Max Weber's terminology), and highlight the role which he ascribed to the quid pro quo condition in the decentralization of 'beneficial' allocations (i.e., the 'civil' ranking of agents according to the relative scarcity of their respective contributions). In isolating the Scottish idea of anonymous coordination, *ADM models are superior shelters of the invisible-hand proposition* (relative to the canonical classical model as 'translated' by Samuelson 1978).

Further, there is no reason to reproach general equilibrium theorists like Arrow and Samuelson for admitting their indebtedness to classical analysis. After all, ADM models are the ultimate generalizations of the Ricardian model of international trade (which introduces opportunity costs for the special case, where they coincide with the costs of labor). Blaug's hostile polemics ("to pile travesty on travesty") are therefore misleading: given that Smith's notion of dynamic efficiency perishes with his mercantile long-run system, it is natural to formulate a substitute notion of social efficiency, one that suits best the transformed invisible-hand proposition. In fact, Pareto efficiency was resorted to precisely because it is *no* perfect substitute of the Scottish notion of efficiency (i.e., not "exactly the same thing"). The great strength of Pareto's brilliant concept is that it incapacitates the theorist's normative judgment, and empowers his hypothetical agents instead, that is, those who inhabit his model, and who must actually tolerate the aggregate outcome of their individual choices (the agents' egalitarian values might play a role via the second fundamental welfare theorem). The normative biases of Mandeville, Hume, Smith, Blaug, or any other economist are neutralized by imposing quasi-unanimity as efficiency criterion: any subset of agents that can agree upon pooling (price-weighted) resources in order to gain significance (a positive weight, however small), is therewith empowered to veto any (re)allocation. Such 'tyranny of the status quo' leaves little room for the normative beliefs of outsiders like us. In this respect, the sterility of ADM reasoning is a strength, not a weakness as argued by Blaug and Hutchison.

The prominent role of existence proofs and efficiency results in ADM analysis does not impair the significance of disequilibrium studies, usually summarized in subsections on stability. The transformed invisible-hand proposition would be stuck in limbo, if approximation routines would in general be unable to detect equilibrium price systems (or ϵ-neighborhoods thereof), even in case that such systems exist and decentralize efficiently.

Naturally, convergent mechanisms are isolated by restricting the class of admissible aggregate excess demand correspondences, that is, by specifying sufficiency conditions in the form of static system characteristics (which are hopefully implied by individual restrictions). As a matter of course, axiomatic reasoning interrelates existence and stability results (as well as results concerned with uniqueness): the greatest conceivable success of general equilibrium analysis would be a sequence of lemmata, theorems, and corollaries that proofs uniqueness and stability with the axiomatic basis of existence proofs, that is, without additional individual restrictions (a hopeless task, as was soon found out).

Blaug regards as fundamentally flawed this interrelation between existence and stability in axiomatic analysis (Blaug 1996). He argues that *static and dynamic analysis ought to be completely separated fields of inquiry*, at least if one aims to remain within the Scottish framework. He even goes as far as to reject Samuelson's correspondence principle, which suggests a link between static and dynamic properties (Samuelson 1947). In concrete, the principle claims that comparative statics is a legitimate method of analysis, if (1) there exist at least one dynamic system (a set of differential equations) that locally converges, if (2) it is possible to derive *static* restrictions on individual characteristics that isolate such an approximation routine, and if (3) individual restrictions sufficient for local stability are those that already imply the theorems of comparative statics (see also Ingrao and Israel 1990: 266). Blaug is critical:

> "Surely, existence and stability are tied up together and to address one is to address the other? Is it not what Samuelson called the 'correspondence principle'? *By no means*; for centuries before Adam Smith and for a half-century or more after Adam Smith, competition meant an active process of jockeying for advantage, tending towards but never actually culminating in an equilibrium end-state." (Blaug 1996: 593, emphasis added)

This critique, however, is fundamentally flawed, for it suggests that stability in general equilibrium analysis usually requires more than a tendency towards market-clearing price systems. The very nature of a convergent algorithm (e.g. asymptotic convergence), in contrast to a finite algorithm, is that the approximation process is "never actually culminating in an equilibrium end-state". Surely, convergence does not revoke the link between dynamics and statics; it *is* the link. In fact, Blaug's claim that the 'process concept of competition' is unrelated to static properties of general equilibrium systems, or that a focus on such characteristics is of little help in coming to grips with Smith's 'true intentions', can be easily disproved. Since his critique is generic, it is sufficient to find a single instance (besides perfect competition) where Smithian dynamics rests on a static assumptions.

Take, for instance, Smith's famous proposition that technological progress and, thus, economic growth (in terms per capita income) results from the 'division of labor' which, in turn, depends on the 'extent of the market' (Smith 2003: 8-32). Thus, progress is associated with increasing returns to scale. As shown in section 2.3, Smith's theorem is at the heart of what Blaug recognizes as Smith's notion of 'dynamic efficiency'. Yet, as is shown by Robbins (1934) as well as by Edwards and Starr (1987), only non-convex production possibility sets can account for the dependence of 'division of labor' on the 'extent of the market' (i.e., effective aggregate demand or, via Say's Law, aggregate supply). More precisely, only if production opportunities are indivisible, the scale of production activities can constrain the utilization of technological knowledge. Thus, *the convexity assumption on opportunity sets dissolves Smith's twin concepts of coordination and development*. Of course, indivisibility is a static property imposed on an economic system. This proves that Blaug's generic critique is mistaken. Further, the example provides an instance where general equilibrium analysis sharpens our understanding of Smith's vision by revealing hidden assumptions.

3.2.3 The ADM-Specification of the Rationality Postulate

As argued in section 3.1.3 (HC2-3 and HC5), the class of ADM models guarantees minimum requirements on individual rationality and information-processing skills (by maximizing the wedge between individual and system requirements). This section surveys the relevant results, and discusses the ADM-specification of the neo-Walrasian rationality postulate (HC2-3). Recall that rationality is a set of individual restrictions that ensures the convergence of choice processes, conditional on the availability of 'relevant information', and that in addition implies existence (and which is also implied by existence). Informational requirements are discussed in the subsequent section. Given the metarule (MR) as formulated in section 3.1.1, which says that system properties must be reducible to individual characteristics, every ADM economy is structured by its population (yet not by the distribution of characteristics). The primitives of the system are the agents' preferences, their endowments, and, if production is allowed for, also their knowledge of how to transform such commodities. Given that "the very basis of economic analysis, from Smith on, is the existence of differences in agents" (Arrow 1986: 389), agents are of arbitrary heterogeneity (including the heterogeneity of beliefs, given by subjective probability distributions on state space).

The original proof of existence for small-square ADM models isolated reflexivity, completeness, transitivity, continuity, monotonicity, and convexity as necessary and sufficient restrictions on individual preferences (Arrow and Debreu 1954). A more narrow but also more common definition confines rationality only to reflexivity, completeness, and transitivity. Given this narrow definition, an equilibrium price system exists, if preferences are rational, monotonic, continuous, and convex. Transitivity suggests that 'agents can consistently make up their mind', that is, for each agent there is an ordering of options from 'best' to 'worst'. It allows to partition individual choice sets by equivalence classes and, therefore, excludes the possibility of cycling choice processes. Completeness assumes that for *any* two alternatives, each agent weakly prefers one to the other such that comparison is always possible. Thus, for each agent there exists a *global* ordering of options. In case where each of the two alternatives is weakly preferred to the other, that is, in case of indifference, it seems natural to assume that choice is random. Reflexivity ensures that the choice process can cope with indistinguishable options.

Evidently, there is nothing in the Scottish outline that supports the consistent global ordering of options as a generic individual characteristic. Fortunately, subsequent results (foremost Mas-Colell 1974, Shafer 1974, Shafer and Sonnenschein 1975, Aliprantis and Burkinshaw 1988) show that - for finitely many agents and commodities - neither transitivity, nor completeness, nor reflexivity are necessary to prove existence. The notion of preference ordering can be dropped altogether if replaced by the rather harmless assumption of irreflexivity, meaning that no consumption alternative is preferred to itself. Stated differently, choice processes converge, even if agents are not rational in the narrow sense defined above!

Monotonicity implies that 'more of each commodity is preferred to less' and, insofar, provides a link between order and preference relations. Since Smith's agents systematically strive for higher per-capita income, monotonicity seems unproblematic with respect to the congruence of Scottish Political Economy and the ADM reasoning. Yet, monotonicity is a highly restrictive assumption, for it rules out satiation points on potentially infinite individual choice sets. No such assumption is supported by Smith's reasoning (or by classical analysis in general). Note, however, that existence and optimality do not rest on monotonicity (i.e., monotonicity is sufficient, yet not necessary). Rather, for at least one general equilibrium to exist, it is necessary (and sufficient) to restrict nonsatiation (Hume's individual imbalance) to arbitrary small neighborhoods of any consumption alternative (Arrow and Hahn 1971: 79). As argued in section 3.1.2, local nonsatiation is a generic minimum requirement, that is, not only necessary to ensure existence in ADM models, but in all classes of models in the protective belt of the neo-Walrasian paradigm.

The convexity assumption, however, poses a deeper problem. It claims that upper contour sets are weakly convex such that individual demand correspondences are weakly convex. During the early stages of axiomatic reasoning, convexity of individual demand was thought necessary to establish a convex aggregate demand correspondence which, in turn, is necessary for Kakutani's fixpoint theorem to apply. Convexity is highly restrictive: it suggests, for instance, that all agents weakly prefer Blends over Single Malts; or that they weakly prefer to mix their Scotch with Porter. It is inconceivable that Adam Smith would accept such a necessary condition for a solution of his coordination problem to exist. Yet, the problem roots even deeper. Nonconvex preferences allow agents to opt systematically for corner solutions or, equivalently, that they specialize with respect to consumption activities. Such behavior is excluded by the convexity assumption.

Also with regard to production, early proofs assumed firm-specific technologies to be convex, an assumption which corresponds to the idea of non-decreasing marginal costs. For a given positive price system, the convex combination of any two production alternatives is as least as profitable as any of the two initial activities. The supply correspondence of each firm is convex-valued. It follows that the aggregate supply correspondence is convex-valued, too. Profit maximizing firms are thus systematically biased to transform various inputs into a variety of outputs. As in the case for consumers choice, convexity constraints the scope of specialization for each firm. As a necessary condition for existence and optimality, it

> "[...] would surely horrified our intellectual forebear, Adam Smith: specialization was central to his vision of the workings of the competitive market, but convexity is the enemy of specialization." (Ellickson 1993: 106)

Moreover, convexity of choice sets (in contrast to the convexity of preferences or technologies) suggests perfect divisibility and, as mentioned above, dissolves Smith's link between specialization and the extent of the market.

As Aumann (1966) has shown, however, sheer largeness is a substitute for convexity as a necessary condition for existence and optimality. The key to his proof is Lemma 5.1 (ibid.: 8, 15). It says that the Lebesgue integral of any set-valued function over an atomless measure space is convex, even if the individual values of that function are nonconvex. In economic terms: for a continuum of agents, the aggregate of nonconvex individual upper contour sets is convex. Thus, it is possible to apply a convex-valued mean demand correspondence even in case of nonconvex preferences. For a continuum of firms, the same logic suggests a convex-valued mean supply correspondence despite of indivisible opportunity sets. Thus, it is possible to establish a convex-valued mean excess-demand correspondence such that a fixpoint theorem applies.

It follows that largeness has a convexifying effect. Yet, if such an effect were restricted to economies populated by uncountably infinite many agents, Aumann's result would be of limited implication. Fortunately, this is not the case. By means of the Shapley-Folkman-Starr theorem (SFS), the convexifying effect can be maintained for large but finite economies (Starr 1969). The key to the proof is its reliance on convex hulls of nonconvex individual choice sets. Accordingly, individual demand and supply correspondences are defined to relate prices to convex hulls of nonconvex best-response sets. Given a finite collection of nonempty and nonconvex individual choice sets defined on a finite-dimensional commodity space, SFS says that any value of the aggregate excess demand correspondence contained in the convex hull of the sum of all individual best-response sets is a convex combination of as many individual vectors as there are agents so that each such vector is contained in the convex hull of a best-response set, and so that the number of such individual vectors not *also* contained in a best-response set does not exceed the dimensionality of commodity space. It can be shown that there exists at least one price system that decentralizes such an allocation.

The gist of SFS is that the number of commodities limits the number of agents who cannot implement their best responses, that is, of agents who signal non-zero excess demands at given prices. Evidently, equilibrium does not exist if the number of commodities exceeds the number of agents. If both parameters are equal, no agent implements optimal choices and equilibrium likewise does not exist. It follows that the number of agents must 'significantly' exceed the number of commodities. The higher the number of agents, the smaller is the fraction of those who cannot establish optimal choices. Thus, the result suggests that for a sufficiently large but finite number of agents there exists a price vector that decentralizes approximate Walrasian allocations.

Note, however, that SFS breaks down in case of infinite commodity spaces. It only applies to the original formulations of ADM models where all agents are assumed to have a common choice set defined as a compact subset of Euclidean commodity spaces. Yet, given that ADM models rely on state-contingent commodities and, thus, predict intertemporal equilibria under uncertainty, the finiteness of the commodity space suggests a terminal point of the system as a whole. Whereas a finite horizon makes sense on the individual level, it is an unconvincing restriction if imposed on the economy as a whole. Rather, it seems natural to specify an infinite-dimensional commodity space, that is, to specify a subset of the sequence space in case of countably infinite many state-contingent commodities, or a Lebesgue space, if the cardinality of the commodity space is that of a continuum (see Mas-Colell and Zame 1991).

It follows that we fall back to Aumann's result and convexity can be dropped only in case of uncountably infinite many agents. In other words, it does not suffice for ADM economies to be competitive; they must be perfectly competitive. The only relevant class of ADM models is the class of 'large scale economies', a notion introduced by Ostroy

(1984) that comprises all economies with infinitely many agents as well as commodities. This, of course, is a drawback. How shall we deal with it? Aumann himself suggested to view such continuous models as productive idealizations:

> "The idea of a continuum may seem outlandish to the reader. Actually, it is no stranger than the continuum of prices or of strategies or a continuum of 'particles' in fluid mechanics. In all these cases, the continuum can be considered an approximation to the 'true' situation in which there is a large but finite number of particles (or traders or strategies or possible prices). The purpose of adopting the continuous approximation is to make available the powerful and elegant methods of the branch of mathematics called 'analysis', in a situation where treatment by finite methods would be much more difficult or even hopeless [...]." (Aumann 1964: 41)

Thus, even if SFS does not apply, it is legitimate to presume that nonconvex 'small square models' implement solutions close to those of continuous models. This view is reinforced by Ellickson:

> "Nonconvexity raises a problem in economies with a finite number of agents, but so does the hypothesis of pricetaking behavior. As Aumann said some time ago, we need realize that perfect competition is an idealization, an abstraction no more realizable than a perfectly elastic collision between billiard balls. Economics does not have to apologize for offering abstractions - that's what models are for, in economics as in any other science - but we do need to recognize when we are abstracting. The neoclassical overemphasis on convexity reveals confusion on that score." (Ellickson 1993: 314)

Yet, things are not so easy. Perfect competition in Aumann's original exposition suggests the distribution of a continuum of agents among finitely many markets. Trivially, each market is 'thick'. If the commodity space is a Lebesgue space, however, thickness is non-generic and Aumann's results may not apply (see Tourky and Yannelis 2001). For perfect competition to apply, individual insignificance has to be supplemented by a 'high' degree of substitutionality. Thus, even in case of thin markets, there must be a sufficiently high number of sufficiently close outside options for all reference options, and for all agents (consumers and firms). The applicability of continuous ADM models, and the productiveness of perfect competition as an ideal type, very much depend on the restrictions necessary to ensure substitutionality.

To evaluate this point, it is convenient to postpone the discussion and, instead, to focus first on an even deeper problem that is invoked by infinite commodity spaces: the Heine-Borel theorem is not generically valid for infinite spaces, that is, closed and bounded choice sets may not be compact. The definition of compactness that applies to finite as well as infinite spaces is that for an open cover of a subset, that is, for a collection of open sets covering over the subset, there is a finite subcover. Thus, compactness is the topological generalization of finiteness. It ensures that choice sets are finite even if commodity space is infinite and, therefore, that best responses are in reach. But even if choice sets are compact, the cognitive capabilities of each agent seem to exceed any reasonable level for a best response to be detected: infinite spaces are coordinate-free, composed of elements with infinite many characteristics. Even if choice sets have finite subcovers, optimal choice suggests that agents are able to compare options by considering infinitely many dates, locations, and states of the world. This, of course, is prohibitively restrictive - not only from the Scottish point of view.

An interrelated problem is that compactness comes at costs. Whereas all natural topological structures are equivalent to the Euclidean topology in finite-dimensional commodity spaces, a variety of comparable topological structures are consistent with the algebraic structure of infinite spaces. Each such topology is a specification of a collection of open (and closed) subsets that is closed under finite unions and intersections. The family of comparable topologies is ordered by increasing fineness. The finer a topology, i.e., the more open sets there are, the more functions are continuous, but the harder it is to find compact sets. Thus, there is a fundamental tradeoff between compactness and continuity. Fortunately, there exists a class of topologies that suggest a viable compromise (see Mas-Colell and Zame 1991). Of particular importance for the present study is that (at least) one of these topologies quite naturally revokes the assumption that agents compare options with infinitely many characteristics.

This leads us to the continuity assumption on preferences by which any relative topology is imposed on consumption sets. Informally, a topology can be thought of as a common structure on agents' mindsets, that is, as a qualitative notion of 'proximity' between consumption alternatives (in contrast to the quantitative notion of 'distance' induced by a metric). Given an open (or closed) set centered on a reference option, any other option contained in that set is regarded as a 'close substitute' from the viewpoint of agents, while any option not contained in the set is 'not close' to the reference option. Evidently, the finer a topology, the more elements can be separated by open (or closed) sets, and the more precise the notion of proximity becomes.

Continuity of a weak preference relation suggests that if an option A is strictly preferred to an option B, then close substitutes of A are strictly preferred to close substitutes of B (and vice versa). Each such element is contained in an equivalence class defined as the intersection of its upper and lower contour sets. Thus, continuity claims that for any reference option both these sets are closed such that also its equivalence class is closed or, equivalently, that its *strict* upper and lower contour sets are open. Imagine that B is weakly preferred to close substitutes of A such that the neighborhood of A is contained in the lower contour set of B. Since the latter is closed, it must - by definition - also contain A. This, of course, contradicts that A is strictly preferred to B. It follows that A must be strictly preferred to close substitutes of B. Equivalently, closed upper contour sets guarantee that closed substitutes of A must be strictly preferred to B.

Given that in case of a finite commodity space there is no choice with regard to the topological structure, continuity is a purely technical assumption. Its role is to ensure continuous individual demand correspondences and, thus, to guarantee continuous aggregate excess demand correspondences such that fixpoint theorems apply. However, in infinite spaces matters are different. First, a weaker notion of continuity is necessary for an equilibrium to exist. In concrete, preferences have to be upper hemi-continuous, that is, only upper contour sets need to be closed. Second, and more important, the continuity assumption is of economic meaning such that different topologies consistent with the linear structure of the commodity space imply different economic interpretations.

Of particular significance is the Mackey topology (Bewley 1972, Brown and Lewis 1981). First, it is a rich source of (weak-star) compact sets and, thus, of (uniformly) convergent sequences. Second, the Mackey topology restricts agents' mindsets to those which are asymptotically myopic, and which disregard insignificant values of continuous probability distributions. Think of two options in a subspace of the sequence space, that is, of two countably infinite consumption sequences, one of which is a zero function while the other is a sequence of ones. Evidently, the standard sup norm topology suggests that - given non-satiation - the strictly positive-valued profile is weakly preferred to the zero function. This is so even if the sequence of ones is arbitrarily postponed, that is, if it succeeds an ever increasing series of zeros.

Yet, if mindsets are structured by the Mackey topology, the two alternatives are indistinguishable whenever the positive-valued sequence is postponed sufficiently far. More precisely, from the viewpoint of each agent the positive-valued sequence asymptotically approximates the zero function for *finitely* many shifts into the future. Hence, the Mackey topology suggests asymptotically myopic agents. As shown by Aloisio Araujo, such myopia is a necessary condition for existence in case of infinite-dimensional commodity spaces and as such the "best assumption of its kind" (Araujo 1985: 455). For an equilibrium to exist, agents must systematically disregard tail-consumption in choosing their optimal consumption profiles as already indicated by Eugen von Böhm-Bawerk (1959). None of them has to take into account infinitely many characteristics in comparing any two options. The continuity assumption significantly reduces the cognitive capabilities necessary for existence.

Equipped with a precise notion of a 'close substitute', it is finally time to evaluate the need to supplement individual insignificance with substitutionality to establish perfect competition in large square economies. As shown, 'closeness' is a topological property and is imposed on agents' mindsets. Thus, it suffices that agents regard options to be close. More precisely, state-contingent commodities may differ significantly with respect to their physical characteristics as well as to the date and state of delivery, but agents may nevertheless view them as close outside options. It follows that substitutionality, being induced by the continuity assumption, does not violate the metarule (MR) invoked at the beginning of this section. It is therefore legitimate to supplement individual insignificance with a notion of substitutionality (necessary for existence in large-square economies). As a byproduct, requirements on individual cognitive capabilities can be significantly reduced (as in the case of Mackey topologies).

In summary, for a solution to a large square model to exit, agents preferences must be locally nonsatiated, and continuous. That's it! Neither need they be ordered, nor need they be convex. Agents are not assumed to be rational in any meaningful (i.e. empirical) sense! Evidently, the rational choice approach - which is *intentionally* unrealistic - is not necessarily restrictive, as maintained by its many critics. Also production opportunity sets need not be convex. Thus, specialization effects are not excluded as in the early existence proofs for small-square economies. Indivisibilities can account for increasing returns to scale. Specialization is thus potentially linked to the extent of the market. All in all, no assumption is prohibitively restrictive from Scottish point of view. Further, no assumption other than that of largeness is imposed on the system itself. Foremost, substitutionality is invoked by a proper choice of individual mindsets, and does not violate MR.

3.3 The Aggregation of Individual Characteristics

3.3.1 'Relevant Information' in case of Asymmetric Information

As in the previous subsection, consider an economy extending through infinitely many dates. For each date, there is a specification of environmental variables, i.e., variables not controlled by agents or, equivalently, variables controlled by Mother Nature. A state of the economy is a complete specification, or complete history, of such environmental variables. To establish that information on a variable at any date does not extent to its future, define an elementary event at a specific date as a set of partial histories, that is, as a set of states extending from the beginning of the economy's physical history to the date in question. At each date, the set of all possible elementary events is partitioned into a family of mutually exclusive and measurable (or observable) subsets (future events are thus unobservable). As usual, it is assumed that the family of all elementary events is a σ-field, that is, an algebra that is closed under countable unions such that the 'law of averages' applies. The σ-field is finer for subsequent dates meaning that observable information accumulates over time (for an excellent survey on information structures, see Radner 1982).

At each date, and for each agent, there exists an information signal, i.e., a correspondence from state space into a σ-field of disjoint 'observations', that generates an individual information partition. Note that information structures restrict individual choice sets as well as the number of admissible trades. If an agent's information partition is coarser than the partition into elementary events, its best response is the same for all elementary events within any specific partition of the information set (ibid.: 928-9). A net trade is admissible, if and only if elementary events are *mutually observable*. In other words, for net trades to occur the information partition of at least two agents must intersect. Further, the beliefs of each agent with regard to environmental variables are summarized by a subjective probability measure on its information structure such that beliefs are conditional on observable information.

Note further that agents in ADM economies do not necessarily act on complete information, that is, they do not need to observe the sequence of σ-fields in all detail. Rather, an individual's information at each date is a partition into alternatives at that date which is *at least* as coarse as the partition into elementary events. Information is said to be incomplete, if information partitions are strictly coarser than the elementary-event partitions. Stated differently, all information partitions contain the event partition or, more intuitively, the finest discrimination of events available to any agent is not fine enough to account for the sophistication of the event partition. Given that date-event pairs are measurable, all information partitions are measurable (so that 'observations' are in fact observable). In this case, information can be thought of as being costly, that is, the acquisition of information depends on the use of cognitive and physical resources. Each agent's information structure is defined as the succession of subsequently finer information partitions. In ADM economies, information structures are fixed, or exogenous, in the sense that they are choice-invariant.

Most importantly, the case of imperfect information is formally equivalent to the case of perfect information in the sense that existence and optimality of a competitive system are now defined relative to a fixed information structure of *arbitrary* coarseness. The notion of such 'constrained equilibria' was pioneered by Stigler (1961). It is difficult to overestimate the importance of this result for the invisible-hand proposition: if they exist, equilibrium price systems decentralize allocations efficiently, conditional on available individual information; or, equivalently, *the validity of the first fundamental welfare theorem is unrelated to informational requirements*. The invisible-hand proposition applies, no matter how 'undifferentiated' (coarse) the individuals' 'worldview' (information structure) is.

However, the increasing fineness of individual information partitions, even if incomplete at each date, is problematic, if applied to ADM economies. It suggest that individuals accumulate information such that beliefs may improve over time. Yet, in an ADM economy all accounts are settled at an initial date before the physical history of the system; there is no incentive to reopen markets after that initial date. It follows that for an equilibrium to be well-defined no information accumulated during the physical history of the system should ever disappoint initial choices. But this is to require that agents act consistently on information they do not yet have. Indeed, individual learning - as specified by the increasing fineness of information partitions over time - is a meaningless notion in ADM models. It follows that the ADM-specification of indefinitely heterogeneous beliefs bears no implications whatsoever. Right because changes in beliefs are irrelevant (as is the question of belief convergence), ADM frameworks can afford to be generous with regard to heterogeneity. In the terminology of Mordecai Kurz (1994): ADM models only accommodate 'exogenous uncertainty'.

A much deeper problem is that ADM models only accommodate symmetric information (Radner 1982). As is already implicit in the preceding discussion, all agents share a common information structure (implied by exogenous information partitions). The most significant implication of symmetric information is that it leaves no rationale for agents to learn from one another. It follows that the common-information property offsets the incentive to infer non-price information from price-information such that the Grossman-Stiglitz result is evaded. This in turn suggests that prices do not communicate 'dispersed bits' of incomplete and heterogeneous information on observable environmental variables. Rather, a price system is just a list of tradeoffs that also constrains individual choices (Grossman 1981: 541).

Recall from section 2.3 that Smith was well aware of the knowledge problem, that is, the problem of how to aggregate dispersed information efficiently. But also recall that while the Scots were well aware of price systems as lists of tradeoffs and 'civil' ranking devices, they were unaware of the function of price systems as communication devices. This function is rather popularized by F.A. Hayek, who wrongly[2] attributed the idea to Smith (and Mandeville) (Hayek 1991a, 1991b). He takes up the line of thought exactly where Smith has left it:

> "The various ways in which the knowledge on which people base their plans is communicated to them is the crucial problem for any theory explaining the economic process, and the problem of what is the best way of utilizing knowledge initially dispersed among all people is at least one of the main problems of economic policy - or of designing an efficient economic system." (Hayek 1945b: 519)

The second part of the sentence - on the parsimonious use of information inherent in competitive systems - was conceptualized and brought to formal precision by the mechanism design literature pioneered by Leonid Hurwicz (1960). Referring to Hayek, Hurwicz introduces the concept of 'informational efficiency' by specifying a message space in terms of bids and asks, and by designing mechanisms which are evaluated by their ability to economize on the message space. It is only the "crucial problem" specified in the first part of the sentence that cannot be accommodated by ADM models. Hayek is quite explicit with respect to the significance of asymmetric information. In introducing dispersed private information, the assumption provides a rationale for price systems to be "systems of telecommunication" (Hayek 1945b: 87):

> "It is with respect to [the knowledge of the particular circumstances of time and place] that practically every individual has some advantage over all others because he possesses unique information of which beneficial use might be made, but of which use can be made only if the decisions depending on it are left to him or are made with his active co-operation." (ibid.: 80)

> "We must look at the price system as such a mechanism for communicating information [...]. The most significant fact about this system is the

[2] In fact, Hayek - who invented dated commodities and the intertemporal perfect-foresight equilibrium (Hayek 1984a) - never claimed any objective originality for himself.

> economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action. In abbreviated form, by a kind of symbol, only the most essential information is passed on and passed on only to those concerned." (ibid.: 86)

Thus, asymmetric information necessitates incentive systems that motivate self-interested agents to feed the system with their private information such that equilibrium price systems are fully revealing and, therefore, decentralize efficient allocations. This relates back to Hurwicz's result. The "marvel" (ibid.: 87) of an equilibrium price system is its informational efficiency due to the codification of non-price information (i.e., on environmental variables and individual primitives) into price information.

None of this is accommodated by the ADM framework. In fact, to generalize the transformed invisible-hand proposition to include the case of asymmetric information, it has to be relocated to a different habitat: it must leave the class of ADM models, and enter the class of sequence economies, where trade plans are executed during the physical history of an economy. In its new shape, the invisible-hand proposition must be based on the proof that equilibrium predictions of stochastic sequence models are fully revealing (sufficient statistics) and, therefore, generically implement ADM allocations (Grossman 1981). Recall that according to the generic tradeoff between individual and system intelligence (section 3.1.3), the switch to sequence economies accelerates the 'relevant information' necessary and sufficient for a general equilibrium to exist. Given a sequence of open spot markets for securities and uncertain flow budget constraints, agents must be endowed with mathematical expectations, i.e., a common conditional expectations operator.

Note, however, that Radner's perfect-foresight equilibria as introduced in section 3.1.3 is defined for symmetric and, thus, global information. Whatever an agent might learn about his stochastic environment, it immediately crops up in all other agents' information partitions. Given such a 'free lunch', each individual can easily ignore the activities of other agents, and independently choose an optimal amount and structure of costly information. The assumption of exogenous information partitions adds no further harm, because net returns of individual information gathering are determined independently. Agents are (somehow) able to infer *common* price expectations from *common* non-price information. As argued before, such perfect-foresight equilibria are based on prohibitively high individual cognitive and information-processing capabilities. Even worse, the problem of excessive requirements escalates once the highly artificial common-information property is revoked or, equivalently, once the class of stochastic sequence models is generalized to include dispersed information:

Primitives are introduced as stochastic processes, i.e., time-indexed random variables. The physical past of the economy at any 'relevant' date is assumed to be long enough for relative frequencies to be settled. In this sense, primitives are stationary. The limiting relative frequencies given (objective probability), a *structural regime* is specified by the joint probability distribution of all random variables. Agents maximize their conditional expected utility by compiling portfolios, each subject to a flow trade constraint (at each date, individual net trades sum up to zero).[3] Given that information is costly and, therefore, to some extent 'undifferentiated', the list of traded Arrow securities (i.e., the Hamel basis of security space) is incomplete (or in short, incomplete information implies incomplete markets). Like in Radner's perfect-foresight equilibria, agents are equipped with common conditional price expectations. Individuals compute random security prices (risk-adjusted probability measures) 'as if' they shared common information (actually, one has to assume 'common knowledge' to cope with higher-order uncertainty). Insofar, agents make optimal conditional forecasts or, equivalently, form *rational expectations*.

Evidently, rational expectations is an equilibrium condition, that is, 'relevant information' according to HC5 in sections 3.1.1-2. In some sense, 'relevant information' suggests that each individual is a copy of Hayek's central planner, i.e., able to compute (stochastic expressions of) perfect-foresight equilibria from a *centralized* non-price information set with respect to the *'true'* joint probability density function of *all* payoff-relevant environmental variables (Grossman 1981, Radner 1979). Thus, a Rational Expectations Equilibrium (REE) is implied by all agents sharing a *common* model (or belief system) that perfectly describes the structural regime, i.e., the data generating mechanism ('Mother Nature'). Such optimal use of information suggests that security prices equal mathematical expectations *on average*. Divergences between security prices and conditional price expectations are unpredictable (the orthogonality property), that is, forecast errors are unbiased (mean of zero) and serially uncorrelated (of spherical correlation) (for more context, see Sheffrin 1983). Since asymmetric information implies the possibility to learn from one another, it accounts for the incentive to invert the common conditional expectation operator, that is, to map from equilibrium prices to observable events (Grossman 1981: 546, 550)

Thus, in REE, security price systems reveal (communicate) all non-price information against the innocuous backdrop of white noise; they make public all dispersed and, thus, private information. Such informational efficiency is what Hayek described as marvelous: all dispersed environmental data is transformed - or rather codified - into incentive-relevant price signals by self-interested agents, who utilize private information in order to reap

[3] I disregard commodity spot markets for the sake of simplicity.

arbitrage opportunities, and who *thereby* 'price in' their "knowledge of the particular circumstances of time and place" (as an unintended consequence). Other agents only need to confront the present values of alternative trade plans to decode such impersonal information, and to detect their respective gradients of motion in light of all relevant data. It is 'as if' individual laws of motion were attuned by a central mind that has access to a comprehensive pool of heterogeneous non-price information. The invisible-hand proposition encapsulated in the first fundamental welfare theorem is therefore extended (Grossman 1981: 555): Since informationally efficient REE implement ADM allocations, i.e., decentralize Pareto-efficient risk-sharing allocations, equilibrium price systems are invoked as (1) lists of tradeoffs, (2) 'civil' ranking devices, and *simultaneously* as (3) complete summaries of decentralized information (sufficient statistic).

Given that security prices are fully revealing, the role of incomplete information as discussed with respect to ADM models must be reconsidered. Since symmetric information is a fundamental property of ADM models, complete information implies that each agent has access to all observable information, that is, each individual information partitions equals Mother Nature's choice set. For rational expectations under asymmetric information this is no longer the case. It suffices that the *union* of heterogeneous information partitions equals the σ-field of elementary events. In this sense, 'relevant information' suggests much weaker individual requirements than in ADM settings. Given that the property of dynamically complete markets is implied by complete information, and given that REE decentralize efficient allocations as by the invisible hand of a fully informed social planer, it follows that the completeness of security space is explained (endogenous), not presumed as in ADM models.

Note further that REE models differ from Radner's perfect-foresight model in that agents have an actual incentive to open financial markets over time (whereas this is artificially imposed on RE models; see Grossman 1981: 545-6). In such a 'marketplace' of information (and beliefs, as discussed below), each agent is able to confront its private forecast with the consensus view as expressed in security prices. If, for instance, a state price is one (and, thus, all other state prices are zero), an agent can infer that 'the market' expects with certainty the occurrence of the underlying event. If the private information of that agent is superior (due to prior information acquisition), there is an incentive to bet against 'the market' by going short on that asset (and going long on some others). Unlike convergence processes in ADM models, usually based on a highly artificial *central* algorithm (the super auctioneer, see Arrow 1959), *convergence to REE can be introduced as an anonymous process*: individuals attempt to 'beat the market' by making optimal use of private information. They strive for excess returns (private interest); in doing so, they feed the system with information and, thereby, contribute to optimal risk sharing (public interest). In this sense, one can say that REE models conform much more closely to Scottish Political Economy, than do ADM models.

Unfortunately, the extension of the transformed invisible-hand proposition to include the informational efficiency of equilibrium prices has no happy ending. As is obvious, individual information partitions have to be extended to include price signals or, more precisely, they have to be augmented by a (sub-)σ-field generated by random prices. Thus, a price-augmented individual information partition is the join of this σ-field and an individual non-price partition as defined above (i.e., the smallest σ-field containing both). In contrast to ADM models, individual learning as out-of-equilibrium activity is meaningful in REE frameworks: if agents with incomplete local information become more 'sophisticated' (i.e., secure private information), they can systematically improve the quality of their forecasts, and sustain a higher level of expected utility. An individual information equilibrium is given whenever the marginal opportunity costs of information gathering equals the expected marginal return on sophistication.

However, given that non-price information is both, costly and asymmetric, augmented information partitions suggest that information acquisition is a *strategic substitute*: each agent may either gather costly information, or free-ride on price signals, that is, on the information-acquisition efforts of other agents. Naturally, self-interested agents tend to opt for the latter strategy. This implies the well-known *Grossman-Stiglitz paradox* (Grossman 1977, Grossman and Stiglitz 1980, Grossman 1981, Stiglitz 1995): the more information is revealed by prices, the lower is the incentive to spend resources on the accumulation of information. Yet, if more agents choose to remain uninformed and, thus, do not feed the system with their private observations, prices reveal less information (are more noisy). By making public private information, price signals are in fact *positive externalities*. Agents cannot appropriate their social contribution, i.e., they cannot secure a return for the (unintended) provision of all-available information. It follows "the Impossibility of Informationally Efficient Markets" (the title of Grossman and Stiglitz 1980): a complete security space is *too efficient* in that it *kills* the incentive to gather information for *all* agents. "Some markets must close until there are few enough prices that an uninformed trader cannot completely free-ride on the informed trader's information" (Grossman 1981: 556). Most importantly, the transformed invisible-hand proposition does not transcend to REE under asymmetric and costly information: informationally inefficient price systems generically do not decentralize constrained Pareto-efficient allocations.

Accordingly, incompleteness is a rational response of the system with the result that informational inefficiency is generic. This points to an interesting difference between ADM and REE models with respect to the relation between market completeness and externalities: whereas completeness in ADM economies rules out externalities, completeness in REE models under asymmetric information invites the greatest possible extent of externalities. This alters the role of institutions. In ADM models, they serve to reduce transaction costs (see the introduction to section 3.2). Optimal institutional solutions account for zero transaction costs. A perfect system of natural liberty as an institutional configuration is *only implied* by the assumption of completeness. The allocation of resources is coordinated by price systems alone. By contrast, *institutional solutions in REE models explicitly support price coordination.* For instance, varying regulations of insider trading immediately affects the informational efficiency of prices. In general, incomplete markets invite rich institutional structures, i.e., interdependent institutional solutions like 'trust' in the "performance of promises" (recall Hume), 'contracts', 'off-exchange trading', 'collateral', 'firms', 'shops', etc. (e.g. Greenwald and Stiglitz 1986, 1990, 1986). Further, whereas a continuum of agents rules out strategic behavior in ADM economies, competitive REE economies still suffer from the incentive to 'game the system' (by free-riding on price signals). Finally, the instantaneous response of security prices to new information is excessive, since excess returns from 'insider trading' vanish immediately, which destroys the incentive to acquire information (Demsetz 1986).

3.3.2 Uniqueness, Stability, and the SMD Results

Given that the transformed invisible-hand proposition (the first fundamental theorem) does not transcend to the class of REE models under asymmetric information and, therefore, does not include the function of price systems as communication devices, we fall back on the class of ADM models. In this sense, the Grossman-Stiglitz result discounts the transformation of the Scottish invisible-hand proposition according to neo-Walrasian standards (this point is taken up in the concluding discussion): the transformed invisible-hand proposition cannot cope with dispersed information; it cannot be extended to genuinely dynamic, stochastic environments; it cannot handle institutional solutions in direct support of price coordination; it does not accommodate individual learning; learning as by an impersonal intelligence is prohibitively simple and artificial (e.g. a feedback relation between a system of differential equations and a system of excess demands).

Still, the list of such negative results is incomplete by far. Whereas existence is indispensable (the consistency check), uniqueness and stability are desirable, to say the least. To prove that the set of equilibrium price systems is continuous (i.e. to suggest non-uniqueness), is to show that individual characteristics do not restrict the number of predictions ('anything goes'), and would render meaningless the existence proof (internal consistency) and, therewith, the first fundamental theorem of welfare economics.

Likewise, if equilibrium price systems cannot be established as attractors or 'centers of gravitation' (generic instability), the transformed invisible-hand proposition breaks down (for a detailed account, see Ingrao and Israel 1990, ch. 11 and 12). Global uniqueness and global stability, of course, are most desirable. Are there individual restrictions that sufficiently restrict aggregate behavior such that equilibrium is globally unique as well as globally stable? By how much do we have to extend the axiomatic basis? How does such an extension affect individual cognitive and information-processing capabilities? What is the best one can still hope for?

Beforehand, let me note that the primary problem is not one of finding sufficiency conditions for uniqueness and/or stability. Given that the role of Arrow-Debreu prices is curtailed relative to REE, disequilibrium behavior is comprehensively described by the interplay of substitution effects (controlled by price systems as lists of tradeoffs) and wealth effects (via price systems as 'civil' ranking devices). Global uniqueness and stability is ensured, if the aggregate (or mean) excess demand relation is subject to the 'law of demand' such that income effects are dominated by substitution effects (or, equivalently, such that positive feedback effects are dominated by negative feedbacks). A sufficiency condition for the 'law of demand' in exchange economies is a continuous (C) aggregate (or mean) excess demand relation, which is homogeneous of zero degree in prices (H) and convex-valued (K), which obey's Walras' Law (W), and which satisfies gross substitutionality (GS) and a boundary condition (B) (Arrow and Hahn 1971: ch. 9, 11; see also Ingrao and Israel 1990: ch. 11, 12). (C), (H), and (W) are necessary for an equilibrium to exist, and follow from the individual restrictions examined in section 3.2.3. In case of small-square economies, (K) is implied by convex preferences. Recall that in large-square economies, a continuum of heterogeneous agents has a convexifying effect. As long as there exist sufficiently many sufficiently heterogeneous agents, (K) applies even in case of individually decreasing subjective and objective opportunity costs. (B) claims that aggregate (mean) excess demand tends to infinity whenever a routine on price space approximates the boundary (i.e. all not strictly positive price systems). Any groping process is repelled by the boundary, bouncing back to interior elements.

For small-square economies, (GS) says that all off-diagonal entries of the Jacobian of an aggregate excess demand function, i.e., the matrix of all its first-order partial derivatives, are positive for all price systems. An infinitesimal increase in the price of any one commodity increases the excess demands for some or all other commodities. Given (W) and (H), the diagonal entries must be negative. It follows that excess demands are decreasing in own prices and all complementarities are ruled out at the aggregate level. Given (GS), it is possible to apply a Lyapunov function, that is, a continuous and positive-definite scalar-function that converges for all admissible initial values and is constant if (and only if) the initial value is an equilibrium. In concrete, (GS) establishes that a system of differential equations that display Walrasian groping allows for the existence of a Lyapunov function which, in turn, establishes global fixpoint stability (Arrow and Hurwicz 1958; Arrow, Block, and Hurwicz 1959). Note that if production is allowed for, (GS) is no longer sufficient to establish global uniqueness (Kehoe 1998).

So if the problem is not one of finding sufficiency conditions, what is it? There are essentially two candidates: (1) Individual restrictions beyond those implied by existence are prohibitively restrictive with respect to cognitive and information-processing capabilities (from the Scottish point of view). (2) Some information about individual characteristics get lost by adding excess demands over all agents and commodities. Evidently, the second problem dominates the first: if the aggregation problem applies, further investigations in the primitive structure of ADM models are futile. For instance, homothetic preferences that establish the 'law of demand' for individual (Marshallian) demand functions by disconnecting income and substitution effects (excluding Giffen goods), may not sufficiently restrict the class of admissible aggregate (mean) excess demand relations, that is, individual restrictions may impose insufficient control over aggregate behavior. Evidently, problem (2) endangers the invisible-hand proposition even in its arid ADM representation.

At the beginning of a sequence of highly influential results, Hugo Sonnenschein asked, if individual restrictions add up to properties of aggregate excess demand beyond those necessary for existence. He shows for strictly positive prices and two commodities that "Walras' Identity and Continuity summarize all of the restrictions on the community excess demand function which follow from the hypothesis that consumers maximize utility and producers maximize profits" (Sonnenschein 1973: 353; see Sonnenschein 1972 for the initial result). It follows that standard restrictions on individuals do not aggregate to (GS) or any other related restriction on the system that could ensure global uniqueness or stability. This result was generalized by Mantel (1974), who extends it to l commodities, but restricts his result to the case of continuously differentiable aggregate excess demand functions that can be additively decomposed to $2l$ individual demand relations. A further generalization is supplied by Debreu (1974). Given (B) such that prices are strictly bounded away from zero, he proves that given any function f from the unit price simplex to the l-dimensional commodity space that satisfies (C), (H), and (W), for every compact subset of the price simplex there exists an exchange economy with l well-behaved agents such that the aggregate excess demand of that economy equals f. Well-behaved agents are defined by strictly convex, monotone, and complete preference preorderings (ibid.: 15-6). The ϵ-qualification induced by (B) suggests that, although prices must by strictly positive for SMD to apply, they may approximate the boundary arbitrary close (Kirman 1989: 130-1).

This, of course, suggests problem (2). It says that by adding up individual excess demands, we lose all structure beyond what is necessary to ensure existence. Given Debreu's specifications, it is impossible to derive (GS) or related properties on aggregate excess demand by imposing restrictions on individual behavior. If the neo-Walrasian program is assumed to rest on the metarule (MR) (section 3.1.1), that is, if the program by definition (or convention) rests on *strict* methodological individualism, the so called Sonnenschein-Mantel-Debreu (SMD) results severely discount the applicability of ADM reasoning, since they deny that decentralized markets aggregate individual rationality beyond what is necessary and sufficient to ensure the possibility of plan consistency. Consequently, the SMD results create serious doubt about the transformation of the invisible-hand proposition. Most importantly, given that problem (2) dominates (1), no additional structure - if introduced in form of individual characteristics - can shield the invisible-hand proposition against the SMD results. For instance, the SMD results also apply to the case of homothetic preferences as shown by Mantel (1976).

Recall that ADM models do not delimit the dispersion of individual characteristics. Naturally, individual restrictions in addition to the axiomatic base of existence and efficiency proofs straiten the heterogeneity of agents (like in the case of homothetic preferences). Imagine a convergent sequence of small-square ADM exchange economies that decreases in the dispersions of primitives (increases in structure). In the limit, all agents have identical preferences and endowments. The limit economy behaves as if there is only one agent (and no-trade theorems apply). Such a system is what Arrow and Hahn name a 'Hicksian economy' (Arrow and Hahn 1971: 220-1), being a mere list of self-sufficient individuals such that Hume's exchange nexus is in fact disintegrated. The aggregation problem is trivially solved at the cost of irrelevance.

Surely, the economy that exists closest to the limit is one with identical preferences and initial endowments that differ only by a scalar. Such collinear endowments suggest a price-independent relative income distribution. It is often taken for granted in applied analysis that the SMD results can be bypassed by such radical homogeneity assumptions (e.g. Acemoglu 2009: 150-1), i.e., that it is possible to restrict aggregate outcomes (model predictions). To show that this is not the case, and that problem (2) indeed dominates (1), it must be shown that the SMD results also extend to the case of identical preferences and collinear endowments. A corresponding proof exists: Kirman and Koch (1986) demonstrated that structural information gets lost, even if one adds up excess demands that *only differ in scale*! With respect to uniqueness and stability, *anything goes*! As later summarized by Kirman, for

> "an arbitrary excess demand function, no matter how ill-behaved and difficult to work with, I can give you an economy in which people are as close as you like to being identical, i.e. they have the same preferences and almost the same income, which will generate this ugly excess demand function." (Kirman 1992: 128)

Is this the last nail in the coffin of ADM models in general, and of the transformed invisible-hand proposition in particular (see Rizvi 2006)? Recall that the SMD results rest on the additive decomposition of aggregate excess demand functions which suggests that individuals act independently of each other or, equivalently, that all interaction is by the anonymous market system. If social interaction is allowed for, that is, if non-market relations are admissible, then it is in fact possible to restrict aggregate demand (Kirman 1989: 137). This, of course, would violate the hard core of the neo-Walrasian program (HC4, see section 3.1.1). Indeed, if the purpose of general equilibrium analysis is to transmute the Scottish vision of social coordination, then (HC4) is an obstructive self-flagellation that hinders progress in the mainline of economic theorizing. This also holds for strict methodological individualism (MR): Hildenbrand (1983) established for a continuum of agents with identical preferences the 'law of demand' *in the mean* by restricting endowment *profiles* to those of increasing density. Thus, global uniqueness and stability would be in reach, if assumptions on the distribution of individual characteristics were admissible.

Yet, although the SMD results dispel any hope for global uniqueness and stability as long as the hard core of the neo-Walrasian program remains stationary, it is possible to establish local uniqueness and stability without resuming to aggregate excess demand relations, at least not directly. In the following, I focus on local uniqueness, yet note that the toolset of differential topology on which the following exposition rests also allows to establish local stability (see Nagata 2004, Ch. 5). It is again Debreu (1970), to whom we owe the proof that local uniqueness is a generic property of smooth ADM-systems. Given that compactness is a common characteristic of the set of equilibrium price systems, the demonstration that equilibria are isolated (i.e., that the set of equilibria is discrete) breaks down to the proof that the set of equilibria is finite. Debreu's proof rests on the ingenious device of 'regular economies', which he introduces in the aforementioned article, and on Sard's theorem as "[t]he key mathematical tool in the proof" (ibid.: 387). The notion of regular economies not only allows to answer how the set of equilibria generally looks like, but also how that set varies, whenever primitives vary (see also Dierker 1982: 796). The latter question is concerned with the continuity of the equilibrium price correspondence and, thus, with the suitability of comparative-statics in ADM settings.

Debreu focuses on continuously differentiable excess demands in large but finite exchange economies, each parameterized by l commodities and n agents (for competitive economies with infinite-dimensional commodity spaces, see Mas-Colell 1986 and Shannon 1999; for production economies see Mas-Colell 1990). Imposing (H) on individual demand, the set of equilibrium price vectors is normalized to lie in the $(l-1)$-dimensional unit simplex (i.e., only relative prices matter). He further imposes a desirability property on *one* individual demand relation that follows from strictly monotonic preferences, and which says that the demand of that individual for any commodity becomes arbitrarily large, if its price is the only one that approaches zero. The restriction expresses the idea that any one agent, say the first ($n = 1$), desires to consume every commodity (Debreu 1970: 388). Finally, Debreu assumes that demand relations are fixed such that only endowment profiles are allowed to vary (for extensions to the case of varying demand, see Dierker 1982).

Debreu's major innovation is a space of economies, that is, a whole set of conceivable specifications of endowment profiles over n agents such that each specification defines an economy. The space of economies is a differentiable ln-dimensional manifold, that is, a topological space locally homeomorphic to a ln-dimensional Euclidean space such that each point of the manifold has a neighborhood that is diffeomorphic to an open subset of the Euclidean space (the homeomorphism is differentiable). Thus, there exist a tangent space at each point of the space of economies which is the image of the Jacobian matrix of the diffeomorphism at zero.

In a next step, Debreu establishes a relation that associates a set of equilibrium price systems with each endowment profile (or economy). The relation is indirect: to make use of the desirability property of the 1st agent, he defines a map whose range contains the parameter space, and whose domain is a differentiable product manifold that is an open set in the ln-dimensional economy space. In concrete, its domain is the Cartesian product of the $(l-1)$-dimensional unit simplex, the one-dimensional income-space of the 1st agent, and the $l(n-1)$-dimensional space of economies with respect to all other agents (note that the sum of the factors' dimensions is again ln). Most importantly, its value is an endowment profile, if and only if the function maps equilibrium price systems (Debreu 1970: 388-9). Thus, the purpose of the auxiliary mapping is to isolate the set of equilibrium price systems for any endowment profile.

It follows that there is a one-to-one correspondence between the inverse image of the auxiliary function for any economy and the image of the equilibrium price correspondence that maps each economy to a set of equilibrium price systems. Furthermore, the inverse image of the auxiliary function and the image of the equilibrium price correspondence are homeomorphic. Thus, it is possible to replace the relation of economies and the set of equilibria by the one of economies and the inverse image of the auxiliary function. Any element of the price manifold is a critical point of the auxiliary function, if its derivative at that point, that is, the projection of vectors tangent to the product manifold at that point onto the tangent space of the parameter space, is not surjective or, equivalently, has a rank less than ln. An endowment profile is a critical value of the auxiliary function, if every point in its inverse image is a critical point. An endowment profile is a regular value of the auxiliary function, if it is not a critical value.

A *regular economy* is specified by an endowment profile that is a regular value of the auxiliary function. Sard's Theorem establishes that the set of critical values of the auxiliary function has Lebesgue measure zero in parameter space (ibid.: 388; Theorem). It follows that the set of regular values is dense. Informally, the set of critical economies is negligibly small while the set of regular economies is 'fat'. Thus, any property of regular economies concerned with the structure of the set of equilibrium price systems is generic, that is, applies 'almost always'. Most importantly, since the dimension of the product manifold equals the dimension of parameter space, the preimage of a regular value constitutes a discrete set. Thus, the set of equilibria is discrete. In other words, *local uniqueness is a generic property of ADM economies*. By applying the desirability property introduced above, Debreu further shows that the discrete set of equilibria is also compact. Given that a discrete and compact set is finite, he is able to show that *finiteness of the set of equilibria is a generic property of ADM economies* as well (ibid.: 389-91).

Finally note that even though Debreu's results are based on differentiable individual demand functions, it is possible to derive them by restricting preferences to the class of smooth relations (Debreu 1972, see also Mas-Colell 1990, Ch. 2). All in all, the extent to which Debreu's and subsequent results mitigate the consequences of the negative SMD results depends on the willingness to substitute local for global uniqueness (and stability).

Yet, even if the impact of the SMD results are mitigated, further negative results already lurk around the corner. They all suggest that Walrasian approximation routines - based on a central algorithm (super auctioneer) - fail in general. Recall that Hurwicz's demonstration that ADM models are most parsimonious with respect to the message space (number of bids and asks sufficient to decentralize efficient allocations) relies on the markets system as a monolithic entity. Even though it could be shown that informational requirements are relatively low *once the system is in equilibrium*, a rich literature emerged suggesting that any convergence mechanism which relied on central algorithm has to process far more information, and to consume far more time than what can be reasonably assumed. The complexity of the allocation problem simply overtaxes any centralized approximation routine. A prominent example is the Global Newton Method (GNM) introduced by Stephen Smale (1976). After Debreu had reestablished calculus tools in his proof of finite equilibria, introduced above, Smale attempted to systematize the use of differential methods in Walrasian analysis. His goal was to find a price adjustment process for an exchange economy, a central algorithm, that detects the price equilibrium after starting at any point sufficiently close in the normalized price simplex. Unfortunately, such a routine strictly defies any economic interpretation (Saari 1985), even though Smale's intention was to free general equilibrium theory from its reliance on fixed point theorems, which themselves are difficult to access economically.

In contrast to tâtonnement processes, which operate by linear and nonlinear price adjustments according to the *law of supply and demand* and insofar allow for disproportional changes in relative excess demands during convergence, GNM asks prices to change in a way that necessarily preserves the initial ratio of aggregate demand for commodities. Most importantly, in addition to all market information from the aggregate excess demand relation, the routine also has to know its Jacobian, the knowledge of how changes in the price of a particular brand of apples (or any other commodity) affect the excess demand for the second last vintage of the iPhone (or any other commodity). Accordingly, whereas tâtonnement processes relate the adjustment of a single price to the excess demand expressed on that market alone, GNM relates any single adjustment to the properties of excess demand in all other markets as well (Hahn 1982: 767). The message space of such dynamics is accordingly significantly larger than the message space preserving equilibrium.

Further, even though it is possible to define static characteristics that ensure local stability, D. G. Saari and C.P. Simon (1978) demonstrated that there are no "locally efficient price mechanisms", i.e., no differential adjustment processes which necessarily converge, *and* that make use of less information than the GNM in the case of three or more commodities. Saari and Simon argue that since globally converging mechanisms need to convert only a subset of equilibria into sinks or attractors, the lower bound on informational requirements is lower than for local convergence. Yet, such requirements remain prohibitively high: in addition to the information revealed by the aggregate excess demand function, most entries of the Jacobian are still needed. Further, these results depend on the statement of the routine as a differential equation. Accordingly, all such updating rules rest on a continuum of information.

In consequence, Saari (1985) studies global convergence by means of a discrete iterative mechanism. The results are remarkable, especially because they are introduced for two or more commodities and thus apply to all exchange and production economies. Even though the introduction of iterative mechanisms does not alter the informational requirements for locally bounded processes (since they are limiting cases of differential dynamics), it can be shown that global routines now ask for more information, and for information of a different kind! In a first step, it is shown that - given the informational requirements of the GNM - global mechanisms cannot be universally convergent, if the routine has to select initial prices from a set that is partitioned into subsets of convergent and nonconvergent elements. Nonconvergence properties are insofar robust.

Next, the routine is equipped with a finite memory such that the information set is broadened by some history of the aggregate excess demand function. It is shown that also this heroic assumption does not improve on the choice among initial prices. In effect, we neither know the kind of information sufficient to ensure convergence, nor do we know its amount. To guarantee efficient mechanisms we have to assume *global* information, that is, we assume the system knows whatever it has to know, including the knowledge of how different efficient allocations translate into different efficient mechanisms that bring them about. Omniscience is indeed a quite restrictive assumption. In fact, the strict adherence to comprehensive routines prohibits any resemblance to the invisible-hand mechanism! Where, for instance, do we have to look for such a centralized memory, a store of aggregated information, even if finite?

The upshot of this process is that the analysis of the invisible-hand mechanism dropped together with the investment in this field of pure theory. This view is neatly summarized by Saari:

> "I have no idea whether Adam Smith's invisible hand holds for the 'real world', but, then, no one else does either. This is because, even though this story is used to influence national policy, no mathematical theory exists to justify it. Quite to the contrary; what we do know indicates that even the simple models from introductory courses in economics can exhibit dynamical behavior far more complex than anything found in classical physics or biology. In fact, all kinds of complicated dynamics (e.g., involving topological entropy, strange attractors, and even conditions yet to be found) already arise in elementary models that only describe how people exchange goods." (Saari 1995: 1)

All this is supported by related results in the field of computable economics, in contrast to computational economics as initiated by Herbert Scarf, mostly associated with K. Vela Velupillai (1996). The goal of this field of analysis is to evaluate the *computational complexity* of economic theories by models of computation based on Turing machines (ideal digital computers), which highlight the resource constraints necessary to calculate standard optimization problems, measured by the time it takes for an algorithm like the super-auctioneer to halt (Velupillai 2005). A problem is tractable, if at least one algorithm exists that solves the problem in polynomial time, that is, if the worst-case number of primitive operations of an algorithm is bounded by a polynomial function of its input. A problem is incomputable, if the number of operations grows exponentially with the input. In this case, computing a solution asks for an uncountably infinite number of operations such that a finite algorithm cannot exist.

The common denominator of all these results is that out-of-equilibrium processes do not converge if they are driven by a *single coherent and comprehensive intelligence*. Even though agents successfully feed the monolithic system with primitive information, it is unable to process all data necessary to compute a set of solutions even if such information were somehow available. Accordingly, it is unable to generate signals to bring about the coherence of independent and individually optimal choices. Given the long list of negative results, Kirman summarizes the implications:

> "It appears that the informational requirements of adjustment processes are so extreme that only economy-specific processes are plausible. This is hardly reassuring for those who argue for the plausibility of the equilibrium notion. Any change in the parameters of an economy would entail a change in the price-adjustment mechanism that would keep the economy stable. If we are interested in describing a stable system, then the arguments for the general equilibrium model as one that is informationally efficient are illusory." (Kirman 2006: 267)

Chapter 4

Concluding Discussion

The following list summarizes and discusses the major results of this article:

1. Bernard Mandeville was introduced as the originator of the invisible-hand proposition: vicious conduct attunes to beneficial social outcomes 'as if' orchestrated by an invisible hand. It was argued that this first statement of the proposition already involved a notion of general equilibrium ("as in Musick Harmony"). Further, Mandeville's invisible-hand proposition relies on 'largeness', and is conditional on a 'proper' institutional design. It was shown that Mandeville took the moral standards of the feudal society as given. Accordingly, he introduced the individual pursuit of happiness as vicious (section 2.1). As morals evolved, and self-interest could escape the burden of religious doctrines (already with Hobbes), the invisible-hand proposition was de-demonized by Hume (and turned against Hobbes). His 'enlightened' invisible-hand proposition claims that institutionally harnessed self-interest aggregates to the common good via the division of labor, *and* that the consequent social diversity feeds back on the institutional setting, e.g., by teaching the lessons of tolerance and benevolence. Hume's higher-order invisible hand coordinates the process of civilization, not just economic allocation (see 2.2).

2. From the onset, the natural poverty of man (the scarcity problem) and the consequent conflict of interest were central to invisible-hand explanations (2.1). The persistent imbalance between excessive needs and meager natural endowments constitutes individual laws of motion. The predominance of self-interest implies their independence. David Hume observed that such individual imbalances provide the incentive for social cooperation. Given social and legal norms that facilitate the exchange nexus by harnessing self-interest (especially property rights), binding budget constraints submit self-interested conduct to the service of strangers: individual improvement is subject to the material needs of others. The quid pro quo condition

ensures that individuals unintentionally serve one another as eager as they serve themselves (see 2.2). In this sense, Mandeville and Hume introduced exchange relations as the predominant putty of large commercial societies.

3. The predominance of exchange relations in large societies implies that social coordination is decentralized and impersonal. The finer the 'partitioning of employments' and, thus, the more prosperous a society, the more it relies on anonymous market interactions, and the more individuals are interlinked via the quid pro quo condition. This was Hume's great counterdraft to the mercantile feudal system of his time. Access to resources is not granted by privilege, but earned on markets. Each individual is weighted by the 'exchange value' of its net contributions, and is therefore ranked in a 'civil' manner, that is, according to market consensus (2.2). This Scottish utopia is taken to the extreme in Arrow-Debreu-McKenzie (ADM) models: adding up binding budget constraints (the quid pro quo condition) over all individuals gives Walras's law; its corollary establishes market interdependence; price systems that decentralize equilibrium allocations weight each individual according to the relative scarcity of its net contribution to society (3.1.2). In fact, the Sonnenschein-Mantel-Debreu (SMD) results imply that ADM models cannot embody much structure beyond Walras's law (3.3.2). In this sense, the predictions of ADM models may be regarded as ideal types of Hume's vision of anonymous social coordination (3.2.2).

4. By contrast, Adam Smith's long-run model predicts socially inefficient outcomes. The distribution of income is not controlled by scarcity relations, but determined by feudal mechanisms like the 'iron law of wages', and the class struggle between Tories and Whigs over net income. Right because Smith's long-run system was designed to describe the then prevailing mercantile feudal system, not to provide a counterdraft thereof, the scope of the invisible hand is rather limited (even if his system of natural liberty applies). Equilibrium commodity prices do not reflect relative scarcities, but are simply derived from equilibrium factor prices (determined by feudal forces) in conjunction with a Leontieff technology, and decentralize inefficient allocations. Finally, short-run convergence on commodity markets is summative, that is, each market converges independently, blinding out market interdependence via income effects (2.3). Neo-Walrasian models are in this sense superior habitats of Hume's vision (3.2.2)

5. ADM models were chosen to host the invisible-hand proposition (in form of the first fundamental welfare theorem), because they maximize the wedge between indi-

vidual and system requirements (3.1.3). Complete markets do not only isolate the allocation problem from institutional considerations by imposing strict anonymity (3.2), they also minimize individual restrictions necessary and sufficient to demonstrate existence (3.1.3). For continuous ADM models such restrictions are strikingly mild: for at least one price system to decentralize equilibrium allocations, arbitrarily heterogeneous agents must be locally insatiable (Hume's individual imbalance) and behave continuously. Preferences must neither be ordered, nor need they be convex (3.2.3). As hosts of the invisible-hand proposition, ADM models provide the normative benchmark for other classes of models in the protective belt of the neo-Walrasian program (3.1.1). Conditions that implement ADM allocations for other model classes (e.g., those predicting rational expectations equilibria) transcend the invisible-hand proposition to the protective belt. In this sense, the class of ADM models is primus inter pares (3.2).

6. In presuming a continuum of agents (competitiveness as a generic property, see 3.2.1), ADM models give a radically stylized expression to the Scottish ideal of an 'unranked society', where no privileges exist and, thus, where no one is able to extract more 'exchange value' than he provides in return (i.e., where nobody 'games the system', see 2.1-2.3). The unranked society, of course, is not necessarily an egalitarian one. Or, equivalently, a uniform wealth distribution is not implied by the insignificance (zero measure) of agents. Rather, the prevalence of almost perfect outside options kills the incentive to manipulate prices irrespective of the distribution of wealth (3.2.1). No matter how far an individual might raise above all others in terms of wealth, he cannot secure any rent by evading the market consensus with respect to the relative scarcity of his net contribution to society ('justice' as the absence of crony capitalism). Perhaps to the surprise of contemporary libertarians, it was indicated that Hume supported egalitarian measures (within limits, of course), so that the second fundamental welfare theorem is not in conflict with Scottish Political Economy (2.2-2.3).

7. In confronting a continuous ADM model with its Pareto-copy, the transformed invisible-hand proposition, which claims that - in the absence of externalities - equilibrium price systems generally decentralize constrained Pareto-efficient allocations, can be derived by means of core equivalence theorems. Subsequent results showed that the invisible hand carries over to the case of a finite, but sufficiently large number of agents: as long as there are more agents than markets, equilibrium price systems decentralize quasi-equilibria such that 'almost all' agents have 'almost no' incentive to depart from core allocations (3.2.1). With respect to the existence and

efficiency of general equilibrium, the transformation of Hume's invisible-hand proposition is a great success, a highlight in the history of economics (for a short discussion of adverse opinions, see 3.2.2). This conclusion is not affected by the fact that ADM models exclusively accommodate the highly restrictive case of symmetric or common information. In this case, price systems function as lists of tradeoffs (guiding substitution effects) and 'civil' or 'just' ranking devices, yet not as communication systems that make private information broadly available by translating environmental data into a code (i.e., into price signals that can be decoded in economic calculation; see 3.3.1). Yet, there is no textual evidence that could substantiate the claim (made by Hayek) that Scottish Political Economy did host the idea of prices as surrogates of dispersed and costly information (see the discussion in section 2.3).

8. Although the common information property inherent to ADM reasoning does not misrepresent past formulations of the invisible-hand proposition, it is indisputably a burden for its present (and possibly of its future). If the proposition could not be extended to the case of dispersed and costly information, then Hume's vision would have to be regarded as internally consistent, yet empirically irrelevant (3.3.1). However, the situation is much worse. Not only are ADM models prohibitively restrictive in requiring informational symmetry, they also fall victim to the Sonnenschein-Mantel-Debreu (SMD) results: no matter how vigorously we restrict individual characteristics, by adding up individual excess demands we lose all structure beyond what is necessary to demonstrate existence and efficiency. It is therefore impossible to restrict ADM economies to those which satisfy sufficiency conditions for global uniqueness and stability (3.3.2). While Walras's law introduces (some) Humeian structure via the rule of the quid pro quo condition, the ADM setting is in general too arid to determine aggregate outcomes, or to ensure convergence in case of arbitrary initial values. Only local uniqueness (and stability) can be shown to be a generic ADM property (ibid.). Yet, whereas local properties of ADM equilibria do legitimize comparative-static analysis (and its textbook appearance), such local analysis is a poor substitute for Hume's vision of an anonymous, yet benevolent intelligence that drives the physical history of commercial societies (2.2).

9. To generalize the transformed invisible-hand proposition to the case of asymmetric information, it must be relocated to a different habitat: it must leave the class of ADM models, and enter the class of Rational Expectations Equilibria (REE). In dynamically complete REE models, security prices decentralize efficient allocations - i.e., they implement ADM allocations - as by the invisible hand of a fully informed social planer. Not only does the switch to REE models allow to introduce price

systems as communication devices, it also allows for less counterfactual convergence processes: in stochastic sequence models, it is admissible to introduce financial markets; disequilibria constitute the incentive to reopen spot markets; in striving for excess returns (private vices), individuals try to 'beat the market' by making optimal use of all affordable information; in doing so, they feed the system with their local information and, therewith, contribute to optimal risk sharing (public benefits) (3.3.1). Instead of the ADM-specific feedback relations between aggregate excess demand functions and centralized approximation routines, REE models display convergence as *fully* decentralized, that is, they do not draw on such super-auctioneers. In this sense, REE models conform much more closely to Hume's vision of an impersonal system intelligence, than do ADM models. Further, it remains unproven that SMD results also extend to the stochastic structure implied by REE.

10. Unfortunately, there is no happy ending for the transformed invisible-hand proposition. Since REE models introduce information-gathering activities as strategic substitutes, security prices cannot be fully revealing in general equilibrium. Dynamically complete markets are to efficient in that they kill the incentive to gather information for all agents. The impossibility of informational efficiency suggest that individuals trade at 'false prices', that is, at security prices that do not decentralize efficient allocations (the Grossman-Stiglitz result) (3.3.1). Further, even if prices would be fully revealing, the 'true' model (of the structural regime) implied by REE-predictions must be learnable. As shown by Mordecai Kurz, however, diverse rational beliefs do not in general converge to a common model. The confrontation of subjective priors with data may convince agents of false beliefs. Non-convergent beliefs, in turn, introduce higher-order price uncertainty, i.e., uncertainty about other agents' beliefs and their consequent trade plans (Kurz 1996: 384-5; not discussed in the paper).

11. However, while there is no happy ending to the neo-Walrasian transformation of the invisible-hand proposition, this does not suggest that it took a bad end. The Grossman-Stiglitz result, for instance, applies foremost to statistically processable information (e.g., information gathered and revealed by rating agencies). Although such costly information is important, much information becomes available in the wake of consumption and production activities (learning-by-doing, tacit knowledge, etc). Furthermore, market selection analysis has shown that even if individual learning is hopeless, the system at large may detect 'true' beliefs. Abstracting from the influence of diverging rates of time preference on the distribution of wealth, Alvaro

Sandroni could show that agents with inferior beliefs lose their wealth to systematically superior decision-makers (for dynamically complete markets, however; Sandroni 2000). If, by chance, any set of agents has ‘true’ beliefs, all wealth concentrates in their hand, and prices are risk-adjusted objective probability measures.

In summary, there is *no end to invisible-hand studies*, neither a happy, nor a bad one. Certainly, the hard core of the neo-Walrasian program must further evolve, for instance, by relaxing its strict methodological individualism (MR in 3.1.1). But this process is already under way. It is however true that invisible-hand studies stopped constituting the dominant trajectory of economics. The best the author of this study can hope for, is to change this state of affairs by focalizing the most important results against the backdrop of Scottish Political Economy.

Part II

Myopia and Optimal Growth: A Rational Reconstruction of F. A. Hayek's Capital Theory

Chapter 5

Introduction

This paper translates F.A. Hayek's informal capital theory into a dynamic equilibrium model. The focus is restricted to Hayek's largely unrecognized contribution in *Utility Analysis and Interest* (UAI), published by *The Economic Journal* in 1936, being restated in *The Pure Theory of Capital* (PTC), first published in 1941. The underlying premise is that Hayek adopts infant versions of modern analytical tools during his time at the London School of Economics (LSE) such that a rational reconstruction of his capital theory by established neoclassical tools is admissible.

The major result is that UAI and PTC contain a generalization of the Ramsey-Cass-Koopmans model. In concrete, Hayek provides the solution to an infinite-horizon deterministic social planner optimization problem in a one-sector economy such that the rate of pure time preference encapsulated in the discount factor increases in prospective utility. The endogeneity of myopia is due to intertemporal complementarities and accounted for by the modified Uzawa aggregator (the Epstein-Hynes aggregator is introduced for expository reasons). It is shown that Hayek remains within the scope of modern capital theory as defined by Roland Becker (2008).

The contribution of Fwu-Ranq Chang (1994) is identified as the most commensurable framework for Hayek's capital theory, because it closely relates the case of increasing myopia to the standard case with time-additive utility. Accordingly, the generalization of the Ramsey-Cass-Koopmans model is introduced as the Hayek-Chang model. From a purely theoretical point of view, the implications of 'increasing myopia' are significant: individual corner solution are exiled; on the aggregate level, convergence is ensured even in case of constant returns to per-capita accumulation. The latter result is of central importance to Hayek and establishes a partial alliance with Frank Knight.

To relate Hayek to the more recent history of economics, which by and large is the history of mathematical economics, I introduce the fundamental analytical tools developed during the past five decades that are necessary to formalize the 'Hayek Problem' in accordance with Neo-Walrasian standards. Since Hayek in his Copenhagen lecture in 1933 - the first indication of his shift to dynamic equilibrium analysis - himself insists that "[e]quilibrium analysis certainly needs, if we want apply it to a changing competitive system, much more exact definitions of its basic assumptions" (Hayek 1999b: 234), the inquiry places special emphasis on axiomatic foundations, that is, on the minimum set of (individual) restrictions proved to be necessary and sufficient to consistently deduce Hayek's system. Naturally, microfoundations are discussed in an extended Arrow-Debreu-McKenzie setting.

It is shown that none of the assumptions made are prohibitively restrictive from a history-of-economics perspective, that is, restrictions are at most as strong as Hayek and his contemporaries believed. In staunch contrast to what the intense debates on the legitimacy and proper role of positive time preference suggest, involving among others Eugen von Böhm-Bawerk (1959), Frank Fetter (1902, 1907, 1927), Irving Fisher (1997a, 1913, 1997b), Joseph Schumpeter (1934), and Knut Wicksell (2002, 1934), the imposition of 'asymptotic myopia' on the agents' mindset arises rather naturally in the infinite-horizon case. It is an instance where a mathematical result clarifies past theoretical disputes ex-post.

In concrete, mathematics sheds light on the controversy among those, like Schumpeter, who deny the rationality of myopia and accordingly defy its place in positive analysis, those who insist on its apodictic and rational nature, like Fetter (and later Mises; see Part III), and those, like Fisher and Hayek, who take an intermediate position and are willing to introduce myopia into the "logic of choice" for pragmatic and empirical reasons. Modern analysis strengthens this latter view: it shows that asymptotically myopic mindsets are necessary for a general equilibrium to exist in the infinite-horizon case; it shows that rational choice and myopia are analytically compatible (against Schumpeter); it also shows that myopia is no characteristic property of rationality (against Mises).

The present inquiry is a novel contribution to the history of economics. As to my knowledge, it is first to recognize Hayek's departure from his Austrian heritage in favor of dynamic equilibrium models. It is the first to provide a formal restatement. Given the extensive perception of Hayek's work, this claim must at first appear as radical indeed. To get an idea of the sweeping neglect of his contribution note that UAI is not even listed in the otherwise comprehensive bibliography of Bruce Caldwell's *Hayek's Challenge* (Caldwell 2005). Also PTC, especially chapters seventeen and eighteen that expand on UAI, is largely ignored with the exception of Lutz (1956), Feldstein (1964), and Becker and Boyd III (1993), who, however, only hint at the relation of Hayek to modern capital theory. Only recently, that is, in 2007, is Caldwell's series *The Collected Works of F. A. Hayek* extended by PTC (volume XII). Even though Lawrence H. White provides an excellent introduction to this "dry treatise" (see also Steedman 1994), he brushes over the central aspects discussed in the present study.

Section 2 places Hayek's contribution into context and provides textual evidence. It is argued that as a result of his productive interactions with the 'Robbins Circle' at the London School of Economics, Hayek comes to endorse dynamic equilibrium reasoning as the main vehicle for real analysis. He moreover develops the idea to employ the predictions of such dynamic equilibrium models as reference trajectories in monetary analysis (to say it in modern parlance).

Since the study reaches out to a heterogeneous audience, section 3 provides some notations and mathematical preliminaries that are useful to enhance understanding of how continuity requirements on preferences involve the choice of topological structures on infinite-dimensional commodity spaces, a choice that allows to impose myopia on individual mindsets. Being a contribution to the history of economics, the paper contains no mathematical innovations.

Section 4 isolates asymptotically myopic mindsets by the choice of the Mackey topology and, thereby, gives a precise notion of Böhm-Bawerk's second 'ground' of positive interest, i.e., the systematic undervaluation of future consumption. Heavy emphasis is placed on economic interpretation. To foster understanding, the Mackey topology is discussed in relation to the product and sup norm topology.

Section 5 introduces recursive Hayekian preferences and defines a pure exchange economy. At this stage of analysis, the axiomatic structure sufficient to support the subsequent inquiry is laid down. The central result is the "need for impatience" as proclaimed by Araujo (1985).

Section 6 exemplifies increasing myopia by means of recursive utility functions. The local net rate of pure time preference, i.e., the degree of myopia, is defined. The modified Uzawa aggregator and the Epstein-Hynes aggregator are introduced. With respect to the latter, it is shown that the steady-state rate of pure time preference is increasing in prospective utility. Note the similarities between Figure 1 and Hayek's own graphical exposition of the problem in UAI and PTC.

For expository reasons, sections 4-6 are couched in discrete time. Section 7 translates Hayek's approach by means of optimal control theory. It contains the Hayek-Chang model. *Inter alia*, it provides the Euler equation for the 'Hayek problem'. A subsection discusses existence, uniqueness, and stability issues. Section 8 provides the concluding discussion.

Chapter 6

Context and Textual Evidence

Besides high entry costs to his arduous work, a reason for the widespread neglect of Hayek's contribution is that deserved attention is drawn away by his inferior business cycle theory (Hayek 1935, 1933b, Colonna 1994). In 1931, he initiated his career at the London School of Economics (LSE) by a series of lectures, where he introduced his refinement of Mises's business cycle theory as the only legitimate approach to the dynamics of aggregate variables (Mises 1912). Two important characteristics of Austrian business cycle theory - including Schumpeter's approach (Schumpeter 1934) - are that (1) recurrent variations in macroeconomic variables are traced back to individual choices (methodological individualism), and that (2) such variations are conceived as monetary disequilibria (yet not necessarily as money-induced disequilibria).

Although the first property suggests the use of Walrasian methods, only a static notion of general equilibrium was available at that time, including Hayek's innovation of intertemporal equilibrium (Hayek 1928, 1984a; see also Kompas 1985 and Robbins 1930). Given the lack of alternative notions of equilibrium, traverse processes and business cycles were both introduced as disequilibrium phenomena instead. In fact, the inter-war debates on business cycle theory show that even the delimited use of stationary states as attractors of out-of-equilibrium behavior was highly contested (Rühl 1994). The most serious doubts were expressed by Adolph Lowe, who played a substantial role in the formation of Hayek's thinking:

> "The business cycle problem is not a reproach *for*, but a reproach *against* the static system, because in it it is an antinomic problem. It is solvable only in a system in which the polarity of upswing and crisis arises analytically from the conditions of the system just as the undisturbed adjustment derives from the conditions of the static system. *Those who wish to solve the business cycle problem must sacrifice the static system. Those who adhere*

> *to the static system must abandon the business cycle problem.*" (Lowe 1997: 267; quoted from Dal-Pont and Hagemann 2010: 9; my emphases)

By contrast, Hayek argues that

> "the incorporation of cyclical phenomena into the system of economic equilibrium theory, with which they are in apparent contradiction, remains the crucial problem of Trade Cycle theory. [...] if we want to explain economic phenomena at all, we have *no means available* but to build on the foundations given by the concept of a *tendency towards an equilibrium.*" (Hayek 1933b: 33-4),

Thus, dynamic analysis is equated with stability analysis, that is, with the convergence properties of a Walrasian system. For instance, a traverse process induced by an innovation to the 'social rate of time preference' is analytically equivalent to the convergence of an arbitrary initial intertemporal price system (or the interest rate under the then-standard single-commodity assumption). In contrast to recontracting processes in Walrasian analysis, Hayek focuses on adaptive flex-price adjustments in physical time.[1] With the benefit of hindsight, that is, in awareness of the cluster of problems still associated with non-tâtonnement approximation (e.g. Fisher 1989), it comes as no surprise that Hayek is unable to provide any sufficiency condition for his system to converge. In this respect, the theoretical foundation of Hayek's early traverse analysis, though very intuitive, is rather shaky.

Hayek's early interest in such adjustment paths is only indirect. They serve as reference paths for monetary disequilibrium processes, and allow him to define deviations that are contingent on the existence of money as a medium of exchange (see Klausinger 2011 for a much more detailed account). More precisely, money as "loose joint in the self-equilibrating apparatus of the price mechanism" (Hayek 2007: 367) is assumed to invite coordination problems by relaxing the strict budget constraints that would otherwise prevail. In this sense, Hayek extends Wicksell's concept of neutrality to the case of *relative* price adjustments (Hayek 1935, 1999a, Koopmans 1933, Myrdal 1939, Wicksell 1898). The business cycle is just an instance of such a coordination problem or, equivalently, a specific (yet prominent) variety of monetary non-neutrality. Thus, whatever the specific weaknesses of Hayek's business cycle theory, *all* potential varieties suffer from the poor analytical foundation of the benchmark process.

[1] Any persistence is due to backward-looking (or learning) agents.

Hayek is aware of this problem. Very early he attempts to escape the Austrian framework for intertemporal analysis as laid down by Böhm-Bawerk (Hayek 1984b), but admittedly fails (Hayek 1984a, 1936). In essence, Hayek claims that Böhm-Bawerk's definitions utilized in his theory of capital and interest are fundamentally related to his narrow notion of stationarity as a state described by time-invariant absolute magnitudes. More precisely, in employing a framework that equates the constancy of primitives with the constancy of economic outcomes, and that fundamentally depends on this equation, Böhm-Bawerk provides a straitjacket that inhibits the expansion of the set of analytically feasible equilibrium notions to include time-variant prices or even genuinely dynamic equilibria. In fact, Hayek argues that Böhm-Bawerk's entire analysis is centered on a "pseudo-problem" (Hayek 1936: 44).

It is therefore possible to define a 'Hayek program' as the quest to disengage disequilibrium analysis from the study of neutrality benchmarks and, thereby, to isolate coordination problems as the conclusive scope of monetary analysis. He does so by engaging more developed equilibrium notions instead that predict variations of outcomes over time. The rational reconstruction that constitutes the present study supports the view that the ultimate goal of the 'Hayek program' is to dispense with static notions of equilibrium, including the intertemporal equilibrium, in favor of a genuinely dynamic perfect-foresight equilibrium.

Seen in this light, Hayek's innovation of intertemporal equilibrium by means of dated commodities (Hayek 1999b: 192-3) is just a first attempt to escape Böhm-Bawerk's straitjacket (Hayek 1984a). Even though essentially static, the class of intertemporal equilibrium models predicts time-variant outcomes and, thus, can at least mimic price variations that are due to changing primitives along the timeline. He insists that stationarity of outcomes is - at most - a special case of a general equilibrium with respect to 'given data':

> "[R]egular self-production of the economy is not at all synonymous with continuity in the flow of the individual processes within it. In fact, under given external conditions, this will never be so." (ibid.: 72)

In PTC, where he speaks of "general equilibria which are not stationary" (Hayek 2007: 41), he is most explicit:

> "But between the concept of stationary state and the problem of dynamics in this sense [i.e., monetary dynamics], there is an intermediate field through which we have to pass in order to go from one to the other,"

and this intermediate field is

> "the general idea of equilibrium, of which the stationary state is merely a particular instance, refers to a certain type of relationship between the plans of different members of a society. It refers, that is, to the case where these plans are fully adjusted to one another, so that it is possible for all of them to be carried out [...]. This is clearly the case where people know exactly what is going to happen for the reason that the same operations have been repeated time after time for a long period. But the concept as such can also be applied to situations which are not stationary and where the same correspondence between plans prevail, not because people just continue to do what they have been doing in the past, but because they correctly foresee what changes will occur in the actions of others. This sort of fictitious state of equilibrium which can be *conceived* to comprise any sort of planned change, is indispensable if we want to apply the technique of equilibrium analysis to all phenomena which are *ex definitione* absent in the stationary state. It is in this sphere alone that we can usefully discuss equilibrium relations extending over time, and in which consequently the pure theory of capital mainly falls, and the latter might almost be said to be identical with the whole of this intermediate field between the theory of stationary state and economic dynamics proper." (ibid.: 43-4)

Unlike Arrow-Debreu-McKenzie type of models, Hayek's intertemporal equilibrium theory does not rest on the assumption that all balances are settled at an initial point in time preceding the physical history of the economic system and, thus, does not rely on a complete set of markets. However, in the absence of a complete market system at a singular point in time, the informational requirements on individuals that are necessary to bring about - *ab initio* - a Hayekian intertemporal equilibrium include the deterministic evolution of all primitives. Accordingly, Hayek identifies 'perfect foresight' as an equilibrium condition (see also Hayek 1937; for a more detailed account, see Butos 1985, 1997).

The equilibrium notion in modern theory that approximates Hayek's early concept most closely is Roy Radner's *Equilibrium of plans, prices, and price expectations*, defined for a sequence of open spot markets (Radner 1972).[2] Hayek develops a non-probabilistic

[2]Note, however, that the role of price systems as communication devices fundamentally depends on the notion of Rational Expectation Equilibrium under asymmetric information. See Grossman 1981

variant, emphasizing the equivalence of general equilibrium with overall plan consistency.[3]

Entering the 'Robbins Circle'[4] at the LSE, a powerhouse of general equilibrium reasoning during the inter-war period, Hayek becomes acquainted with the new tools associated with the so-called 'Ordinalist Revolution' (Hicks and Allen 1934), and with Hicks's elaborations on economic dynamics culminating in *Value and Capital* (1939), which is a milestone in general equilibrium analysis and the link between the LSE hub and the craft activities associated with the Cowles Commission. The most important non-LSE influences are Irving Fisher, Frank Knight, and Knut Wicksell.[5] Altogether, these economists play a significant role in the formation of Hayek's thinking.

As Hicks points out, the Circle developed its commitment to approach actual market phenomena by means of general equilibrium analysis before Hayek entered the scene:

> "We seemed, at the start, to share a common viewpoint, or even, one might say, a common faith. Some of us, especially Hayek, have in later years maintained that faith; [...] The faith in question was a belief in the free market, or 'price-mechanism' - that a competitive system, free of all 'interferences' by government or by monopolistic combinations, of capital or of labour, would easily find into 'equilibrium'. Hayek, when he joined us, was to introduce into this doctrine an important qualification - that money (somehow) must be kept 'neutral' in order that the mechanism should work smoothly. That [...] was to cause us quite a lot of trouble." (Hicks 1982: 3)

Ever since, the research on Hayek focused on that 'lot of trouble', that is, on his attempts to integrate monetary theory into general equilibrium analysis, but particularly on his business cycle theory. By contrast, the present study exclusively focuses on the Circle's impact on Hayek, especially with regard to his choice of equilibrium benchmarks for monetary neutrality. Given the impressive intellectual calibers at the LSE, but especially because of his intense and harmonic interactions with Hicks and Robbins, it is

[3]In fact, the concept of 'dated commodities' is inessential to his outline. Instead, financial assets may ensure an optimal intertemporal allocation. Such 'capital markets' relate more closely to Hayek's analytical narratives.

[4]In short, the 'Robbins Circle' is the general-equilibrium based antipode to Cambridge, UK, comprising - besides Lionel Robbins as primus inter pares - young guns like Roy R. D. Allen, John R. Hicks, Abba Lerner, Nicolas Kaldor, and George Shackle. As to my knowledge, Hicks invented the term; see Hicks (1982)

[5]Hayek's admiration of Knut Wicksell survives his gradual transformation without any discount and even grew considerably, as is evident throughout PTC. Unsurprisingly, from all those who had an impact on Hayek's education as an economist, Wicksell is the most Walrasian.

argued that Hayek is a net profiteer. UAI and PTC, especially chapters seventeen and eighteen, tell witness that he strongly benefits from his new colleagues.

For an example of how the Hayek program benefits from modern 'utility analysis', recall Böhm-Bawerk's 'first ground' of positive interest, that is, "the relation of supply and demand as it exists at one point in time and that relation as it exists at another point in time" (Böhm-Bawerk 1959: 265-6). Even in case of a zero rate of pure time preference, i.e., the absence of Bawerk's 'second ground', the rate of interest is positive in case of increasing consumption profiles, since relatively abundant future goods suffer a discount or, equivalently, relatively scarce present goods earn a premium. By definition, however, Böhm-Bawerk's equilibrium notion cannot accommodate the case of increasing consumption profiles. Thus, the 'first ground' must be treated as a disequilibrium phenomenon and, as such, is analytically distinct from the 'second ground' that does survive in long-run equilibrium.

Now, given the concept of intertemporal indifference curves, the 'first ground' is easily involved by assuming strong convexity. 'Modern' utility analysis does not only provide a precise definition of the smoothing bias, but also ensures its survival in general equilibrium. In this sense, it is possible to unburden out-of-equilibrium analysis by broadening the scope of equilibrium behavior (optimal choice). Modern analysis has shown that intertemporally optimizing agents (or a social planner), described by utility functions that involve a discount factor as well as the intertemporal elasticity of substitution, are fundamental ingredients to dynamic equilibrium analysis.

The impact of his LSE colleagues is already evident in Hayek's Copenhagen lecture in December 1933, first published in German in 1935, and reproduced in English as *Price Expectations, Monetary Disturbances and Malinvestments* in 1939 (Hayek 1999b). Here, Hayek sides with "the modern development of equilibrium analysis", characterized by "the efforts of the younger men in our subject [...] directed towards bridging the gulf between 'static' and 'dynamic' analysis" (ibid.: 234). He identifies such young guns in "all countries with a great theoretical tradition" (ibid.), but where in 1933 is this tradition greater than in Hayek's beloved Great Britain? In addition to the Robbins Circle, Friedrich Lutz is a sure candidate (see below).

In Copenhagen, Hayek further claims that

> "[t]he main difficulty of the traditional approach is its complete abstraction from time. A concept of equilibrium which essentially was applicable only to an economic system conceived as timeless could not be of great value. Fortunately in recent times there have been considerable changes on this

very point. It has become clear that, instead of completely disregarding the time element, we must make very definite assumptions about the attitude of persons toward the future." (ibid.: 234-5)

This passage shows that the 'Hayek problem' is still acute as he enters the LSE. It further indicates that pure time preference - or simply myopia - as well as the smoothing bias play a pivotal role in his analysis. He explicitly withdraws his early approach to economic dynamics:

> "What we all seek is [...] not to jump into something entirely new and different but a development of our fundamental theoretical apparatus which will enable us to explain dynamic phenomena. Not very long ago I myself still believed that the best way to express this was to say that the theory of the trade cycle at which we were aiming ought to be organically superimposed upon the existing theory of equilibrium. I am now more inclined to say that *general theory itself ought to be developed so as to enable us to use it directly in the explanation of particular industrial fluctuations*. As has recently been shown very convincingly by Dr. Lutz, our task is not to construct a separate theory of the trade cycle, that is of a construction of a detailed scheme which fit all actual trade cycles, but rather a development of those sections of general theory which we need in the analysis of particular cycles - which often differ from one another very considerably." (ibid.: 233-4; italics mine)

Thus, the existing body of equilibrium analysis is to be extended to include economic dynamics. In Hayek's own words, he opts for "[e]quilibrium analysis" applied "to a changing competitive system" (Hayek 1999b: 234). Further, as far as business cycles are not the result of coordination problems, but rather the outcome of individually optimal adjustments to changing primitives, they fall within the scope of equilibrium analysis (the "general theory") as well.

His reference to Friedrich Lutz (1932) supports this view. Lutz argues that stability analysis inherent in static general equilibrium analysis suffices to grasp analytically the dynamics of the business cycle:

> "Statics is really but a branch of dynamics, as Marshall put it. [...] the mere fact that business cycle theory deals with processes of change does not mean that it is part of a dynamic theory rather than the static one." (Lutz 1932: 10, n.2; quoted from Dal-Pont and Hagemann 2010: 17)

In the words of Lucas, Lutz opts "for modeling [...] dynamics as response to static excess demands" (Lucas 1981: 286). Note the similarity to Hayek's pre-LSE approach to dynamic analysis in that both treat business cycles as a subbranch of stability analysis. Yet, whereas Hayek's early approach treats money as the 'single cause' of economic fluctuations, Lutz gives up the idea that the business cycle can be traced back to a unique cause. Instead, he treats such fluctuations as flex-price out-of-equilibrium adjustments to changing primitives (Rühl 1994: 188-9).

Thus, Lutz's business cycle theory focuses on what Hayek's early analysis identifies as hypothetical benchmarks for monetary neutrality (like the traverse process introduced above). Even though Lutz's argument is defective, it directs Hayek's attention to the idea that business cycle theory is (partly) a subbranch of general equilibrium analysis. Given that he aims at time-variant equilibrium systems by broadening the notion of 'stationary primitives', and given that the specification of 'stationary primitives' as stationary stochastic processes is congruent with this goal, it is tempting to view the Hayek-quote above as an indication to a prototype 'Real Business Cycle' (RBC) theory.

Of course, Hayek does not deliver the theoretical foundations that could legitimate a rational reconstruction by means of competitive Dynamic Stochastic General Equilibrium models. What he does, however, is to provide material sufficient to model a infinite-horizon deterministic social planner optimization problem as currently employed in capital theory (Becker 2008). UAI and chapters seventeen and eighteen of PTC contain that material (see also Hayek 1945a). Note, however, that Hayek's dynamic narratives are exclusively supported by the local techniques of differential calculus as developed by Fisher and Hicks/Allen.

Only equipped with a strictly convex intertemporal indifference curve and a nonconvex transformation curve, Hayek delineates a "saving path" by visualizing a succession of two-period optimization problems and, in fact, arrives at a generalized Keynes-Ramsey rule (Hayek 1936: 50, 57; Hayek 2007: 216, 225, 228). More precisely, he portrays transitional dynamics, or a dynamic equilibrium, as a potentially continuous succession of intersections of a transformation curve with the tangent indifference curve. The succession of two-period optimization problems suggests the use of recursive methods. In fact, recursive tools are imposed on any attempt to rationally reconstruct Hayek's dynamic equilibrium by his specification of next-period utility as a measure for infinite consumption tails (e.g. Hayek 1936: 49-50). Given a time-indexed sequence of indifference curves, Hayek 'models' increasing myopia by successively steeper sloping (tangents to) indifference curves at the intercept with the bisector. The economic rationale behind time-variant myopia is some kind of intertemporal complementarity that is excluded by time-separable preferences.

Chapter 7

Mathematical Preliminaries

In infinite-dimensional spaces, myopic preferences are invoked by the proper choice of the topological structure of commodity spaces, which gives rise to alternative notions of 'closeness' and 'approximation' of consumption profiles. Such a procedure allows to identify the restrictions that are necessary to ensure (and understand) myopia, Böhm-Bawerk's second cause of interest (called 'pure' time preference). The ultimate strength of this approach roots in the nature of the relationship between linear and topological space structures underlying infinite-dimensional topological vector spaces (note that linear spaces are of algebraic nature). Arbitrary combinations of the two structures are generally not a topological vector space: the two structures must conform, namely, the linear operations must be continuous.

In finite-dimensional spaces, which characterize early Arrow-Debreu-McKenzie type of models, there is only one topological structure that is consistent with the linear structure. Finite-dimensional vector spaces and finite-dimensional topological vector spaces are therefore equivalent in that every linear transformation of a finite-dimensional topological vector space is a homeomorphism. There is no topological choice to make and topological notions, like continuity and convergence, are introduced for technical convenience. However, in infinite-dimensional spaces different topological structures conform to a given linear space structure, and linear transformations are generally not homeomorphisms.

The burden of an additional margin of choice mirrors a potential benefit: topological assumptions can carry economic interpretation. In concrete, an informed topological choice can reduce the general class of preferences, i.e., those induced by the norm topology, to those that discriminate against non-myopic preferences. Thus, *given* rational preferences, myopia may or may not be enforced on preferences. It therefore pays off to delve a bit deeper into mathematics and to give some precise notions. Even though the subsequent material is standard in modern analysis, it may be new to historians of economics. They may also consult any introduction to real analysis like Royden's classic

(Royden 1968), or more accessible literature like Aliprantis and Border (2006).

For an index set I of arbitrary cardinality, a topology τ on X is a collection of open sets $\mathcal{O} = \{O_\alpha\}_{\alpha \in I} \subset P(X)$ such that if $O_\alpha \in \mathcal{O}$, then $\bigcup_{\alpha \in I} O_\alpha \in \mathcal{O}$ (closed under arbitrary unions), if $\{O_1, \ldots, O_n\} \in \mathcal{O}$, then $\bigcap_{\alpha=1}^{n} O_\alpha \in \mathcal{O}$ (closed under arbitrary intersections), and $\emptyset, X \in \mathcal{O}$. The finite unions and finite intersections of open sets are therefore open. A *topological space* is the couple $\langle X, \tau \rangle$. A topology introduces a weak notion of closeness, independent of a metric. For $x, y, z \in X$, we say that y is closer to x than is z, if y is contained in an open set around x not containing z. It is always the agent's perspective that imposes structure on topological space. Any two profiles are close, iff agents regard them as such. To isolate the topological structure of a topological vector space, say Y, define a topological space X such that $X \subset Y$ and $\tau = \mathcal{O}_X = \{O \cap X : O \in \mathcal{O}_Y\}$ (subspace topology).

A set $N \subset X$ is called a neighborhood of an element x, if $\exists O_\alpha \in \tau$ such that $x \in O_\alpha \subset N$. A *neighborhood system* of x, denoted by $\mathcal{N}_x$, is the collection of all such neighborhoods at x. A topology (and thus a neighborhood system) is generated by its *base* $\mathcal{B} = \{B_\alpha\}_{\alpha \in I}$ such that $\bigcup_{\alpha \in I} B_\alpha = O_{\alpha_i}$, iff each $x \in X$ is contained in some $B \in \mathcal{B}$ and if $x \in B_1 \cap B_2$ for $B_1, B_2 \in \mathcal{B} \rightarrow B_3 \in \mathcal{B}$ such that $x \in B_3 \subset B_1 \cap B_2$. Informally, a neighborhood base approximates any open set by a large number of small open balls. For instance, in any metric space the set of all ϵ-neighborhoods (for all different values of ϵ) is a base for a topology. It allows to reduce topological properties to statements about the 'building blocks' generating that topology. Finally, a *subbase* is the collection of all finite intersections of neighborhoods that forms a base for a topology (a minimum set of open sets that span the entire topology). If X is a space that has a topology $\mathcal{T}$, then we say that X is locally convex, if there exists a base $\mathcal{B}$ for the topology that consists of convex sets.

An important topological property is that of continuity. A mapping f between the topological spaces $\langle X, \tau_X \rangle$ and $\langle Y, \tau_Y \rangle$ is τ-*continuous*, iff the inverse image of every open set in X is open in Y, that is, if $O \in \tau_Y \Rightarrow f^{-1}(O) \in \tau_X$, $\forall O \in \mathcal{O}_Y$. The mapping f is a rule that associates to each point $x \in X$ a point $y \in Y$. Continuity implies that it makes open sets in X correspond to open sets in Y. Thus, points that are 'close together' in the first topological space (in terms of preferences) are mapped to points that are also 'close together' in the second topological space. Given $\langle X, \tau_X \rangle, \langle Y, \tau_Y \rangle$, and $\langle Z, \tau_Z \rangle$ such that $f : \langle X, \tau_X \rangle \rightarrow \langle Y, \tau_Y \rangle$ and $g : \langle Y, \tau_Y \rangle \rightarrow \langle Z, \tau_Z \rangle$ are both continuous, it follows that $g \circ f \rightarrow \langle Z, \tau_Z \rangle$ is continuous. If $f : \langle X, \tau_X \rangle \rightarrow \langle Y, \tau_Y \rangle$ is a continuous bijection and has a continuous inverse function f^{-1}, then it is a *homeomorphism*. Stated differently, for $f : \langle X, \tau_X \rangle \rightarrow \langle Y, \tau_Y \rangle$ and $f^{-1} : \langle Y, \tau_Y \rangle \rightarrow \langle X, \tau_X \rangle$, $f \circ f^{-1} = I_X$ and $f^{-1} \circ f = I_Y$ (I is an identity map). The homeomorphism forms an equivalence relation on the class of all topological spaces (i.e., is reflexive, symmetric, and transitive).

A crucial role of topologies is to define the class of continuous functions: the more open sets there are, the more mappings are continuous. In general, there more sets are open in the domain of a function, and the fewer sets are open in its codomain, the more functions are continuous. For instance, if the domain of a function $f : \langle X, \tau_X \rangle \to \langle Y, \tau_Y \rangle$ has the discrete topology, that is, $\tau_X = P(X)$ such that all possible subsets of X are open,[1], then each function on X is continuous, i.e., if $O \in \tau_Y$, then $f^{-1}(O) \in \tau_X$. Equally, the trivial topology on Y, $\tau_Y = \{\emptyset, Y\}$, ensures that all functions are continuous: since the only open sets are the empty set and the massive Y, only $f^{-1}(\emptyset) = \emptyset$ and $f^{-1}(Y) = X$ have to be open in X. This is true by definition (each topology is at least trivial). Whereas a domain equipped with the discrete topology is at least as fine as any topology on the codomain, and thus satisfies even the most demanding requirements for continuity, the trivial topology on the codomain imposes the least demanding requirements for continuity on the domain.

The discrete and trivial topologies are extreme cases of the *family of topologies*, denoted by $\mathcal{T} = \{\tau_\alpha\}_{\alpha \in I}$, is partially ordered by set inclusion. Most importantly, $\mathcal{T}$ is the economist's choice set. For any two topologies $\tau_1, \tau_2 \in \mathcal{T}$ on X, if $\tau_1 \subset \tau_2$ such that $O_i^1 \subset O_j^2, \forall i$ (and not necessarily $\forall j$), we say that τ_1 is 'weaker' (coarser) than τ_2, and that τ_2 is 'stronger' (finer) than τ_1. Obviously, the relative weakness of a topology is accounted for by its lack of open sets (neighborhoods). Topologies only allow for qualitative statements about distance: any two vectors are topologically distinguishable, and are in this sense distanced, if they can be separated by open sets. Think of a set of open sets centered at an arbitrary point $x \in X$. Take a distinct point $y \in X$ and count the number of open sets you have to cross from x to y. The weaker a topology is, the more probable it becomes that x and y are in the same open set and, therefore, topologically indistinguishable. In the trivial topology, for instance, where the empty set and the entire space are the only open sets, all vectors in that space are topologically indistinguishable. In turn, the more open sets around x there are, the more precise becomes the notion of closeness.

[1]The discrete topology is induced by the discrete metric, denoted by g, which states that the distance between all distinct vectors is of value one and that non-distinct vectors are of zero distance. It is therefore possible to impose $\forall x \in X$ open balls $B_x(\epsilon) = \{x \in X : g(y, x) < \epsilon\}$ such that

$$B_x(\epsilon) = \begin{cases} \{x\} & \text{if } 0 < \epsilon \leq 1 \\ X & \text{if } \epsilon > 1 \end{cases}.$$

Thus, each singleton is an open set. Since the finite union of open sets is open, all possible subsets in $P(X)$ are open.

To make more ambitious statements about closeness without resuming to a metric (which discriminates against the topology we are looking for), it is necessary to impose a *uniform structure*: given two vectors $x, y \in X$, a uniform structure is the non-empty system Θ of entourages U of the Cartesian product $X \times X$ such that if $U \in \Theta$, then $U \supset \{(x, x) : x \in X\}$ (every vector is close to itself), and $\{(x, y) : (y, x) \in U\} \in \Theta$ (if x is close to y, then y is close to x). If in addition $U \in \Theta$, $V \in X \times X$ and $U \subset V$, then $V \in \Theta$ and $U \cap V \in \Theta$ (relaxing the degree of closeness gives another notion of closeness; combing two degrees of closeness provides another degree of closeness). Finally, if $U \in \Theta$, there exists $V \in \Theta$ s.t., whenever $(x, y) \in V$ and $(y, z) \in V$, then $(x, z) \in V$ (to every degree of closeness, there exists another one that captures 'twice as close').

Uniform properties, preserved under uniform homeomorphisms, fall between topological and metric properties. Even though stronger than the topological notion of closeness, uniform spaces are weaker than metric spaces, since they do not depend on the quantification of closeness. Each entourage is a family of neighborhoods, one for each vector in X. Two neighborhoods from the same entourage are of the same size and thus allow to relate profiles in terms of closeness. In uniform space, it is therefore possible to formalizes the notion of 'x_1 is as close to x_2, as is y_1 to y_2'. A property is uniform on a set if that property holds at every point in the set with common estimates. A topology is induced by the uniform structure by defining subsets $O_\alpha \in \mathcal{O}$ to be open, iff for every $x \in O_\alpha$ there exists an entourage U such that $U(x) \subset O_\alpha$. Every linear topological space is also a uniform space.

We now turn the the costs of strong topologies. In this regard, the relations between open, closed and compact sets are crucial. In infinite-dimensional spaces, the Heine-Borel theorem (in $\mathbb{R}^n$ with Euclidean topology, compact sets are precisely those that are closed and bounded) is not valid. The generic approach to compactness makes use of the topological notion of an open cover. A collection $\mathcal{F} \in \mathcal{B}$ of open sets in a topological space X is an open cover of X, if $X \subset \mathcal{F} = \{F_\alpha\}_{\alpha \in I}$. X is *compact*, if every open cover $\mathcal{F}$ has a finite subcover, $\mathcal{F}' = \{F_1, F_2, \ldots F_N\}$, such that $X = \mathcal{F}' = \bigcup_{i=1}^{N} F_{\alpha_i}$. Compact sets do not ever run away to infinity! In fact, a nice property of compact sets is that they behave like points. Note that the notion of compactness is intimately related to the notion of being a closed set. This follows from the definition of closed sets as complements of open sets, i.e., $C_\alpha = \complement F_\alpha$, such that $\bigcup_{\alpha \in I} F_\alpha = \complement \left(\bigcap_{\alpha \in I} C_\alpha \right) = X$ (using De Morgan's laws).[2]

An equivalent definition of compactness is that X is compact, if for every collection of closed sets $\mathcal{C} = \{C_\alpha\}_{\alpha \in I} \in X$ such that $\bigcap_{\alpha \in I} C_\alpha = \emptyset$, there exists a finite collection of

[2]Note that $\emptyset$ and X are always open *and* closed, since for all topological spaces $\complement\emptyset = X$ and $\complement X = \emptyset$.

closed subsets $\mathcal{C}' = \{C_1, C_2, \ldots C_n\}$ such that $\bigcap_{i=1}^{n} C_{\alpha_i} = \complement\left(\bigcup_{i=1}^{N} F_{\alpha_i}\right) = \emptyset$. Instead of a finite subcover of open sets whose union is still equal to X, we look for a finite number of closed sets whose intersection is still equal to the empty set (any topological space is equipped with the empty set). Since all finite collections of closed sets with empty intersection are associated with the empty set, the topological space X isolates all finite collections of closed sets that have the finite intersection property (each finite subset has a nonempty intersection). It can be further shown that each closed subset of X has a finite subcover, or equivalently, that closed subsets of compact spaces are compact.

So far, closed sets are given as complements of open sets. An equivalent definition of a closed set is that of a container of *limits*, the possible solutions of choice problems. Implicit in the notion of a limit is the estimation of the error of successive approximations. Only by showing that the error can be made arbitrarily small can we establish that a limit exists. The notion of convergence allows for a computational perspective: choice is the outcome of an approximation algorithm, an iterative routine that generates a sequence of trial or approximate solutions that converge to a 'final solution'. In general topological spaces, for $x_n, x \in X$, a sequence $\{x_n\}_{n \in I}$ converges to x, written $\{x_n\} \to x$, if for any neighborhood $F(x) \in \mathcal{B}$ there exists a threshold n_0 (depending only on F) such that $x_n \in F$ for all $n \geq n_0$. Think of a collection of open sets centered on x. Convergence suggests that there exists an iterative method such that for an initial guess x_0 'sufficiently close' to x, the subsequent iterates are contained in successively smaller open sets around x. Thereby, the threshold n_0 tells us which approximate solution of the algorithm comes sufficiently close. Convergence requires that such a threshold exist so that there are at most finitely many initial guesses which are not sufficiently close. Note that for general topological spaces it is impossible to make quantitative statements about the approximation of limits, that is, statements about the speed of convergence.

Now let X be a compact space with an infinite (uncountable) subset S which has no limit point. $\forall x \in X$, take an open set $F(x)$ around x. There is an open cover of X composed of open neighborhoods $F(x)$ such that

$$F(x) \cap S = \begin{cases} \emptyset & \text{if } x \notin S \\ \{x\} & \text{if } x \in S \end{cases}.$$

Thus, for $x \in S$ we ensure that neighborhoods around x are small enough to be a neighborhood of just one point in S. For $x \notin S$, neighborhoods are small enough not to contain any point in S. Since X is compact, the open cover $\{F(x) : x \in X\}$ must admit a finite subcover and we can discard the rest. We are left with a finite number of sets which contain at most one point in S. Thus, as a subset of a compact space, S must be finite. Evidently, compactness is a topological generalization of finiteness.

Given that all compact subsets of Hausdorff spaces are closed, we arrive at the *Bolzano-Weierstrass property*: an infinite subset on a compact space must contain a limit point, or equivalently, every net has a convergent subnet.[3] Hence, the assumption of compactness eliminates all infinite sets that contain no limit point, and leaves us with only those sets that admit convergence. Further, compactness is a topological invariant, i.e., preserved under homeomorphisms. Finally, a continuous real-valued function defined on a compact space achieves its maximum and minimum values and is thus bounded. The more subsets of a topological space are compact, the more probable it is to find routines that approximate limit points, independent of initial positions. However, the more convergent sequences exist, the weaker becomes the notion of convergence. It simply does not mean much for a routine to converge, if we introduce convergence as a generic characteristic of topological spaces.

We are finally prepared to highlight the tradeoff involved in topological choices. Economic analysis exploits the *fundamental tradeoff* introduced by the ordered family of topologies ($\mathcal{T}$): *the finer the topology, the more open sets there are, the more functions on that space are continuous. Yet, the finer the topology, the fewer compact sets and, hence, the fewer convergent subnets there are.*

Having introduced the basics of topological structures, I now turn to the vector structure. From the economic point of view, the linear structure of commodity space reflects the linearity of market choices: any linear combination of two such choices gives a third choice. Linearity also extends to the price system: the market value of the sum of any two market choices equals the some their respective market values. *normed vector space* $(X, \|\cdot\|) := \{X, \mathbb{R}, +, \cdot\} \cup \{\|\cdot\|\}$ is a vector space X over a field $\mathbb{R}$ equipped with a norm $\|\cdot\| : X \to \mathbb{R}^+$ such that (1) $\|x+y\| \leq \|x\| + \|y\|$ (triangle inequality), (2) $\|\alpha x\| = |\alpha| \, \|x\|$ (homogeneity), and (3) $\|x\| = 0 \Leftrightarrow x = 0$. (3) states that the norm separates distinct vectors. The field imposes algebraic structure on X or, more precisely, it is the set of real numbers together with the two vector operations $(x, y) \mapsto x + y : X \times X \to X$ (addition) and $(\alpha, x) \mapsto \alpha x : \mathbb{R} \times X \to X$ (scalar multiplication) such that the well-known set of

[3] A net is a function $x : I \to X$, where the index set I is a directed set, but not necessarily $\mathbb{N}$; it is a topological generalizations of sequences, which are restricted to metric spaces. In *Hausdorff* spaces (so-called T_2-spaces), any two distinct vectors are separable, i.e., for $x, y \in X$ there are disjoint open sets O_1 and O_2 such that $x \in O_1$ and $y \in O_2$ (points can be separated by neighborhoods). Also the Hausdorff property can be expressed from a more constructivist perspective: any two vectors $x, y \in X$ are separable by some neighborhoods, if there is a continuous routine assigning values only in $[0, 1]$ that assumes zero value on an open set and the value one on the other (a characteristic function indicating membership to a set). Note that T_2 implies T_1 (the Tychonoff property) such that each singleton is a closed set.

axioms on the field of reals hold. An endomorphism $f : X \to X$ is a linear operator, if for every $x, y \in X$ and for every $\alpha, \beta \in \mathbb{R}$ it satisfies $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ (homogeneity and additivity). In general, linear operators are transformations that preserve the algebraic structure under homeomorphisms.

A subset $S \subset X$ is a subspace of X, if the two operations $(+)$ and $(\cdot)$ never associate vectors in S with vectors in $\complement S$. Informally, it is impossible to escape the subspace by means of linear vector operations. The span of a subset S is the smallest vector subspace containing it. In general, the dimension of a vector space X is given by the cardinality of its basis B, the linearly independent spanning set such that every vector in X can be expressed as a linear combination of vectors in B in a unique way. For a Banach space $(X, \|\cdot\|)$, the infinite series $B = \{b_i\}_{i\in\mathbb{N}}$ is the so-called *Schauder basis*, if for each $x \in X$ there exists a unique sequence of scalars $\{\alpha_i\}_{i\in\mathbb{N}}$ such that $\lim_{n\to\infty} \|x - \sum_{i=1}^{n} \alpha_i b_i\| = 0$. In this case, we write $x = \sum_{i\in\mathbb{N}} \alpha_i b_i$ and say that B is norm convergent. Thus, each element in a Banach space is generated by countably infinite linear combinations of the basic vectors.

If the linear operations $+$ and $\cdot$ are both continuous with respect to the underlying topology, the notion of a normed vector space generalizes to that of a topological vector space (linear topology). Note that whenever vector addition is continuous, the topology on X is translation invariant such that if $O \subset X$ is open, then $O + x$ is open $\forall x \in X$. It therefore suffices to provide a base around the zero vector, a local base denoted by $\mathcal{B}_0$, to generate a linear topology. It is assumed that $(X, \|\cdot\|)$ is *locally convex*, that is, $\mathcal{B}_0$ consists of absolutely convex and absorbing open sets (an absolutely convex set is balanced and convex). Local convexity suggests that for each $V_1 \in \mathcal{B}_0$ there exists some $V_2 \in \mathcal{B}_0$ such that $V_2 + V_2 \subset V_1$ (for each ϵ-ball, there is a $\frac{\epsilon}{2}$-ball; for each $\frac{\epsilon}{2}$-ball, there is a $\frac{\epsilon}{4}$-ball;...). A linear topology is locally convex, iff it is generated by a family of seminorms $\{p_i\}_{i\in I}$, with $p_i : X \to \mathbb{R}^+, \forall i$ characterized by properties (1) and (2) above. In particular, a locally convex topology is generated by a family of gauges, each gauge given by $p_V(x) = \inf\{\alpha > 0 : x \in \alpha V\}$ (p_V enlarges the zero-centered set V such that it just contains x). For each $x \in X$, denote by $V_p(\epsilon) = \{x \in X : p(x) \leq \epsilon\}$ the closed ϵ-balls of p centered at zero and containing x. For $\epsilon > 0$ and $i \in I$, $\mathcal{B}_0 = \bigcap_{i,\epsilon} V_{i,\epsilon}$, where $V_{i,\epsilon}$ is the ball induced by p_i.

The family of seminorms can be taken to be the single norm, if the seminorms also separate distinct points, i.e., satisfy property (3) above. In this case the norm function acts as a gauge and we say that the vector space is generated by the *norm topology* τ. Here, the topological notion of distance is given by the norm-induced metric $d(x, y) = \|x - y\|$ for each pair $x, y \in X$. Because of property (3), normed spaces are Hausdorff locally convex. Since τ is translation invariant, it follows that $(X, \|\cdot\|)$ is endowed with an uniform structure. Accordingly, τ is also called the uniform topology and is fine enough to admit *uniform convergence*. We write $\{x_n\}_{n\in I} \to x$ uniformly, if there is an open epsilon-ball on x, denoted by $B_x(\epsilon)$, with $\epsilon > 0$ and $N \in I$, such that for all $n > N$ we have $\|x - x_n\| < \epsilon$. If the normed space X is also a complete metric space such that all nets converge to some limit in X, it is a *Banach space*. Note that the norm topology is the strongest topology which preserves the algebraic structure under homeomorphism. Accordingly, it is known as the *strong topology*.

Most spaces utilized in economic analysis are Riesz spaces. A *Riesz space*, denoted by the pair $(E, \geq)$, is a vector space E over the reals that is equipped with a partial order $\geq$ so that (1) $x \geq y \to x + z \geq y + z, \forall z \in E$ and (2) $x \geq y \to \alpha x \geq \alpha y, \forall \alpha \in \mathbb{R}^+$, and that is also a lattice so that for each pair $x, y \in E$, $x \vee y = \sup\{x, y\} \in E$ and $x \wedge y = \inf\{x, y\} \in E$. The ordering is pointwise, i.e., $x \geq y \in E$, if $x_i \geq y_i, \forall i \in I$. Each element $x \in E$ is decomposed as the difference of its positive part $x^+ = x \vee 0$ and its negative part $x^- = (-x) \vee 0$, that is, $x = x^+ - x^-$. Note that both parts are positive. The absolute value of x is defined by $|x| = x^+ + x^-$. Given properties (1) and (2), which suggest the compatibility of $\geq$ with the algebraic structure of E, the subset $E^+ = \{x \in E : x \geq 0\} \subset E$ is a positive cone that induces $\geq$ on E, i.e., E^+ satisfies (a) $\forall \alpha \geq 0$, $\forall x \in E^+ \to \alpha x \in E^+$, (b) $x, y \in E^+ \to x + y \in E^+$, and (c) $E^+ \cap -(E^+) = \{0\}$.

A sequence $\{x_\alpha\}_{\alpha\in I}$ converges in order to $x \in X$, written $x_\alpha \xrightarrow{o} x$, if there a sequence $\{y_\alpha\}_{\alpha\in I} \in X$ satisfying $y_\alpha \downarrow 0$ and $|x_\alpha - x| \leq y_\alpha, \forall \alpha$. A function $f : E \to F$ between two arbitrary Riesz spaces is order continuous, if $x_\alpha \xrightarrow{o} x$ in E implies $f(x_\alpha) \xrightarrow{o} f(x)$ in F. The norm on Banach lattices is order continuous, if every decreasing sequence that converges in order to zero also converges to zero in norm (i.e., $\bigwedge_{\alpha\in\mathbb{N}} x_\alpha = 0 \to f(x_\alpha) = \|x_\alpha\| \downarrow 0$). For a linear functional $f : E \to \mathbb{R}$, if $x \geq 0$ implies $f(x) \geq 0$, f is positive; if $x > 0$ implies $f(x) > 0$, f is strictly positive. If $f([x, y])$ is a bounded subset of $\mathbb{R}$ for each oder interval $[x, y]$, f is order bounded. If one of the lattice operations $(x, y) \mapsto x \vee y$, $(x, y) \mapsto x \wedge y$, $x \mapsto x^+$, $x \mapsto x^-$, and $x \mapsto |x|$ is uniformly continuous with respect to the topology on $\mathbb{R}^{\mathbb{N}}$, all lattice operations are and τ is locally solid so that for any $V \subset \mathcal{B}_0$, $y \in V$, if $x \in V$ and $|y| \leq |x|$. Most importantly, if a linear topology is locally solid, the positive cone of the space is τ-closed. Further, if τ is locally solid, $x_\alpha \uparrow$, and $x_\alpha \xrightarrow{\tau} x$, then $x_\alpha \uparrow x = \sup\{x_\alpha\}$. Thus, τ is a *locally convex-solid topology* on a Riesz space so that $\mathcal{B}_0$ consists of all τ-closed, convex, and solid neighborhoods centered at zero.

In economic analysis it is common to obtain Hausdorff locally convex spaces by means of *dual systems*. A dual system is a pair of vector spaces $\langle X, X'\rangle$ together with the duality $(x, x') \mapsto \langle x, x'\rangle$ from $X \times X' \to \mathbb{R}$. The duality is a bilinear functional such that $x' \mapsto \langle x, x' \rangle$ is linear for each $x \in X$, and $x \mapsto \langle x, x' \rangle$ is linear for each $x' \in X'$. It separates the points of X and X', that is, if $\langle x, x' \rangle = 0$, then $x = 0$ for each $x' \in X'$, and $x' = 0$ for each $x \in X$. Evidently, each vector space of the pair $\langle X, X' \rangle$ is a set of linear functionals: each $x \in X$ defines the linear functional $x' \mapsto \langle x, x' \rangle$, and each $x' \in X'$ defines the linear functional $x \mapsto \langle x, x' \rangle$. Note that the set of all linear functions on X is its algebraic dual, denoted by X^*. X' is a subspace of X^*, the set of all linear functionals on X that are also continuous with respect to the norm topology. In general, X' is called the continuous or *topological dual* of X, separating elements in X. If X' is the continuous dual of a normed space it is a normed space called norm dual. The dual norm of a continuous linear functional on X is defined by $\|x'\|_{X'} = \sup\{|x'(x)| : x \in X, \|x\| \leq 1\}$. The norm dual X' is a Banach space, if the X is a Banach space. If $\langle X, X' \rangle$ is a Riesz dual system, X is a Riesz space, $X^\sim$ denotes its order dual, that is, the set of all order bounded linear functionals on X. The subset of all order continuous linear functionals is called the order continuous dual of X, denoted by $X_n^\sim$.

In general, according to the definition of the continuous dual, each $x' \in X'$ is continuous with respect to the norm topology. The weakest topology for which each $x' \in X'$ is still a continuous function is called the *weak topology* on X, denoted by $\sigma(X, X')$. $\sigma(X, X')$ is induced by the family of separating seminorms $\{p_{x'}(x) : x' \in X'\}$, with each $p_{x'}(x) := |x'(x)| = |\langle x, x' \rangle| \in \mathbb{R}, \forall x \in X$, and is therefore Hausdorff locally convex. Note that $x_\alpha \xrightarrow{\sigma(X,X')} x \in X$, iff $\langle x_\alpha, x' \rangle \to \langle x, x' \rangle \in \mathbb{R}$ for each $x' \in X'$. Since convergence depends on x', the weak topology is known as the *topology of pointwise convergence* on X' (or weak convergence). Likewise, the *weak-***topology* on X', denoted by $\sigma(X', X)$, is a locally convex Hausdorff topology generated by $\{p_x(x') : x \in X\}$ with $p_x(x') := |x(x')| = |\langle x, x' \rangle| \in \mathbb{R}, \forall x' \in X'$, so that $x'_\alpha \xrightarrow{\sigma(X',X)} x' \in X'$, iff $\langle x, x'_\alpha \rangle \to \langle x, x' \rangle \in \mathbb{R}$ for each $x \in X$. The weak-*topology is the weakest topology on X' so that all linear functionals $x \in X$ on X' are still continuous. Pointwise convergence is equivalent to convergence in the product topology on a product space. Therefore, since $\sigma(X, X')$ ($\sigma(X', X)$) is the topology of pointwise convergence on X' (X), it is likewise the subspace topology on X (X') induced by the product topology τ on $\mathbb{R}^{X'}$ ($\mathbb{R}^X$) such that $X \subset \mathbb{R}^{X'}$ ($X' \subset \mathbb{R}^X$) with $\sigma(X, X') = \tau_X = \{X \cap O : O \in \tau\}$ (and $\sigma(X', X) = \tau_{X'} = \{X' \cap O : O \in \tau\}$). Note that $\mathbb{R}^{X'} = \prod_{x'_i \in X'} \mathbb{R}_i$ is the space of real-valued functions on X' so that for $\{(\mathbb{R}_i, \tau_i)\}_{x'_i \in X'}$ we have $\tau = \prod_{x'_i} \tau_i$ (where X' figures as an index set). The coordinate projections on $\mathbb{R}^{X'}$ are the maps $\{\pi_{x'_i} : \mathbb{R}^{X'} \to \mathbb{R}\}_{x' \in X}$ defined by $\pi_{x'}(x) = x(x'), \forall x \in \mathbb{R}^{X'}$ (similar for $\mathbb{R}^X$). The product topology is the weakest topology for which these projections are still continuous.

If $\langle X, X' \rangle$ is a dual pair, then $(X, \sigma(X, X'))'$ is X' and $(X', \sigma(X', X))'$ is X. Further, let X'_1 and X'_2 be total subspaces of X^*. Then $\sigma(X, X'_1) \subset \sigma(X, X'_2)$, iff $X'_1 \subset X'_2$. In general, the larger X', the stronger is $\sigma(X, X')$ on X (the results for weak-*topologies are similar). The stronger the topology on X, the more closed sets there are. The larger X', the more continuous linear functionals exist, and the more sets in X can be separated. On the other hand, the stronger the topology, the fewer compact sets there are. Further, for a given topological dual X', there are topologies on X stronger than the weak topology that still gives X' as the dual. Instead of enlarging X' (X) for a fixed type of topology on X (X'), i.e., the weak (weak-*) topology, X' (X) can be fixed and we look for those topologies on X (X') that are stronger than the weak (weak-*) topology but still compatible or consistent with the dual pair $\langle X, X' \rangle$. A locally convex topology τ on X is *consistent* with $\langle X, X' \rangle$, if $(X, \tau)' = X'$. Consistent topologies on X' are defined similarly. The weak and weak-*topologies are the weakest consistent topologies by definition. However, we are interested in all topologies consistent with the dual pair.

The generic restrictions that isolate consistent topologies of a dual system are approached by means of polar of sets. For a dual pair $\langle X, X' \rangle$, the *polar* of a nonempty subset A of X, denoted by A°, is the subset of X' such that $A^\circ = \{x' \in X' : |\langle x, x' \rangle| \leq 1, \forall x \in A\}$. Let $\{A_i\}$ be a collection of nonempty subsets of X. Note that (1) if $A_1 \subset A_2$, then $A_2^\circ \subset A_1^\circ$. (2) If $\epsilon \neq 0$, then $(\epsilon A)^\circ = \epsilon^{-1} A^\circ$. (3) $\bigcap_{i \in I} A_i^\circ = \left(\bigcup_{i \in I} A_i\right)^\circ$. Similarly, if A is a subset of X', then the polar ${}^\circ A$ is the subset of X such that ${}^\circ A = \{x \in X : |\langle x, x' \rangle| \leq 1, \forall x' \in A\}$. The corresponding dual statements of $(1)-(3)$ are true for subsets of X'. The polar of the norm bounded, absolute convex and $\sigma(X, X')$-closed unit ball $U = \{x \in X : \|x\| \leq 1\} \in (X, \|\cdot\|)$ is the norm bounded, absolute convex and $\sigma(X', X)$-closed unit ball $U' \in X'$: $U^\circ = U'$ and $(U')^\circ = U^{\circ\circ} = U$ (where $U^{\circ\circ} = (U^\circ)^\circ$ is the bipolar of U). Note that if the unit ball of the norm dual is weak-*-closed and norm bounded, it is weak-*-compact.

The subset $A \in X$ is $\sigma(X, X')$-bounded, iff it is pointwise bounded, that is, iff for each $x' \in X'$ the set $\{|\langle x, x' \rangle| : x \in A\} < \infty \in \mathbb{R}$ (similar definition for $A \in X'$). Let $A \in X'$ be $\sigma(X', X)$-bounded so that for each $x \in X$ the real-valued function $q_A(x) = \sup\{|\langle x, x' \rangle| : x' \in A\}$ is a seminorm on X. Denote by $\mathfrak{S}$ a collection of nonempty $\sigma(X, X')$-bounded sets $A \in X'$. Then, the $\mathfrak{S}$-*topology* on X is the locally convex topology generated by the family of seminorms $\{q_A : A \in \mathfrak{S}\}$. Note that ${}^\circ A = \{x \in X : |\langle x, x' \rangle| \leq 1\}$ provides the unit ball in X (independent of $\|\cdot\|$) so that $q_A(x) = \sup\{|\langle x, x' \rangle| : x' \in A\} = \inf\{\epsilon > 0 : x \in \epsilon^\circ A\} = p_{A^\circ}(x)$. Thus, we may equivalently say that the neighborhood base at zero is generated by the collection $\{\epsilon^\circ A : A \in \mathfrak{S}, \epsilon > 0\}$. $\mathfrak{S}$ can be extended to the smallest saturated superset (the saturated hull) of ${}^\circ A$ so that $\hat{\mathfrak{S}} = \{\epsilon A : A \in \mathfrak{S}, \epsilon > 0\}$, which generates the same topology on X. Thus, for $A_1, \ldots, A_n \in \hat{\mathfrak{S}}$, the zero-centered neighborhood base for the $\mathfrak{S}$-topology is given by

$\mathcal{B}_0 = \bigcap_{i=1}^{n} {}^\circ A_i$.

Given the $\mathfrak{S}$-topology, it is possible to isolate those topologies that are consistent with the dual system. According to the Mackey-Arens theorem, a locally convex topology τ on X is consistent with $\langle X, X' \rangle$, iff τ is the $\mathfrak{S}$-topology for a collection $\mathfrak{S}$ of absolute convex and $\sigma(X', X)$-compact subsets of X' with $\bigcup_{A \in \mathfrak{S}} A = X'$. The strongest such topology is called the *Mackey topology*, denoted by $\tau_{MA}(X, X')$. Likewise, the Mackey topology on X' is denoted $\tau_{MA}(X', X)$. Thus, any topology τ on X is consistent with the dual pair, iff $\sigma(X, X') \subset \tau \subset \tau_{MA}(X, X')$. Likewise, any τ' on X' is consistent, iff $\sigma(X', X) \subset \tau \subset \tau_{MA}(X', X)$. All consistent topologies on X have the same bounded sets. Like the $\sigma(X, X')$- and $\sigma(X', X)$, the norm topology on X is consistent with the dual system by definition.

Chapter 8

Axiomatic Foundations

8.1 Commodity Spaces and Myopic Mindsets

Like contemporary capital theory, Hayek tackles the problem of intertemporal allocation in an infinite-horizon setting. Therefore, it is possible to exploit the mathematical fact that - unlike the finite-dimensional case - an infinite dimensional vector space is generally endowed with more than one natural topology. Even though the formal toolset has to be significantly augmented, as is obvious from the previous section, the variety of potential topological structures allows us to specify alternative individual mindsets common to all agents populating the artificial economy. In particular, it is possible to isolate myopic mindsets and to give a precise notion of Böhm-Bawerk's second 'cause' of interest, i.e., the systematic undervaluation of future needs.

The case of infinitely many commodities arises whenever an infinite variation in any of the properties describing commodities is allowed for. This characteristics could be physical properties, locations of delivery, and the date of delivery. Following Hayek, we restrict analysis to the case of commodities with infinite variation in the time of delivery, but which are otherwise homogeneous (or bundles of fixed composition). For expository reasons, time is assumed to be discrete so that we are in need of an infinite set of 'time periods'. The natural choice is $\mathbb{N}$. Thus, the *objects of choice* are (countably) infinite consumption sequences $x = \{x_t\}_{t\in\mathbb{N}} = (x_1, x_2, \ldots)$, where x_t denotes the quantity of consumption at date t. Given a sequence x, its T-head is defined by ${}^{(T)}x = (x_1, \ldots, x_T, 0, 0, \ldots)$ and the T-tail by $x^{(T)} = (0, \ldots, 0, x_{T+1}, x_{T+2}, \ldots)$. Note that - in contrast to T-tails - T-heads of sequences have a finite support by definition. $\bar{x} = (x, x, \ldots)$ is a constant program.

As soon as at this stage, we arrive at the *absolute* advantage of intertemporal equilibrium analysis as proclaimed by Hayek. Whereas Böhm-Bawerk's semi-neoclassical toolset restricts his equilibrium analysis to the study of constant sequences, rendering the distinction between heads and tails redundant, intertemporal general equilibrium analysis allows us to model choice sets that accommodate a large variety of consumption profiles. Böhm-Bawerk apparatus, so Hayek's critique, restricts the notion of equilibrium or stationarity to constant outcomes, whereas equilibrium in the Fisher-Hicks framework only suggests the stationarity of the system's primitives - imposing no independent restrictions on the shape of equilibrium profiles.

The *basic sequence space* is $\mathbb{R}^{\omega}$, the Riesz space of all real-valued functions on $\mathbb{N}$, that is, functions of the form $x : \mathbb{N} \to \bigcup_{t\in\mathbb{N}} \mathbb{R}_t$ such that $x(t) \in \mathbb{R}_t, \forall t \in \mathbb{N}$, and that for each $t \in \mathbb{N}$ the canonical projection is $\pi_t : \prod_{t=1}^{\infty} \mathbb{R}_t \to \mathbb{R}_t$ defined by $\pi_t(x) = x(t)$, where $x(t)$ will be denoted by x_t. The limit of a sequence, denoted by x_∞, is a real number infinitely often assumed on $\mathbb{N}$. Given the topological spaces $\{(\mathbb{R}_t, \tau_t)\}_{t\in\mathbb{N}}$, the product topology τ is the weakest topology on $\mathbb{R}^{\omega}$ such that each π_t is a continuous surjective open mapping. That is, if $U \subset (\mathbb{R}_t, \tau_t)$ is open, then the preimage $\pi_t(U) = \mathbb{R}_1 \times \ldots \times \mathbb{R}_{t-1} \times U \times \mathbb{R}_{t+1} \times \ldots$ is open in $\left(\mathbb{R}^{\mathbb{N}}, \tau\right), \forall t \in \mathbb{N}$. Note that the set U imposes a restriction on $(\mathbb{R}_t, \tau_t)$ so that the open cylinder set $\pi_t(U)$ is the set of all element in $\mathbb{R}^{\omega}$ in which the t-th component is restricted to U while all other components remain unrestricted. Thus, $\{\pi_t(U)\}_{t\in\mathbb{N}}$ is a subbase for τ. A set $V \in \mathbb{R}^{\omega}$ is open, iff it is the union of finite intersections of the sub-basic open sets $\pi_t(U), \forall t$. Accordingly, the base of τ is $\mathcal{B} = \{\prod_{t=1}^{\infty} U_t : U_t \in \tau_t$ and $U_t = \mathbb{R}_t$ for cofinitely many $t \in \mathbb{N}\}$ such that the basic open set is of the form $V = U_1 \times \ldots \times\ U_T \times \mathbb{R}_{T+1} \times \mathbb{R}_{T+2} \times \ldots$.

Obviously, open sets in $\mathbb{R}^{\omega}$ give rise to a neighborhood system that imposes no restrictions on the tail of sequences so that any two tails of consumption profiles are topologically indistinguishable. Informally, for agents whose choice sets are defined on an infinite commodity space equipped with the product topology, the tails of all sequences are close to one another. Conversely, agents are only able to identify differences between sequences with respect to their heads. It is 'as if' agents discount future differences. In this sense, they are *myopic.* This gives a precise idea of how the choice of a topological structure introduces a positive rate of pure time preference. Note, however, that the product topology is not the only topology that provides a notion of myopia, and it by far does not provide its most plausible notion. Further, to be able to evaluate if and to what extent myopia is prohibitively restrictive, we still need to know how it relates to preference relations, and if additional individual restrictions are involved.

In the product topology, the fact that the open sets of only finitely many time periods are restricted has direct implications for the convergence properties on $\mathbb{R}^{\omega}$. Suppose that in contrast to the definition of basic open sets above, all component space are possibly restricted. This would give as the box topology which is the default topology of the product topology. Assume a sequence of constant sequences $\{x_n\}_{n\in\mathbb{N}}$ with $x_n = (1/n, 1/n, \ldots)$.

Obviously, any reasonable notion of convergence would suggest that the sequence converges to the null sequence $x_0 = (0, 0, \ldots)$. Choose $U_t(x_0) = (-1/t, 1/t), \forall t \in \mathbb{N}$ such that the basic open set (box) is $V(x_0) = (-1, 1) \times (-1/2, 1/2) \times (-1/3, 1/3) \ldots$. In the box topology, no x_n remains within the neighborhood of x_0, that is, $\{x_n\}$ does not converge. This shows how restrictive the box topology is.

By contrast, in the product topology τ, where cofintely many component spaces are unrestricted, there is some $N \in \mathbb{N}$ for which $\{x_n\}_{n \geq N} \in V(x_0) = (-1, 1) \times (-1/2, 1/2) \times \ldots \times (-1/T, 1/T) \times \mathbb{R}_{T+1} \times \mathbb{R}_{T+2} \times \ldots$. Boxes are not open in the product topology so that it is weaker than the box topology. Informally, convergence is more probable, if agents discount future differences. In τ, convergence is accordingly controlled by the behavior of the heads of sequences. Intuitively, it is reasonable to presume that the isolation of a 'best element' is, say, a lot easier, if we do not have to take into account infinitely many characteristics of each alternative. In general, for $\{x_n\}$ to converge in τ it is necessary and sufficient that all sequences eventually intersect a sub-basic open set and, therefore, that all projections converge on all coordinate spaces, i.e., $\{x_n\}_{n \in \mathbb{N}} \xrightarrow{\tau} x \in \mathbb{R}^\omega$, iff $\{x_{t,n}\}_{n \in \mathbb{N}} \xrightarrow{\tau_t} x_t \in \mathbb{R}_t, \forall t \in \mathbb{N}$. Evidently, convergence in the product topology is the same as pointwise convergence of sequences.

Price systems, denoted by p, aggregate individual myopia. In our example with constant sequences, they should in fact precisely reflect the social rate of pure time preference. Since commodities are physically indistinguishable, it is legitimate to introduce the notion of an unique short-run rate of interest as an intertemporal price relation. The price space is generally introduced as the topological dual of the commodity space. If we would choose the basic sequences space $\mathbb{R}^\omega$ as our commodity space, it comes as no surprise that the corresponding price space would be the subspace of all sequences in $\mathbb{R}^\omega$, denoted by φ, which have cofinitely many zero coordinates, i.e., $\varphi = \{p \in \mathbb{R}^\omega : p_t = 0, \text{ except for finite support}\}$.

This indicates the weakness of the notion of myopia as introduced by the product topology, and disqualifies $\mathbb{R}^\omega$ as a commodity space. In particular, the problem with such a strong notion of myopia is that it excludes the set of all strictly positive price systems, i.e., the set $p^+ = \{p \in \mathbb{R}^\omega : x > 0 \rightarrow p(x) > 0 \in \mathbb{R}, \forall x \in \mathbb{R}^\omega\}$, because each functional p is representable by its positive T-head ${}^{(T)}p = (p_1, \ldots, p_T, 0, 0, \ldots)$ such that $x_t > 0$ implies $p(x_t) = 0, \forall t > T \in \mathbb{N}$. Since individual myopia suggests that each agent disregards any difference between sufficiently late outcomes, 'the market' imposes zero weight on the tails of positive consumption programs. Neither on the individual level, nor in the aggregate, does such a notion of myopia provide a productive account of the subjective opportunity costs of 'late' consumption in terms of 'early' consumption.

To evade this problem, we restrict attention to a sufficiently general subspace of $\mathbb{R}^\omega$. Accordingly, the *commodity space* is $(\ell_\infty, \|\cdot\|_\infty) = \{x \in \mathbb{R}^\omega : \|x\|_\infty = \sup\{|x_t| : t \in \mathbb{N}\} < \infty\}$, the Riesz space of all bounded continuous real functions on $\mathbb{N}$, equipped with the sup norm. The exclusion of unbounded sequences from the commodity space and thus from individual choice sets is a restrictive assumption. Also from a Hayekian point of view, we want choice to be bounded by price systems that aggregate information about the availability of endowments. This is the first instance of a restriction that is missing in Hayek's narratives. Note, however, that from the viewpoint of the economy, that is, in contrast to the individual perspective, the choice of ℓ_∞ is unproblematic. With this respect to the economic system as a whole, the assumption of boundedness states the 'niggardness of nature', on the escape of which the central themes of economic reasoning immediately depend since at least Adam Smith.

The sup norm is always a seminorm and, therefore, generates a linear topology, denoted by τ', that is locally convex-solid. Thus, $(\ell_\infty, \|\cdot\|_\infty)$ has a zero-centered base consisting of neighborhoods that are simultaneously closed, solid, and absolutely convex. Of course, ℓ_∞ also inherits the product topology τ from $\mathbb{R}^\omega$. But whereas τ is the topology of pointwise convergence, sequences converge uniformly in the sup-norm topology τ', that is, $\{x_n\}_{n\in\mathbb{N}} \xrightarrow{\tau'} x \in \ell_\infty$, iff $\sup\{|x_n - x| : n \in \mathbb{N}\} \longrightarrow 0$. More generally, a sequence converges uniformly to the limit sequence $x \in \ell_\infty$ whenever for each $\epsilon > 0$ there exists some number n_0 such that $|x_n(t) - x(t)| < \epsilon, \forall t, n \geq n_0 \in \mathbb{N}$. Think of each ϵ-neighborhood of x as a t-invariant basin of attraction for a sequence of trial solutions generated by an iterative method for finding the root of a distance function, triggered by a sufficiently close initial guess. Uniform convergence requires that there exists a threshold n_0, depending on ϵ only, such that the termination conditions are satisfied at x_{n_0}, for all t. Hence, the ϵ-neighborhood determines a degree of tolerance, i.e., an approximation error that is regarded as sufficiently small so that the routine is allowed to cut short.

With each ϵ we associate an entourage, that is, a collection of neighborhoods of equal size, one for each consumption profile in ℓ_∞. The smaller ϵ is chosen, and thus the less generous the requirements for convergence, the larger is the threshold n_0. Likewise, a small threshold implies a high degree of tolerance. In the sup norm topology we can specify that the neighborhood system of the limit sequence is given by a collection of corridors $\mathcal{V}(x) = \{(x - \epsilon(n_0), x + \epsilon(n_0))\}_{n_0\in\mathbb{N}}$ with $\epsilon = \sup\{|x_{n_0} - x|\}$ such that $V_{n_0}(x) = \bigcup_{t\in\mathbb{N}}(x(t) - \epsilon(n_0), x(t) + \epsilon(n_0)) \in \mathcal{V}(x), \forall t, \forall n_0 \in \mathbb{N}$. The requirement that sequences converge uniformly for each $\epsilon > 0$ suggests that they converge even if the basin of attraction is restricted to the immediate neighborhood of the limit sequence. In general, if a sequence converges uniformly, it also converges pointwise. But the converse is not true. In pointwise convergence, where the interval $(x(t) - \epsilon(t, n), x(t) + \epsilon(t, n))$ also depends on t, the value at each t does converge to the value of the limit sequence, but the convergence rates can vary widely enough to make it impossible to control convergence at two different parts of the domain simultaneously. But in uniform convergence we have simultaneous control of the convergence over the entire infinite domain.

Hence, uniform convergence is the stronger notion (equivalently, the uniform or sup norm topology is finer than the product topology). Accordingly, some sequences do converge in the product topology, yet not in the sup norm topology. For instance, if we choose

$$x_n = (\underbrace{1, 1, \ldots, 1}_{n}, 0, 0, \ldots),$$

then $\{x_n\}_{n\in\mathbb{N}} \overset{\tau}{\to} e$, where e denotes the unit vector $\mathbf{1} = (1, 1, \ldots)$, but it does not converge in τ', since the sup norm takes into account that for any index $n_0 \in \mathbb{N}$, there is a subsequent zero-valued tail such that $\|x^{(n_0)}\|_\infty = \sup\{|x_t| : t \geq n_0 + 1\} = 0$, and $|x_n(t) - x(t)| = 1$, for $t = n > n_0$. No matter how large the index grows, a sequence with a finite support cannot escape the neighborhood of the null sequence and, hence, cannot come close to sequences with infinite support. From the viewpoint of the agent, such consumption programs differ in kind, not in degree. In contrast to the case of pointwise convergence, the behavior of the tail cannot be neglected, because control is over the entire domain simultaneously. In fact, the tail dominates the behavior of the entire sequence.

The converse case is of special significance in economic analysis, particularly in the study of myopia. Thus, we choose

$$\hat{x}_n = (\underbrace{0, 0, \ldots, 0}_{n}, 1, 1, \ldots).$$

Informally, from the viewpoint of an agent, the consumption of the unit sequence is subsequently postponed into the future. How does he cope with it? Unsurprisingly, we have $\{\hat{x}_n\}_{n\in\mathbb{N}} \overset{\tau}{\to} (0, 0, \ldots)$, that is, the further a unit program is postponed into the future, the more similar it becomes to a zero-valued alternative from the viewpoint of an agent, the viewpoint given by the product topology. From the agent's perspective, all $\hat{x}_n$ differ from the zero-valued sequence only in degree, and not in kind. Note that we still say nothing about preferences. But myopia roots in the topological structure of the commodity space and is only 'activated' by the standard continuity assumption on the preference ordering. Evidently, it is the topological structure that is behind the 'elasticity of substitution'.

Yet, such myopic mindsets do not necessarily carry over to the sup norm topology. In our example, where we would like to see $x = (0, 0, \ldots)$, we have $\|\hat{x}^{(n_0)}\|_\infty = sup\,\{|\hat{x}_t| : t \geq n_0 + 1\} = 1$ instead, and thus $d(\hat{x}_n, x) = |\hat{x}_n(t) - x(t)| = 1$, for $t = n > n_0$.

Eventually, no sequence can escape the immediate neighborhood of e. Thus, the initial unit program and any postponed copy of it are 'almost' perfect substitutes. i.e., in cofinitely many periods. In general, the sup norm topology cannot discriminate against non-myopic mindsets. We are therefore exposed to the following problem: The topology of pointwise convergence is generically myopic, yet it is prohibitively weak as argued above. By contrast, the norm topology allows for non-myopic mindsets, but is of uniform structure and thus the more efficient tool in economic application. Are there intermediate topologies that combine the strengths of τ and τ', but avoid their weaknesses? Fortunately, there are some such topologies. One of them is the Mackey topology, which plays an important role in the study of myopic topologies. It will prove useful to devote some attention to the Mackey topology in isolation.

The Mackey topology on ℓ_∞ is the strongest locally convex-solid and weak-* compact topology still consistent with the Riez dual system $\langle \ell_\infty, \ell_1 \rangle$, where ℓ_1 is an ideal of the order dual of ℓ_∞ and gives the price space composed of absolutely summable sequences, that is, $\ell_1 = \{p \in \mathbb{R}^\omega : \|p\|_1 < \infty\}$ with $\|p\|_1 = \| \{p_t\}_{t \in \mathbb{N}} \|_1 = \sum_{t=1}^{\infty} |p_t|$ (the Manhattan distance). Note that $\ell_1 \subset \ell_\infty \subset \mathbb{R}^\omega$. In contrast to ℓ_∞, the Banach lattice ℓ_1 is separable and has the Schur property, i.e., $p_n \xrightarrow{\sigma(\ell_1, \ell_\infty)} p$ implies $p_n \xrightarrow{\|\cdot\|_1} p$, so that the collections of $\sigma(\ell_1, \ell_\infty)$-compact and $\|\cdot\|_1$-compact subsets $\{A\} \in \ell_1$ coincide. The polar of each subset A, i.e., $^\circ A = \{x \in \ell_\infty : |\langle x, p \rangle| \leq 1\}$, is a weakly compact unit ball in ℓ_∞. The collection $\mathfrak{S} = \{A_i \in \ell_1 : A_i \text{ is } \sigma(\ell_1, \ell_\infty)\text{-compact and absolutely convex and solid}, \forall i \in I\}$, where I is an index set, gives way to locally convex-solid $\mathfrak{S}$-topologies on ℓ_∞, all with uniform structure, each generated by the family of seminorms $\{q_A : A \in \mathfrak{S}\}$ with $q_A(x) = \sup\{|\langle x, p \rangle| : p \in A\} = \inf\{\epsilon > 0 : x \in \epsilon^\circ A\}$. If also $\bigcup_{A \in \mathfrak{S}} A = \ell_1$, then a $\mathfrak{S}$-topology is consistent with $\langle \ell_\infty, \ell_1 \rangle$. Note that every consistent topology is Hausdorff. Further, all locally convex topologies consistent with a given dual pair have the same collection of closed convex sets. Thus, the Mackey topology, denoted by $\tau_{MA}(\ell_\infty, \ell_1)$, is the strongest such topology.

Note that according to Riesz's lemma the closed unit balls in infinite-dimensional spaces cannot possibly be compact, i.e., closed and pointwise bounded (or weakly bounded), with respect to the sup norm topology. Given any norm topology, local compactness in fact characterizes finite dimensionality. The advantage of the Mackey topology in economic analysis is related to the Banach-Alaoglu theorem, which states that the closed unit balls of the norm dual is weak-* bounded and, thus, compact in the weak-*topology. Hence, the Mackey topology is a rich source of (weakly) compact sets and, in consequence, of uniformly convergent sequences. Further, while ℓ_∞ is not Hausdorff in the sup norm topology, it is Hausdorff in the Mackey topology so that each sequence converges to a unique limit. Since we are foremost interested in myopic mindsets, it is of particular significance that - in contrast to the sup norm on ℓ_∞ - the Mackey topology is order continuous.

Thus, a sequence $\{x_n\}_{n\in\mathbb{N}} \xrightarrow{\tau_{MA}(\ell_\infty,\ell_1)} x \in \ell_\infty$, if there is a sequence $\{a_n\}_{n\in\mathbb{N}}$ satisfying $a_n \downarrow 0$ and $|x_n - x| \leq a_n, \forall n$, where $a_n \in c_0 = \{x \in \mathbb{R}^\omega : x_\infty = \lim_{t\to\infty} x_t = 0\} \subset \ell_\infty, \forall n$. In words, c_0 is the (separable) space of null sequences (with norm dual ℓ_1). Thus, we can focus on the subset of decreasing null sequences, and choose any one of them to isolate converging sequences in $(\ell_\infty, \tau_{MA}(\ell_\infty, \ell_1))$. Such a filter leaves us with those sequences that satisfy $d(x_n, x) = \sup\{a_n(t) \cdot |x_n(t) - x(t)| : t, n \in \mathbb{N}\} \xrightarrow{\tau'} 0$, all t, which evidently is a much more restrictive notion of convergence. It follows that in $\tau_{MA}(\ell_\infty, \ell_1)$, the T-tail $x_n^{(T)} = (0, \ldots, 0, x_{T+1}, x_{T+2}, \ldots) \to 0, \forall n$. Resuming to the example above so that

$$\hat{x}_n = (\underbrace{0, 0, \ldots, 0}_{n}, 1, 1, \ldots),$$

we follow Epstein (1987a) and choose $a_n(t) = t^{-\frac{1}{2}}, \forall n$, so that for $x = (0, 0, \ldots)$ we have $d(\hat{x}_n, x) = \sup\left\{t^{-\frac{1}{2}} \cdot |\hat{x}_n(t)|\right\}$. Evidently, $d(\hat{x}_n, x) \xrightarrow{\tau'} 0$, whenever $t \to \infty, \forall n$. Here, the Mackey topology dispels the dominance of tails that otherwise dominates convergence in the sup norm topology. Informally, the agent's mindset discounts the future; it is *asymptotically myopic*.

Even though order and pointwise convergence coincide in $\mathbb{R}^\omega$, Mackey convergence is more restrictive than pointwise convergence in τ. Consider, for instance, the sequence

$$\check{x}_n = (\underbrace{0, 0, \ldots, 0}_{n}, n, n, \ldots),$$

so that any postponement of the program comes by the permanent benefit of an increase in tail-consumption (ibid.). In fact, tail-consumption levels are unbounded as n approximates infinity. Mackey convergence would disqualify as a productive tool in economic analysis, if it *categorically* discriminated against the choice of $\check{x}_n$. Yet, this is not the case. Again, for $a_n(t) = t^{-\frac{1}{2}}$, all n, so that $d(\hat{x}_n, x) = \sup\left\{t^{-\frac{1}{2}} \cdot |\hat{x}_n(t)|\right\}$, even though $t^{-\frac{1}{2}}$ still imposes a discount on tail-consumption, there is a counteracting force since the tail itself grows larger and larger so that $\check{x}_n^{(T)}$ does not converge to $x = (0, 0, \ldots)$.

8.2 A Hayekian Pure Exchange Economy

The economic system is given by the Riesz pair $\langle \ell_\infty, \ell_1 \rangle$ and the duality $\langle \cdot, \cdot \rangle : \ell_\infty \times \ell_1 \to \mathbb{R}$ such that $\langle x, p \rangle = p(x) \equiv p \cdot x, \forall x \in \ell_\infty$ and $\forall p \in \ell_1$. The *pure exchange economy*, denoted by $\mathcal{E}$, is defined by

$$\mathcal{E} = \{\Lambda, \succeq_i, \omega_i, i = 1, \ldots, A\},$$

where $\Lambda = \ell_\infty^+ = \{x : x \geq 0\}$ is a τ-closed, convex cone and gives the consumption (opportunity) set, common to all A agents indexed by $i \in I$ so that all carry the same mindset (note that if Λ is τ-closed, it is also τ_{MA}-closed). $\succeq_i$ on Λ denotes the i-th agent's binary preference relation such that $\succeq_i \subset \Lambda \times \Lambda$, and $\omega_i \in \Lambda$ denotes the i-agent's initial endowment. The economy is competitive, that is, A is sufficiently large so that equilibria in $\mathcal{E}$ approximate the equilibrium properties of the continuum economy. An equilibrium for $\mathcal{E}$ is a pair $((x_1, \ldots, x_A), p) \in (\Lambda)^A \times \ell_1, p \neq 0$, such that

(E1) $\sum_{i=1}^A x_i = \sum_{i=1}^A \omega_i$.

(E2) For each i, x_i maximizes $\succeq_i$ on $\{x \in \Lambda, p \cdot x \leq p \cdot \omega_i\}$.

With respect to $\succeq_i$, it is assumed that

(P1) $\succeq_i$ is reflexive, complete, and transitive (rational choice),

(P2) $\succeq_i$ is monotone, i.e., $x \geq y$ implies $x \succeq y, \forall x, y \in \Lambda$,

(P3) $\succeq_i$ is convex, i.e., if $y_1 \succeq_i x$ and $y_2 \succeq_i x$, then $\alpha y_1 + (1-\alpha) y_2 \succeq_i x, \forall x, y \in \Lambda, \forall \alpha \in [0, 1]$ s.t. $\{y \in \Lambda : y \succeq_i x\}$ is convex for each $x \in \Lambda$,

(P4) $\succeq_i$ is continuous, i.e., $\{y \in \Lambda : y \succeq_i x\}$ and $\{y \in \Lambda : y \preceq_i x\}$ are both τ_{MA}-closed,

for all $i \in I$. The derived strict preference relation $\succ_i$ is defined by $x \succ_i y$, if $x \succeq_i y$ but not $y \succeq_i x, \forall i$. For any two programs $x, y \in \Lambda$, if $x \succeq_i y$ and $y \succeq_i x$, y is in the equivalence class of x, i.e., $[x]_i = \{y \in \Lambda : y \sim_i x\}$, where $\sim_i$ denotes the indifference relation of the i-th agent. Since the upper and lower contour sets are closed, so are the indifference sets. It also follows that the upper and lower contour sets given by the strict relation are necessarily open.

Given transitivity, no program can be a member of two equivalence classes and the set of equivalence classes forms a partition on Λ, i.e., $\Lambda = \bigcup [\cdot]_i$. Think of a routine, a choice process, that runs on Λ, taking into account the wealth constraints, such that for any initial choice it arbitrarily selects an alternative consumption program, identifies the program that is 'weakly preferred to', rejects the other program, and iterates until it detects a maximizer (the unique maximizer in the case of strict convexity). We cannot assure that the routine is finite since wealth sets are infinite. Accordingly, it does not help to impose a memory by assuming that rejected programs are related to the empty set. However, (P1)-(P4) ensure that the choice process is convergent, or, equivalently, ensure that the algorithm generates convergent sequences.

Assumptions (P1)-(P3) are based on the algebraic structure of Λ. Reflexivity, is a technical assumption that prohibits our routine to halt in case it compares identical programs or, in fact, programs in the same indifference sets. Completeness is problematic. It suggests the ability to rank alternative programs globally and, thus, imposes unreasonable high cognitive skills on each agent. However, it can be shown that for sufficiently many heterogeneous agents completeness is not necessary for existence. Transitivity discriminates against cycling choice processes and is unproblematic from a Hayekian point of view. Like Smith, he endows his agents with 'self-love' and 'Reason' so that the ability to rank alternatives consistently seems permissible. In fact, the reduction of Reason to transitivity signifies the strength of the axiomatic method, since its notion of rationality is rather weak for sufficiently large economies.

(P2) relates the binary relation to the partial order and thereby suggests that for each agent 'more is better'. Thus, the routine is able to discriminate between programs that only differ in quantity. (P3) provides Böhm-Bawerk's first cause of interest, the fundamental property that tails of increasing profiles suffer a subjective discount. In UAI, and even more so in PTC, Hayek makes use of the smoothing bias in its strict version (Hayek 2007: Ch. 12). For instance, he extensively discusses smoothing activities as induced by innovations to the investment opportunity set, with each innovation characterized by the period of its occurrence (ibid.: Ch. 22, 23).

It is important to realize that (P3) has nothing whatsoever to do with the notion of myopia. Even non-myopic agents discount future consumption in increasing programs, because by definition they imply time-variant intertemporal scarcities. In such a case, time favors the future, that is, relatively 'late' consumption is less scarce and, just therefore, suffers a discount. Accordingly, 'late' consumption carries a premium over 'early' consumption in decreasing programs. Myopic mindsets are rather induced by (P4).

Continuity implies that if there is a specific sequence $\{y_n\} \to y$ such that $y_n \succeq x, \forall n \in \mathbb{N}$, then $y \succeq x$ with $y \in \{y \in \Lambda : y \succeq x\}, \forall x \in \Lambda$ (similarly for the lower contour set).

The sequence does not reverse, or, equivalently, the choice process does not jump, in approximating the limit y, which, therefore, is regarded as being close to the reference program x. The condition that the upper and lower contour sets are both closed relates to the topological structure and guarantees that the sequence cannot switch between the two sets. We thereby ensure that there exists a maximizing program in each agent's permissible choice set. A glance at Hayek's indifference curves in UAI and PTC suggests that the underlying preference relation is continuous.

Whereas continuity is a purely technical assumption on finite-dimensional commodity spaces, on which all natural topological structures are equivalent to the Euclidean topology, continuity in the infinite-horizon case carries economic interpretation (Brown and Lewis 1981; Mas-Colell and Zame 1991). We gain a degree of freedom with respect to the topological structure; a margin of choice between individual mindsets. (P4) specifies the Mackey topology so that agents are asymptotically myopic. Since $\tau \subset \tau_{MA} \subset \tau'$ on ℓ_∞, it follows that τ-continuous preferences are also τ_{MA}-continuous, and that τ_{MA}-continuous preferences are also τ'-continuous. It also follows that a τ'-convergent sequence is also τ_{MA}-convergent, and a τ_{MA}-convergent sequence is also τ-convergent. In both cases, the reverse does not hold.

The systematic undervaluation of future needs as reflected in a positive pure rate of time preference, that is, Böhm-Bawerk's second cause that is accountable for intensive dispute in the history of economics, appears almost casually. According to Koopmans, who is the first to show that the topological structure on the sequences space ℓ_∞ imposes behavioral restrictions on continuous preferences, "conditions hardly stronger than those that appear needed to *define* impatience in a meaningful way are sufficient to *prove* that there are zones of impatience [on the payoff space]" (Koopmans 1960: 288; his emphases). Bewley (1972) attributes to Werner Hildenbrand the notion of asymptotic myopia on ℓ_∞ as well as the insight that τ_{MA}-continuous preferences are generally asymptotically myopic. Brown and Lewis (1981) introduce the concepts of weakly myopic and strongly myopic preference relations on ℓ_∞, denoted by $\succeq_{WM}$ and $\succeq_{SM}$, respectively.

To simplify notation, we follow Becker and Boyd III. (1993) and define a projection operator π such that the T-th iterate is π^T and $\pi^T x = (x_1, x_2, \ldots, x_T) =^{(T)} x$, as well as a shift operator S such that the T-th iterate is S^T and $S^T x = (x_{T+1}, x_{T+2}, \ldots) = x^{(T)}$. We can thus write $(\pi^n 0, S^n x)$ for $x_n = (\underbrace{0, 0, \ldots, 0}_{n}, x_{n+1}, x_{n+2}, \ldots) \in \{x_n\}_{n \in \mathbb{N}}$.

($\succeq_{WM}$) A preference ordering $\succeq$ on ℓ_∞ is *weakly myopic*, if $\forall x, y, \bar{c} \in \ell_\infty$, where $\bar{c} = (c, c, \ldots)$ is a constant program, $x \succeq y$ implies $x \succeq y + (\pi^n 0, S^n \bar{c})$, for sufficiently large $n \in \mathbb{N}$. Accordingly, the weakly myopic topology on ℓ_∞, denoted by τ_{WM}, is the topology on which all τ_{WM}-continuous complete preference relations are weakly myopic.

($\succeq_{SM}$) A preference ordering $\succeq$ on ℓ_∞ is *strongly myopic*, if $\forall x, y, z \in \ell_\infty$, where $z = (z_1, z_2, \ldots)$ is a bounded program, $x \succeq y$ implies $x \succeq y + (\pi^n 0, S^n z)$, for sufficiently large $n \in \mathbb{N}$. Accordingly, the strongly myopic topology on ℓ_∞, denoted by τ_{SM}, is the topology on which all τ_{SM}-continuous complete preference relations are strongly myopic.

In both cases, myopia is defined as a continuity requirement. We have $\tau_{SM} \subset \tau_{WM}$ so that a τ_{WM}-convergent sequence is also τ_{SM}-convergent. Of particular significance for the present study is that $\tau_{SM} = \tau_{MA}$ w.r.t. $\langle \ell_\infty, \ell_1 \rangle$, as shown by Brown and Lewis (ibid.: Theorem 4a). In words, the Mackey topology is the finest strongly myopic locally convex topology consistent with the economic system, or, equivalently, all strongly myopic mindsets are asymptotically myopic. Further, Araujo shows that for any topology stronger than the Mackey topology, including τ_{WM}, individually rational Pareto optimal allocations may fail to exist so that (P4) is in fact a necessary condition for existence and as such "the best assumption of its kind" (Araujo 1985: 455). Referring to Brown's and Lewis's result that $\tau_{SM} = \tau_{MA}$, Araujo provides the economic interpretation that a solution to the global allocation problem in the infinite horizon case depends on asymptotically myopic mindsets so that he proclaims "the need for impatience" (ibid.).

It follows that there is some vindication for the Fetter-Mises stance on the role of pure time preference as a *conditio since qua non*. But they are right for the wrong reasons, since they suggest that myopia is a categorical characteristic of *real* human choice (whereas the results only show that myopia is a equilibrium condition and say nothing about its empirical content), and that the rate of pure time preference is the sole determinant of rates of marginal intertemporal substitution. Clearly, the Schumpeter-Friedman stance on myopia as an irrational and, thus, illegitimate ingredient to rational choice does not survive, since it cannot be generalized to the infinite horizon case. We still have to wait, however, to be able to evaluate Hayek's position.

The interesting feature of Hayek's approach is that the rate of pure time preference is endogenous. It depends on the size and the time shape of alternative consumption programs due to intertemporal complementarities imposed on preferences. To formalize his contribution, we resort to Koopmans' innovation of recursive preferences (Koopmans

1960). Therefore, the permissible class of preferences is further restricted by augmenting (P1)-(P4) with the so-called 'Koopmans axioms':

(K1) $\succeq_i$ is stationary, i.e., $(\pi z, Sx) \succeq (\pi z, Sy)$, iff $Sx \succeq Sy, \forall x, y \in \Lambda, \forall z \in \pi\Lambda$,

(K2) $\succeq_i$ is limited non-complementary over time, i.e., $(\pi z, Sx) \succeq (\pi z', Sx)$, iff $(\pi z, Sy) \succeq (\pi z', Sy), \forall x, y \in \Lambda, \forall z, z' \in \pi\Lambda$,

(K3) $\succeq_i$ is sensitive, i.e., $\exists x \in \Lambda$ and $\exists z, z' \in \pi\Lambda$ s.t. $(\pi z', Sx) \succ (\pi z, Sx)$,

for all $i \in I$. (K1) suggests that the ordering of any two admissible programs which coincide in the incipient period is invariant to the consumption level during that period. It implies that the preference order is sustained, if the timing of all successive outcomes is advanced by one period. From repeated application it follows that the ordering is independent of calendar time. As argued above, stationarity is unproblematic from a Hayekian viewpoint. In general, (K1) isolates the impact of changes in the rate of pure time preference from changes in tastes.

(K2) suggests that the ordering of any two feasible programs with common tails is invariant to the replacement of their common tail by another one, as long as their new tails still coincide. It implies that 'future' consumption is weakly separable from incipient consumption (no habit persistence). It is important what (K2) does not exclude: *incipient consumption is not weakly separable from future consumption.* Thus, preferences are not additively separable. By repeated application such that the reference period is successively pushed forward along the time axis, it follows that preference between tails of alternative programs is independent of head-consumption, but preference between the heads of alternative programs may depend on tail-consumption.

(K3) excludes indifference between incipient consumption alternatives, and suggests that in the preference relation is sensible to finite differences between programs. It excludes the trivial case in which all programs are equivalent and, thus, all programs are optimal. If (P1)-(P4) and (K1)-(K3) hold simultaneously, then $\succeq \subset \Lambda \times \Lambda$ is a *recursive preference relation.* Note that if

(P2') $\succeq_i$ is strictly monotone, i.e., $x \geq y, x \neq y$ implies $x \succ y, \forall x, y \in \Lambda$,

$\forall i \in I$, then recursive preferences are obtained by substituting (P2') for (P2) and dropping (K3), which follows from strict monotonicity, so that (P1), (P2'), (P3), (P4), (K1), and (K2) hold.

8.3 The Degree of Myopia

If Λ is separable connected, properties (P1) and (P4) guarantee the existence of a continuous utility function $U : \Lambda \mapsto \mathbb{R}_+$ representing $\succeq_i$ on $\Lambda \subset \ell_\infty$, iff $x \succeq_i y$ implies $U(x) \geq U(y), \forall x, y \in \Lambda, \forall i \in I$. U is asymptotically myopic in the sense that if $U(x) \geq U(y)$, then $U(x) \geq U(y) + U(\pi^n 0, S^n z), \forall x, y \in \Lambda, \forall z \in \pi\Lambda$. (K1)-(K3) together ensure that $\succeq_i$ is represented by a *recursive utility function*, involving a variable pure rate of time preference. That is, there is a felicity function $u : \mathbb{R}_+ \mapsto \mathbb{R}$ and a time aggregator $W : u(\mathbb{R}_+) \times U(S\Lambda) \mapsto \mathbb{R}$, which summarizes the prospective utility derived from Sx in a single index, such that $\forall x \in \Lambda$,

$$U(x) = W(u(\pi x), U(Sx)) = W(u(\pi x), W(u(\pi^2 x), W(u(\pi^3 x), W(u(\pi^4 x), \ldots)))), \quad (8.1)$$

called the 'Koopmans Equation' (Koopmans 1972, Becker and Boyd 1993).

The aggregator is non-decreasing in both of its arguments, i.e., for $x, x' \in \Lambda, u(\pi x) \geq u(\pi x')$ and $U(Sx) = U(Sx')$, we have $W(u(\pi x), U(Sx)) \geq W(u(\pi x'), U(Sx'))$. Likewise, for $u(\pi x) = u(\pi x')$ and $U(Sx) \geq U(Sx')$, we have $W(u(\pi x), U(Sx)) \geq W(u(\pi x'), U(Sx'))$. Further, W obeys the Lipschitz condition of order one, which is a continuity proposition and accordingly reflects asymptotic myopia with respect to the aggregator, i.e., $\exists \delta > 0$ s.t. $|W(u(\pi x), U(Sx)) - W(u(\pi x), U(Sx'))| \leq |U(Sx) \leq U(Sx')|, \forall \pi x \in \mathbb{R}_+, U(Sx), U(Sx') \in S\Lambda$ with $\delta = \sup\{W_2(u(\pi x), U(Sx))\}$. W_2 is the partial with respect to the second coordinate in W (similar for W_1) so that W must be differentiable. Future utility is discounted by *at least* δ, if $\delta < 1$, so δ generally imposes a bound on W (Boyd 2006).

The marginal utility of an incremental increase of t-th period consumption, denoted by $U_t(x)$, is given by

$$U_t(x) = W_2(\pi x, U(S^1 x)) W_2(\pi^2 x, U(S^2 x)) \ldots W_2(\pi^{t-1} x, U(S^{t-1} x)) W_1(\pi^t x, U(S^t x)),$$

taking into account that the benefit accruing in the t-th period is measured by the derivatives of the indexes of prospective utility in preceding periods. The incremental benefit is subjectively discounted to its present value. The *marginal rate of intertemporal substitution* is defined by the relation

$$\frac{U_t(x)}{U_{t+1}(x)} = \frac{W_1(\pi^t x, U(S^t x))}{W_2(\pi^t x, U(S^t x))W_1(\pi^{t+1}x, U(S^{t+1}x))}, \tag{8.2}$$

which measures the subjective opportunity cost of substituting future consumption for present consumption. Due to the stationarity assumption we have $1 + \sigma_{t,t+1}(x) = 1 + \sigma_{1,2}(x)$. To isolate pure time preference from the smoothing bias, it is assumed that the program is locally stationary, i.e., $x_t = x_{t+1}$, such that the discount rate measuring myopia, denoted by β, is given by

$$\beta_{t,t+1}(x) = \frac{1}{W_2(\pi^t x, U(S^t x))} = \frac{1}{1 + \tilde{\sigma}_{t,t+1}} \leq \delta < 1, \tag{8.3}$$

where $\tilde{\sigma}_{t,t+1} = W_2(\pi^t x, U(S^t x)) - 1 > 0$ is the *local net rate of pure time preference.* Evidently, it is a dependent variable. For globally stationary programs, we have $\sigma_{t,t+1}(\bar{x}) = \tilde{\sigma}_{t,t+1}$. Hayek's fundamental contribution in UAI highlights this relationship. Whereas Fisher is first to endogenize Böhm-Bawerk's second cause, Hayek is - as to my knowledge - the first to specify the *positive* relationship between the pure rate of time preference and prospective utility, i.e., $\partial\tilde{\sigma}_{t,t+1}/\partial U(S^t x)$ due to the assumption that $W_2 = \partial W/\partial U > 0$, whereby he assumes globally stationary programs. The higher the sensitivity of incipient utility to incremental shifts in prospective utility, the larger is the discount, which acts as a second counterforce to intertemporal income effect: in case of an increase in tail-consumption, not only does the subjective resistance to postpone consumption increase due to the smoothing bias, but also because the 'degree of myopia' increases (a cardinal property).

Obviously, the stability or convergence properties of the system are strengthened. Anticipating the Keynes-Ramsey equation, Hayek's system converges even if the marginal productivity of investment is constant and, thus, all burden of adjustment is on the pure rate of time. Recall that the variability in the degree of myopia is induced by intertemporal complementaries. The first utility function in modern literature capturing this relationship is provided by Uzawa (1968) in continuous time, so that the aggregator in (8.1) can be specified by a discrete time version of the modified Uzawa aggregator W_U, defined by

$$W_U(u(\pi x), U(Sx)) = [v(\pi x) + U(Sx)] \exp[-u(\pi x)],$$

where v is a felicity function and u a discounting function. One of Epstein's modification is the use of two felicity functions indicating that the net rate of pure time preference is independent of present consumption.[1] Epstein and Hynes (1983) set $\upsilon(\pi x) = -1$ to isolate the impact of consumption on the rate of pure time preference so that we have

$$W_{EH}(u(\pi x), U(Sx)) = [-1 + U(Sx)] \exp[-u(\pi x)].$$

Consumption does not seem to generate direct utility (Becker and Boyd 1993). W_{EH} yields the EH utility function

$$U(x) = -\sum_{t=1}^{\infty} \exp\left[-\sum_{\tau=1}^{t} u(x_\tau)\right], \tag{8.4}$$

since

$$\left(-1 - \sum_{t=2}^{\infty} \exp\left[-\sum_{\tau=2}^{t} u(x_\tau)\right]\right) \exp[-u(\pi x)] = -\sum_{t=1}^{\infty} \exp\left[-\sum_{\tau=1}^{t} u(x_\tau)\right].$$

Given its recursive nature, (8.4) can be decomposed such that

$$U(x) = -\sum_{t=1}^{T} \exp\left[-\sum_{\tau=1}^{t} u(x_\tau)\right] + U(S^T x) \cdot \sum_{t=1}^{T} \exp\left[-\sum_{\tau=1}^{t} u(x_\tau)\right],$$

with

$$U(S^T x) = -\sum_{t=T+1}^{\infty} \exp\left[-\sum_{\tau=T+1}^{t} u(x_\tau)\right].$$

Note that prospective utility changes only due to variations in the subjective rate of discount. The isolation of utility from the direct impact of consumption allows us to relate the shape of consumption unambiguously to the rate of time preference. It follows that marginal utility can be specified by

$$\begin{aligned} U_T(x) &= -u'(x_T)\left(-1 - \sum_{t=T+1}^{\infty} \exp\left[-\sum_{\tau=T+1}^{t} u(x_\tau)\right]\right) \exp[-u(x_T)] \\ &= -u'(x_T)(-1 + U(S^T x)) \exp\left[-u(\pi^T x)\right] \end{aligned}$$

so that the current value is $U_T(x) = -u'(x_T)(-1 + U(S^T x))$. Accordingly, the marginal rate of substitution is given by

$$\frac{U_T}{U_{T+1}} = \frac{u'(x_T)(-1 + U(S^T x))}{u'(x_{T+1})(-1 + U(S^{T+1} x))}.$$

[1] Uzawa's original aggregator is $W(u(\pi x), U(Sx)) = [\upsilon(\pi x) + U(Sx)] \exp[-\delta(\upsilon(\pi x))]$ with $\delta' > 0$. Thus, the discount rate also depends on current consumption. Epstein substitutes a distinct felicity function and thereby dissolves the dependency of the discount rate from current consumption. This step is a technical simplification without significant loss in economic content. See also Epstein and Hynes 1983: 618.

For a globally constant program $\bar{x} \in \Lambda$, the gross rate of pure time preference - (8.3) - can be specified by

$$\beta_{T,T+1}(\bar{x}) = \frac{1 + \sum_{t=T+1}^{\infty} \exp\left[-\sum_{\tau=T+1}^{t} u(\bar{x}_\tau)\right]}{1 + \sum_{t=T+2}^{\infty} \exp\left[-\sum_{\tau=T+2}^{t} u(\bar{x}_\tau)\right]}.$$

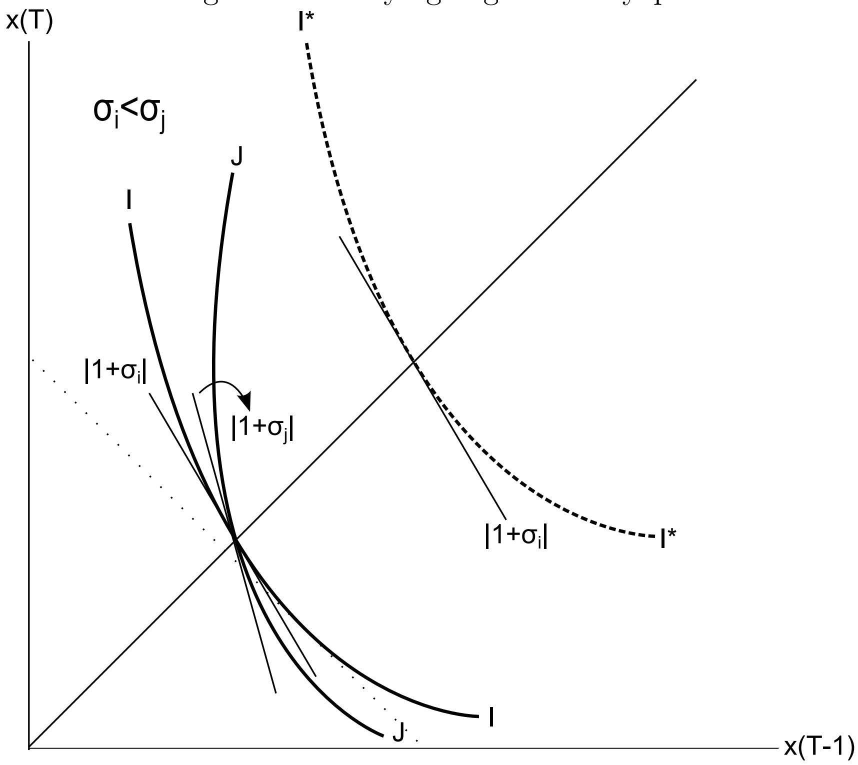
The degree of myopia as the approximate percentage rate at which the marginal utility of consumption decreases over the time horizon is given by

$$\ln \beta_{T,T+1}(\bar{x}) = \ln(1 + \tilde{\sigma}_{T,T+1}(\bar{x})) \approx \tilde{\sigma}_{T,T+1}(\bar{x}) = -\sum_{\tau=T+1}^{t} u(\bar{x}_\tau) + \sum_{\tau=T+2}^{t} u(\bar{x}_\tau) = -u(\bar{x}_{T+1}).$$

Because all alternative programs are assumed to be globally constant, $\bar{x}_{T+1}$ completely summarizes the behavior of $\bar{x}^{(T)}$. Thus, the *steady-state rate of pure time preference* is given by

$$\tilde{\sigma}(\bar{x}) = U(S\bar{x}) \text{ such that } \beta(\bar{x}) = \frac{1}{1 + U(S\bar{x})}, \forall \bar{x} \in \Lambda. \tag{8.5}$$

Figure 8.1: Varying degrees of myopia



This is Hayek's claim that the steady-state rate of pure time preference varies with the level of stationary tail-consumption. In contrast to Fisher (1997b), who argues within the classical tradition that a low prospective consumption level deprives the incentive to forgo early consumption in exchange of more present consumption, Hayek argues that a high prospective consumption level deprives the incentive to add to it. He brings in a satiation-effect: for $\bar{x}_n \in \Lambda, n \in I$, $\bar{x}_n = (n, n, \ldots) \uparrow \infty$ and $\beta(\bar{x}_n) \downarrow 0$ such that $d(U(\bar{x}_{n+1}), U(\bar{x}_n)) \downarrow 0$, or, equivalently, $\partial^2 U / \partial \bar{x}^2 = \frac{\partial^2 U}{\partial^2 U(S\bar{x})} \frac{\partial^2 U(S\bar{x})}{\partial \bar{x}^2} < 0$, since $\frac{\partial^2 U}{\partial^2 U(S\bar{x})} < 0$ and $\frac{\partial^2 U(S\bar{x})}{\partial \bar{x}^2} > 0$. Hayek's utility function exhibits decreasing returns to scale in total consumption. See the case of two periods in Figure 1 (Epstein and Hynes 1983: 617).

Chapter 9

Optimal Growth: The Hayek-Chang Model

9.1 The Hayek Problem

At this stage, we encounter a minor tradeoff. Even though Hayek's graphical expositions suggest a discrete-time analysis, his approach to optimal growth is in continuous time, introduced by a lengthy chapter on the instantaneous rate of interest (Hayek 2007: Ch. 13). So far, however, analysis is couched in discrete time, mostly to foster economic intuition. Therefore, we either slightly depart from Hayek's original approach, or we follow him at the cost of loosening the link to the axiomatic basis introduced above. From a formal point of view, the choice is clear. Continuous time analysis asks for Lebesgue spaces, algebraically structured by a σ-algebra and equipped with a measure. In concrete, the space of equivalence classes of essentially bounded measurable functions, denoted by $\mathcal{L}_\infty$, has to be substituted for ℓ_∞ (with implications for the price space). Evidently, from a formal viewpoint, analysis benefits from continuity in the dimensionality of time.

But as a contribution to the history of economics, the paper takes the opposite route. Not only does a continuous-time approach provide a more precise picture of Hayek's contribution, it also allows to embed it into modern capital and growth theory, which approach the deterministic Ramsey problem by means of optimal control theory (e.g. Acemoglu 2009, Becker 2008). Further, in contrast to some regularity assumptions, the entire economic structure introduced so far survives the switch to continuous time. The literature that approximate Hayek's theory most closely and on which the following is largely based is Chang (1994).

The modified Uzawa aggregator introduced in the previous section reproduces Hayek's theory most closely so that the utility function is given by

$$U(x) = \int_{t=0}^{\infty} \upsilon(x_t) \exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right] dt, \tag{9.1}$$

such that $\upsilon' \geq 0, \upsilon'' \leq 0$ with respect to the instantaneous payoff function υ, and $u \geq 0, u' \geq 0, u'' \leq 0$ with respect to the discount function u, with both, υ, u, being twice continuously differentiable. To simplify calculations, the initial period is $t = 0$ (instead of 1 as in the previous section) such that $\exp[-u(x_0)] = 0$. Following Hayek, (9.1) provides the behavior of the *social planner*.

The twice continuously differentiable aggregate production function, denoted by F, which summarizes efficient intertemporal production opportunities, is given by $F(K(t), 1)$, where $K(t) \geq 0, \forall t$, denotes the stock of capital. Hayek assumes labor to be stationary, so labor input is normalized to 1. Pure scale variations are thereby excluded such that the degree of homogeneity is factually insignificant. The global opportunity set is assumed to be convex. Accordingly, the production function is concave such that $F' \geq 0$, $F'' \leq 0$. Further, F satisfies the Inada conditions. With Hayek, it is further assumed that the rate of depreciation is zero. Accordingly, the differential equation determining the motion of the state variable is simply given by

$$\dot{K}(t) = I(t) = S(t) = F(K(t), 1) - x(t), \tag{9.2}$$

where, as usual, I denotes investment and S denotes saving. Evidently, (9.2) is an equilibrium condition.

To apply Bellman's *Principle of Optimality*, which allows for a recursive formulation of the problem, the discount factor at time t is decomposed such that, for a small $\Delta t \leq t$,

$$\exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right] = \exp\left[-\int_{\tau=0}^{\Delta t} u(x_\tau)d\tau\right] \cdot \exp\left[-\int_{\tau=\Delta t}^{t} u(x_\tau)d\tau\right].$$

Given the histories $\left\{{}^{(\Delta t)}x_n\right\}_{n\in\mathbb{N}}$ and $K(\Delta t) = K(0) + \Delta K$, the present optimal value at Δt is

$$\begin{aligned}
&\max_{\{x(t)\}_{\Delta t \leq t < \infty}} \int_{\Delta t}^{\infty} \upsilon(x_t) \exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right] dt \\
&= \exp\left[-\int_{\tau=0}^{\Delta t} u(x_\tau)d\tau\right] \left\{\max_{\{\cdot\}_{\Delta t \leq t < \infty}} \int_{\Delta t}^{\infty} \upsilon(x_t) \exp\left[-\int_{\tau=\Delta t}^{t} u(x_\tau)d\tau\right] dt\right\} \\
&= \exp\left[-\int_{\tau=0}^{\Delta t} u(x_\tau)d\tau\right] \cdot V[K(0) + \Delta K]
\end{aligned}$$

so that, even though histories affect future choice through the discount factor, the past can be factored out and separated from future consumption (Chang 1994: 428-9).

To simplify notation take $\beta(t) = \exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right]$ and apply the Principle of Optimality such that

$$V[K(0)] = \max_{\{x(t)\}_{0\leq t\leq \Delta t}} \int_{t=0}^{\Delta t} \beta(t)\upsilon(x_t)dt + \beta(\Delta t)V[K(0) + \Delta K] \tag{9.3}$$

$$\Leftrightarrow \max_{\{x(t)\}_{0\leq t\leq \Delta t}} \frac{\int_{t=0}^{\Delta t} \beta(t)\upsilon(x_t)dt + \beta(\Delta t)V[K(0) + \Delta K] - V[K(0)]}{\Delta t} = 0.$$

The *Hayek Problem* is given by

$$V[K(0)] = \max_{\{x(t)\}_{t\in\mathbb{N}}} \int_{t=0}^{\infty} \upsilon(x_t) \exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right] dt \tag{9.4}$$

subject to

$$\dot{K}(t) = F(K(t), 1) - x(t)$$

and

$$K(t) \geq 0, \forall t, K(0) > 0,$$

which solves for the value function V, i.e., for the optimal value of the dynamic maximization problem starting at time $t = 1$ with the state variable $K(1)$.

For $\Delta t \to 0$ equation (9.4) transforms to the *Bellman equation* for the Hayek Problem:

$$\max_{\{x(t)\}_{0\leq t\leq \Delta t}} \{\upsilon(x_0) - u(x_0)V[K(0)] + \lambda[F(K(0), 1) - x_0]\} = 0, \tag{9.5}$$

with the costate $\lambda = V'[K(0)]$ being the shadow price of capital. For $x_0 = x$ and $K(0) = K$, the *first-order condition* of (9.5) with respect to the control variable is given by

$$\upsilon'(x) - u'(x)V(K) - V'(K) = 0 \tag{9.6}$$

It easily follows that the *current-value* costate variable is specified by

$$\mu = V'(K) = \upsilon'(x) - u'(x)V(K), \tag{9.7}$$

so that the present-value, denoted by the standard λ, is given by $\lambda = \beta(t)\mu(t)$.

With Chang it is assumed that (9.5) is strictly concave in x, i.e., the *second-order condition* is

$$\upsilon''(x) - u''(x)V(K) < 0, \tag{9.8}$$

Since the curvature of the payoff function is defined by $-x\upsilon''(x)/\upsilon'(x)$ and the curvature of the discount function is defined by $-xu''(x)/u'(x)$, it follows that whenever $\upsilon'(x) > 0$ and $u'(x) > 0$, then $\mu > 0$, iff

$$-\frac{\upsilon''(x)}{\upsilon'(x)} \leq -\frac{u''(x)}{u'(x)} \tag{9.9}$$

with $u'' < 0$ as assumed above. (9.9) says that the curvature of the discount function has to be at least as concave as the instantaneous payoff function. Given (9.8) and (9.9), we have

$$\upsilon'(x) \geq -\upsilon''(x)\left[-\frac{u'(x)}{u''(x)}\right] > u'(x)V(K),$$

and by (9.7) it follows that $\mu > 0$.

For the Hayek Problem approached by the current-value Hamiltonian:[1]

$$\hat{H}(t, x, K, \mu) = \upsilon(x(t)) + \mu(t)\left[F(K) - x(t)\right]. \tag{9.10}$$

The FOCs of the model are:

$$\hat{H}_x(t, x, K, \mu) = \upsilon'(x(t)) - \mu(t) = 0 \Leftrightarrow \upsilon'(x(t)) = \mu(t), \tag{9.11}$$

$$\hat{H}_K(t, x, K, \mu) = u(x(t))\mu - \dot{\mu}, \tag{9.12}$$

[1]The for discounted infinite-horizon optimal control, the Hamiltonian is given by

$$H(t, x, K, \lambda) = \exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right]\{\upsilon(x(t)) + \mu(t)\left[F(K) - x(t)\right]\}$$

with the costate equation

$$\frac{\partial[\beta(t)\mu(t)]}{\partial t} = H_K(t, x, K, \lambda) = -\lambda(t)F'(K).$$

$$\Leftrightarrow \dot{\mu} = -\mu \left[F'(K) - u(x(t))\right] \Leftrightarrow \frac{\dot{\mu}}{\mu} = -\left[F'(K) - u(x(t))\right]$$

$$\hat{H}_\mu(t, x, K, \mu) = \dot{K}(t) = F(K(t)) - x(t), \tag{9.13}$$

and the transversality condition

$$\lim_{t\to\infty} \left[\exp\left[-\int_{\tau=0}^{t} u(x_\tau)d\tau\right] \mu(t)K(t)\right] = 0. \tag{9.14}$$

Differentiating (9.7) in time yields $\dot{\mu}(t) = \left[\upsilon''(x(t)) - u''(x(t))V(K(t))\right]\dot{x}(t)$ (see (9.8)). Dividing by $\mu(t)$ defines *capital gains* by

$$\frac{\dot{\mu}(t)}{\mu(t)} = \frac{\left[\upsilon''(x(t)) - u''(x(t))V(K(t))\right] \cdot \dot{x}(t)}{\upsilon'(x(t)) - u'(x(t))V(K(t))}.$$

Extending by $x(t)$ and substituting the expression into (9.12) yields the *Euler equation* for the Hayek Problem

$$\frac{\dot{x}(t)}{x(t)} = \frac{1}{\eta(x(t))}\left[F'(K) - \sigma(x(t))\right] \tag{9.15}$$

where

$$\eta(x(t)) = -\frac{\left[\upsilon''(x(t)) - u''(x(t))V(K(t))\right] \cdot x(t)}{\upsilon'(x(t)) - u'(x(t))V(K(t))} > 0 \tag{9.16}$$

is the *elasticity of intertemporal substitution* (see Epstein 1987b, eq. (6), and Chang 1994, eq. 12) , and

$$\sigma(x(t)) = u(x(t)) + u'(x(t))\dot{K}(t) \tag{9.17}$$

is the *rate of pure time preference.* The first term on the r.h.s. is the discount rate at a given consumption level, while the second term is its instantaneous rate of change due to the implicit saving decision. Since the marginal productivity of capital is variable, equation (9.15) is the Keynes-Ramsey type of equation for Hayek's generalized model in UAI, chapter four, and PTC, chapter eighteen.

9.2 Existence, Uniqueness, and Stability

In this section we follow Hayek and assume that $v > 0, v' > 0, v'' < 0$ and $u > 0, u' > 0, u'' < 0$. The *stationary point* of the Hayek-Chang model, denoted by the pair $(\bar{x}^*, K^*)$, is defined by $\dot{x} = \dot{K} = 0$ so that

$$\dot{K} = F(K) - x = 0 \Leftrightarrow \bar{x} = F(K) \tag{9.18}$$

and

$$\dot{x} = F'(K) - u(x) = 0 \Leftrightarrow F'(K) = u(\bar{x}) \tag{9.19}$$

hold simultaneously, where K^* is the *modified golden-rule* stock of capital. Since capital is permanent and population stationary, (9.18) says that in long-run equilibrium all output is consumed. Accordingly, its graph, denoted by θ_1 in Figure 2, is equivalent to the graph of the production function. The upper bound of accumulation, denoted by K_{max}, is reached whenever the diminishing marginal productivity of capital falls to zero, i.e., $F'(K) = 0$. This is Schumpeter's optimal capital stock, since he insists that $u(\bar{x}) = 0$ (Schumpeter 1934). Given the interval $\left[0, \bar{K}\right]$, $u(\bar{x}) > 0$ gives an interior solution.

Equation (9.19) states the equality of the objective marginal benefits of accumulation, given by the marginal productivity of capital, and the subjective marginal costs, measured by the rate of pure time preference. In contrast to the standard Ramsey problem with time-additive utility, the graph of (9.19), denoted by θ_2, is not vertical but sloping downwards, i.e.,

$$\frac{\partial \bar{x}}{\partial K} = \frac{F''(K)}{u'(\bar{x})} < 0,$$

since $F''(K) < 0$ and $u'(\bar{x}) > 0$ (Chang 1994: 432). The higher the level of stationary consumption or, equivalently, of long-run output, the higher is the degree of myopia. The higher the discount, the higher is the equilibrium rate of return to capital, and the lower is the optimal capital stock K^*.

As shown in Figure 9.1, it follows that for $\bar{x} > 0$ the modified golden-rule level of capital for the Hayek Problem is strictly smaller than the optimal stock in case of the canonical Ramsey model, that is, in case of a constant degree of myopia (indicated by the dashed vertical line). This is what we would expect from a satiation effect.

The unique intersection of θ_1 and θ_2 defines the stationary point $(\bar{x}^*, K^*)$. As shown by Chang (Theorem 3.1), a higher recursivity, that is, a discount factor $\tilde{u}(\bar{x}) > u(\bar{x})$, suggests a leftward shift of θ_2, the new position being given by $\tilde{\theta}_2$.

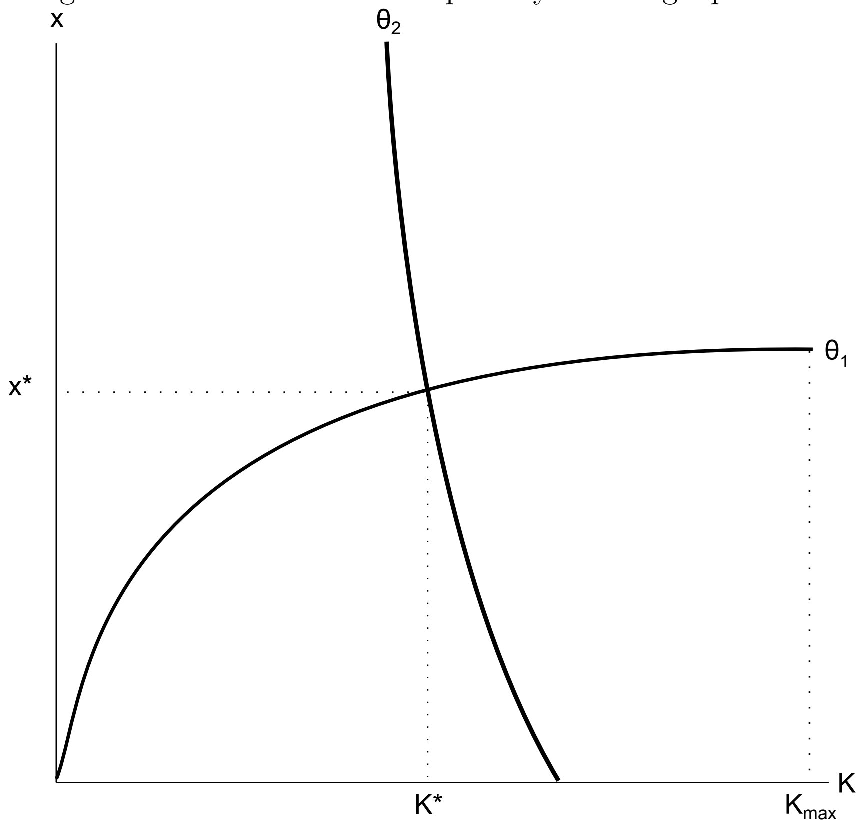
Together with (9.13), equation (9.15) determines the motion or *transitional dynamics* of Hayek's system. Applying the implicit function theorem yields

$$R(x,K) = F'(K) - u(x) - u'(x)\left[F(K) - x\right] = 0 \tag{9.20}$$

such that K can be written as a function of x as capital gains are always zero, and such that

$$\Leftrightarrow F'(K) = u(x) + u'(x)\left[F(K) - x\right]. \tag{9.21}$$

Figure 9.1: Existence of a unique Hayek-Chang equilibrium

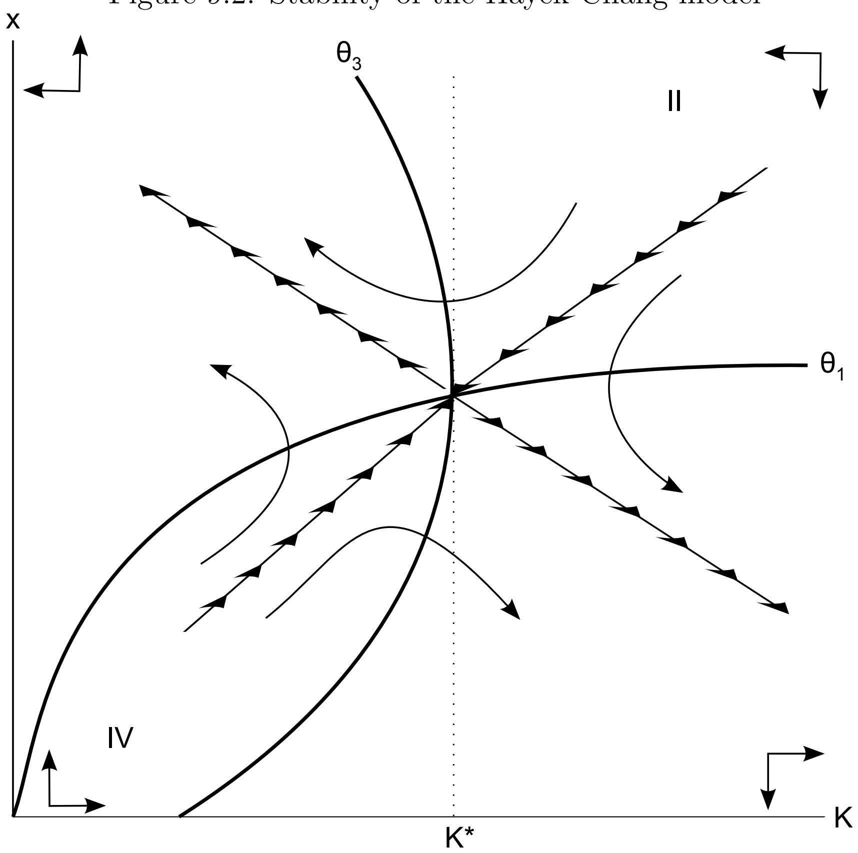


Note that in case of capital accumulation ($\dot{K} > 0$), (9.15) suggests positive capital gains and consumption growth. Likewise, in the case of capital consumption or decumulation ($\dot{K} < 0$) we have capital losses and negative growth. Informally, (9.20) set the capital gains (losses) back to zero and ask for a consistent capital stock. See the phase diagram in Figure 9.2 for the graph of (9.20), denoted by θ_3. Its slope is given by $R_x \frac{\partial x}{\partial x} + R_K \frac{\partial K}{\partial x} = 0 \Leftrightarrow$

$$\frac{\partial K}{\partial x} = -\frac{R_x}{R_K} = \frac{u''(x)\left[F(K) - x\right]}{F''(K) - u'(x)F'(K)}. \tag{9.22}$$

R_K measures the decreasing efficiency of accumulation, composed of the rate at which the marginal benefits diminish, given by $F''(K) < 0$, and the addition to the rate at which these marginal benefits are discounted, $u'(x)F'(K) > 0$, so that $R_K < 0$. R_x summarizes by how much this additional discount diminishes, depending on the speed of convergence.

Figure 9.2: Stability of the Hayek-Chang model



Since $R_K < 0$ and $u''(x) < 0$, the sign of $\partial K/\partial x$ depends on the sign of $\dot{K} = I = S$. If $\dot{K} > 0$, then an incremental increase in consumption lowers the marginal discount $u'(x)$ such that - by (9.21) - a lower marginal productivity suggests a higher capital stock. It follows that θ_3 is increasing whenever $\dot{K} > 0$. In contrast, if $\dot{K} < 0$, then an incremental addition to consumption and the associated decrease of the marginal discount suggest a higher marginal productivity and, thus, a lower level of capital such that θ_3 is decreasing. Thus, θ_3 is bell-shaped (Chang 1994: 436-8).

As usual, θ_1 and θ_3 divide the first quadrant of the (x, K)-plane into sectors I-IV. The vertical arrows in Figure 9.2 indicate that

$$\dot{x} < 0, \text{ if } (x, K) \text{ is to the right of } \theta_3 \;\&\; \dot{x} > 0, \text{ if } (x, K) \text{ is to the left of } \theta_3.$$

The horizontal arrows show that

$\dot{K} < 0$, if (x, K) is above θ_1 & $\dot{K} > 0$, if (x, K) is below θ_1.

The transversality condition (9.14) restricts the initial guess $(x(0), K(0))$ to sections II and IV. Since $\partial K/\partial x = 0$ for $x = \bar{x}^*$, $(\bar{x}^*, K^*)$ is a *saddle-point equilibrium* such that the system given by (9.13) and (9.17) converges on a unique path, which is an optimal accumulation program for $(x(0), K(0)) \in$ IV, and an optimal decumulation program for $(x(0), K(0)) \in$ II.

We can isolate the peculiarities of the Hayek-Chang model by means of Hayek's specification of the stock-flow relationship in UAI and chapter seventeen of PTC. The context is given by the debate on the proper configuration of Böhm-Bawerk's three causes of interest. On the one hand, Mises and Fetter argue that myopia (the seconnd cause) unidirectionally determines the rate of interest. On the other, Knight (1931) insists that marginal productivity alone (the third cause) determines the interest rate. Given this context, Hayek formulates two questions (Hayek 1936: 53):

> "What is the relative influence of the productivity element and the psychological attitude respectively in determining the rate of interest while the process of saving continues?"

> "At what point will that process come to an end, and on what will the rate of interest depend in that final stationary state?"

With regard to the first question, Hayek sides with Knight, "with whose more recent statements on this point I find myself in *complete* agreement" (Hayek 2007: 225, fn. 4, my emphasis) He thereby refers to Knight's review of Fisher's *Theory of Interest* (Knight 1931). There, Knight provides a neat statement of the *Knight-Hayek position* that is quoted in full:

> "It will not do to say without more ado that if two things are interconnected the control relation between them is mutual. If forces originating in or impinging upon *a* are always effective, *a* moving accordingly and carrying *b* with it, while forces arising in or impinging on *b* are powerless to produce movement, *b* maintaining its relation to *a*, whether the latter is in motion or in rest, then it is surely justifiable to say that *a* determines *b* and not *b*, *a*." (ibid.: 42)

In concrete:

> "The elasticity of saving is one thing and the elasticity of the supply of 'capital' another. To have any picture of the supply, it is necessary to take into account the fact that the saving flowing into the capital market in any short interval of time is an addition to all the capital previously saved and invested in that market up to the beginning of the interval in question. The 'elasticity of supply' must be measured with reference, not to variation in the amount saved in any time unit, in correspondence with variations in the interest rate, but with reference to the variation in the total amount to which the saving of that time is being added. [...] The elasticity of supply of capital must be taken as practically zero; at any given moment, or within any short time-period, the supply is an almost completely fixed quantity." (ibid.: 47)

In a nutshell, Knight and Hayek view the study of transitional dynamics as an exercise in short-run analysis with given stocks, a view that reflects the state of the art at that time, such that a sufficiently small horizon suggests constant returns to investment. To ensure dynamic equilibrium, subjective rates have to adapt to that given marginal productivity of capital. In this sense, productivity dominates preferences and, thus, the rate of interest. Irrespective of whether we have sympathy for this viewpoint, it led Hayek to an interesting result concerning the stability properties with increasing pure time preference.

Also with respect to the second question, Hayek sides with Knight that the degree of substitutionality between investment opportunities increases over time. In fact, this view is common among early neoclassical economists. It follows that there exists a counterforce to the 'stock effect' on marginal productivity: whereas an increasing stock of capital decreases marginal productivity, investment decisions adapt such as to change the composition of capital and, thereby, to counter the fall in return on capital invested.

Naturally, the second question is taken up first. For the sake of argument, think of a modified production function $f(K)$, only depending on capital such that constant returns prevail. It is convenient to set $\bar{r} = f'(K) = 1$, where r denotes the instantaneous rate of interest. Thus, it is possible to restate (9.18) and (9.19) such that

$$\bar{x} = f(K) \tag{9.23}$$

and

$$u(\bar{x}) = 1. \tag{9.24}$$

Evidently, also in the long run the subjective discount is dominated by the marginal productivity of capital, and so is the interest rate. Existence and uniqueness is ensured by $u' > 0$. Whenever $u(\bar{x}) > 1$ the stationary level of consumption must fall to ensure equilibrium, while it has to increase whenever $u(\bar{x}) < 1$. The level of long-run consumption determines the modified golden-rule capital stock for a given technology. A stationary point exists and is unique.

This leads us to Hayek's first question. Given increasing myopia,

> "it is evidently possible that the time-preference in a position where present and future income are equal [i.e., the rate of pure time preference], would become as high as the technical productivity of capital. In this case the ultimate stationary equilibrium would be reached with a positive rate of interest, equal to the constant productivity of investment. The psychological attitude would merely determine at what income this point would be reached." (Hayek 1936: 54)

Restate (9.15), (9.20), and (9.22) such that

$$\frac{\dot{x}(t)}{x(t)} = \frac{1}{\eta(x(t))} \left\{ \bar{r} - u(x) - u'(x(t)) \left[f(K(t)) - x(t) \right] \right\}, \tag{9.25}$$

$$R(x, K) = 1 - u(x) - u'(x)\dot{K} = 0, \tag{9.26}$$

$$\frac{\partial K}{\partial x} = -\frac{R_x}{R_K} = -\frac{u''(x)}{u'(x)}\dot{K}, \tag{9.27}$$

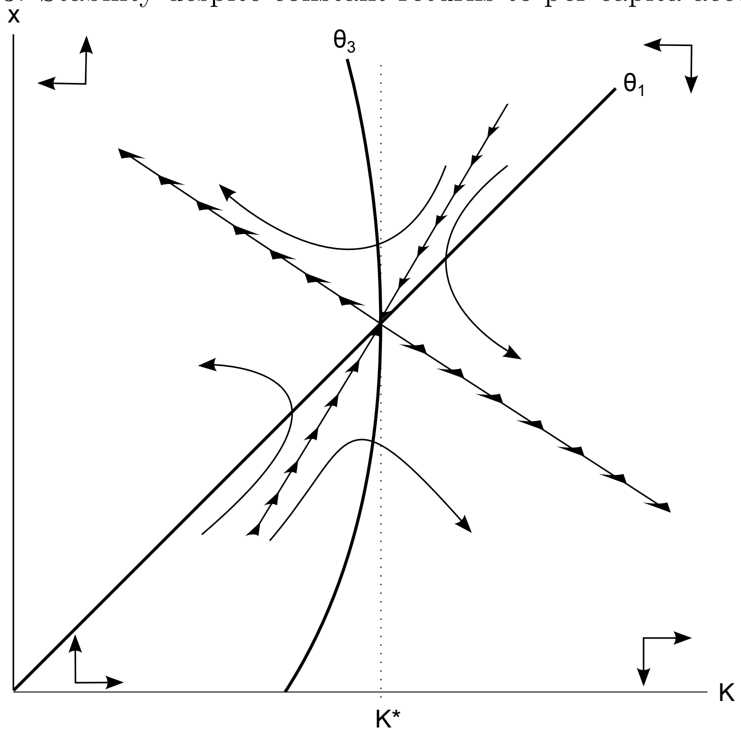
where u''/u' is the curvature of the discount function. The slope of θ_3 is still bell-shaped, but less bended. See Figure 9.3. Still, $\partial K/\partial x = 0$, iff $x = \bar{x}^*$. Even in the absence of diminishing returns to capital, the system is globally stable since $u' > 0$.

Equation (9.25) summarizes *Hayek's central result*, his answer to the first question above. Given constant returns to investment, the instantaneous rate of interest equals this given rate in dynamic equilibrium and all burden of adjustment is on the degree of myopia. We have

$$\dot{x(t)} < 0, \text{ if } \bar{r} < \sigma(x(t)) = u(x(t)) - u'(x(t))|\dot{K}(t)|,$$

$$\dot{x(t)} > 0, \text{ if } \bar{r} < \sigma(x(t)) = u(x(t)) + u'(x(t))|\dot{K}(t)|.$$

Figure 9.3: Stability despite constant returns to per capita accumulation



If $(x(0), K(0))$ is in section IV of Figure 9.3 so that $\dot{x} > 0$, $u' > 0, u'' < 0$ guarantee that the degree of myopia grows at a diminishing rate until it equates the interest rate. The second term on the right hand side of each inequality diminishes over time since it decreases in both components.

> "While the process of saving still continues, the rate of interest will be determined solely by the productivity of investment, and the psychological attitude will merely determine how much will be saved at every moment in order that the marginal rate of time-preference may adapt itself to the given and constant productivity rate. The only role 'time preference' plays in this particular case is that it determines how long it will last until a stationary position is reached." (Hayek 1936: 53)

Chapter 10

Concluding Discussion

By introducing intertemporal complementarities, Hayek extends the scope of general equilibrium analysis to the case of non-stationary preferences. This is not just an unrecognized byproduct of his effort to escape the early Austrian heritage, but a central aspect of his analysis. The present study argues that UAI and the relevant parts of PTC show his willingness to significantly expand the scope of general equilibrium analysis to frame all time-variant economic phenomena that do not result from coordination problems. It is further argued that post-WWII economics innovated toolsets that accommodate his logical narratives with no losses in information. It follows that the widespread speculation that Hayek would have *generally* declined mathematical economics is not well founded.

In some sense, Hayek's model with constant marginal productivity is the antagonist of the standard Ramsey-Cass-Koopmans model. Whereas the latter assumes a constant rate of pure time preference such that convergence is solely due to adjustments in marginal productivity, Hayek follows Knight and assumes constant marginal productivity such that convergence is completely controlled by variations in the rate of pure time preference. The Hayek-Chang model is a generalization of both that allows to relate Hayek's contribution to the mainline of modern capital and growth theory. Considering the diffusion of (stochastic variants of) Ramsey-type models to subbranches of macroeconomics far beyond these two fields, we gain a good idea of how close Hayek's intuition carried him to the bread-and-butter business of modern economics. Seen in this light, and keeping in mind the deficiency of his toolset, the formalization of Hayek's *Utility Analysis and Interest* and *The Pure Theory of Capital* highlights the strength of his economic intuition. Indeed, the fact that Hayek made productive use of general equilibrium theory may increase the weight of his pretense-of-knowledge argument.

From the viewpoint of pure theory, Hayek's contribution is significant. In a competitive economy with heterogeneous agents, if myopia is increasing in consumption and standard regularity assumptions prevail, then agents choose interior solutions to intertemporal allocation problems. By contrast, time-additive utility suggests that, if not the constant rate of time preference equals the rate of interest by chance, agents choose either to lend or to borrow all their prospective income in long-run equilibrium. With respect to the aggregate level, increasing myopia excludes explosive solutions even in case of constant risk-free rates of return. In general, that is, if the marginal productivity of capital is allowed to vary as well, the modified golden-rule stock of capital is smaller than in case of the standard Ramsey problem. Naturally, increasing myopia as an additional force of convergence comes at the cost of a relatively low stationary level of consumption.

This is not to say that Hayek ever was at peace with general equilibrium analysis. He clearly was not, not even at the LSE (Hayek 1937). Hayek's interests guided him to the limitations of general equilibrium theory rather soon, and his skeptical viewpoint indeed constitutes much of his fame today. In a nutshell, he retained the view that empirical time-series ought to be explained by disequilibrium analysis as they result from coordination problems. However, given a dynamic equilibrium as reference path for disequilibrium processes, his early business cycle theory cannot be maintained. In fact, he intended to present a modified version of his integrated monetary theory in a second volume to PTC, but never delivered. With the benefit of hindsight, after Dynamic Stochastic General Equilibrium (DSGE) models came to dominate business cycle theory, it seems evident how his answer could look like, and why his intuition eventually faced strongly diminishing returns on his limited toolset.

In fact, Hayek comes to expect more from Walrasian theory than did Schumpeter, who since his *Wesen und Hauptinhalt der theoretischen Nationalökonomie* (1908) is known as the most Walrasian Viennese, but who during his long career insisted that general equilibrium theory cannot be of *any* use in dynamic analysis.

Part III

The Foundation of Ludwig von Mises's Business Cycle Theory: Real Analysis as a Chain of Tautologies

Chapter 11

Introduction

11.1 Plan of the Study

This paper addresses Ludwig von Mises's business cycle theory at maturity, as advanced in his opus magnum *Human Action*, first published in 1949. In this work, which is one of the last treatises written in the field of economics, Mises embedded the business cycle theory which he initially developed in *Theorie des Geldes und der Umlaufmittel* (published in 1912) into the broad context of his methodological convictions. Whereas the initial outline of his cycle theory strongly relied on Böhm-Bawerk's capital theory, its mature version is built upon a significantly altered framework of real analysis. The paper

- describes and evaluates the impact of Mises's praxeology on his conceptualization of real analysis (i.e., on concepts like 'rationality', 'time preference', the 'originary rate of interest', 'saving', and 'investment');
- provides a simple model - or rather "some unpleasant arithmetics" - to depict and clarify Mises's real analysis;
- draws implications for his monetary disequilibrium analysis and its core prediction that 'any money-induced traverse by necessity reverses' (which is a kind of long-run neutrality postulate);
- argues that this core prediction is the only one that survives Mises's endeavor to deprive his real analytical device of almost all empirical content (by substituting a chain of tautologies for equilibrium analysis proper);
- concludes that Mises's mature business cycle theory is a regression, if compared with its infant version (and with contributions of contemporary economists within or close to the Austrian tradition, like F. A. Hayek and Knut Wicksell).

Section 2 introduces Mises's approach to time preference, starting with Böhm-Bawerk's initial exposition of the theory of interest. Section 3.1 provides a simple 'model' of Mises's real analysis. Section 3.2 analyses the traverse as a genuine disequilibrium phenomena and the role of forced saving therein. Section 4 discusses Mises's business cycle theory. Section 5 concludes.

11.2 Remarks on Praxeology

Even though some remarks on Mises's methodology are necessary to carry out the argument, the paper only provides a crude sketch of Mises's praxeology, no more than is necessary to provide a comprehensive account of his idiosyncratic motives and transcendental views that guided his specific theory choices. Thus, praxeology is introduced to facilitate the comprehensibility of the present study. It is not introduced as a reference point for the evaluation of Mises's real analytical apparatus. His approach is rather appraised for its productivity, that is, its ability to answer a variety of relevant theoretical questions or, equivalently, to generate a rich set of specific testable theories. Any assessment of Mises's methodological convictions is only indirect. The author's presumption is that a methodological approach is to be declined, if it is consistently applied, but yet finds expression in relatively unproductive analytical frameworks. This paper is concerned with the capability of Mises's framework to explain the world, but leaves it to the reader to draw implications with respect to his methodology. Most methodological remarks are contained in the rest of this introductory section.

Mises defined praxeology as the "general theory of human action" (Mises 2009: 3), and human action as "proposeful behavior" (ibid.: 11). To "clarify the definition" and "prevent possible misinterpretations", he provides a connotative definition by adding an equivalence class of definientia: "Or we may say: Action is will put into operation and transformed into agency, is aiming at ends and goals, is the ego's meaningful response to stimuli and the conditions of its environment, is a person's conscious adjustment to the state of the universe that determine his life." (ibid.). Praxeology as the study of "*homo agens*" (ibid.: 14) is supposed to provide access to categorical properties of individual action, like the "law of marginal utility" (ibid.: 119), and, on that basis, to generate "laws of social cooperation"; its scope is to explain the "regularity and invariance" of observable human *inter*action (ibid.: 2). Praxeology is an aprioristic approach, that is, its propositions are deduced from non-contingent statements that are epistemologically available independent of experience. As a believer in Immanuel Kant's notion of synthetic apriori judgments, Mises regarded intuition as insight about external phenomena apart from perception. Introspection gives us access to irreducible propositions that are not hypothetical and, thus, not falsifiable, but rather inescapable and undeniable.

Accordingly, the application axioms of praxeology to economic behavior defines the field of economics. Mises's definition of economics as a subfield of praxeology accounts for the peculiarities of his real and monetary analysis. In general equilibrium analysis, a set of independent axioms is introduced that specifies agents that inhabit a model. Existence demonstrations check for internal consistency. This 'modern' approach conflicts with Mises's axiomatic theory. Here, axioms are dependent and hierarchical. The 'axiom of human action' (humans act!) dominates all other axioms. Equivalently, all other axioms are derived as implications of the action axiom. Rationality, for instance, is a direct consequence of the action axiom:

> "Human action is necessarily always rational. The term 'rational action' is therefore pleonastic and must be rejected as such." (ibid. 18)

Mises's action axiom even determines quantitative relations. With respect to the rate of (pure) time preference, which plays a prominent role in the subsequent study, he argued that it must be positive just because humans act:

> "Time preference is a categorical requisite of human action. *No mode of action can be thought of* in which satisfaction within a nearer period of the future is not - other things being equal [in the sense of constant consumption] - preferred to that in a later period. The very act of gratifying a desire implies that gratification at the present instant is preferred to that at a later instant." (ibid.: 481; my emphasis)

The list of such derived axioms could be extended:

$$\text{action} \Leftrightarrow \text{rationality} \Leftrightarrow \text{positive rate of pure time preference}$$

$$\Leftrightarrow \text{positive 'originary' rate of interest} \Leftrightarrow \ \ldots$$

With a few additional auxiliary assumptions (like 'money exist', 'a banking system exist', etc.), we eventually arrive at Mises's notorious proposition: 'any money-induced traverse *by necessity* reverses'. As will become clear, his real analysis severely suffers from this praxeological approach.

Whereas progressive scientific programs are characterized by the increasing differentiation of concepts, and eventually mature to sophisticated classification systems, Mises deliberately traveled in the opposite direction. Since all action is rational, it is futile to define irrationality. Since time preference is always positive, it is futile to model economies with perishable consumption goods and to ask what happens, if aggregate consumption is expected to fall.

Chapter 12

The Riddle: A Positive Rate of Interest in Stationarity

Mises's difficulty with economic dynamics roots deep, going all the way back to Böhm-Bawerk's exposition of the Austrian theory of interest. There, Böhm-Bawerk posed the question of why there is interest at all, of why capital earns an "excess value" arising from "the fact that the sum of the products created with its help is regularly of greater value than the sum of the costs of the goods expended in the course of production" (Böhm-Bawerk 1959: 6). He rules out the most obvious answer to his question - capital is physically productive and scarce - by the distinction between original factors and produced means of production (capital goods). Only original factors in form of common labor and the raw forces of nature (land as 'virgin soil'; Wicksell 1934: 145) are productive and "constitute the twin fountainheads from which flow all our goods. There is no other spring. There is no possibility of a third elemental source" (Böhm-Bawerk 1959: 80). Capital goods store the result of past productive combinations of the services of original factors. In consequence, the physical productivity of capital goods does not guarantee value productivity. If, in equilibrium, the services of labor and land are paid according to their marginal product, and if capital has no productive power of its own (ibid.), the cost of producing or purchasing capital goods eat up the entire revenue generated by their use. In long-run competitive equilibrium, the value of total product should be accordingly exhausted by outlays to such original factors.

Mises retained this perspective: the marginal value productivity of capital is introduced as a riddle. Whereas it may be "all right for the businessman" to regard capital as a distinct productive factor in his accounting schemes, it is "a serious mistake for the economists to agree with the businessman's superficial view. They erred in classifying 'capital' as an independent factor of production along with the nature-given material resources and labor. The capital goods [...] are not an independent factor. They are the joint products of the cooperation of the two original factors - nature and labor - expended in the past. They have no productive power of their own" (Mises, 2009: 490). It indeed comes as no surprise that both, Böhm-Bawerk and Mises, faced difficulties in identifying a permanent source of net income to capital. Obviously, the formulation of the interest problem leads attention to stationary states. What Böhm-Bawerk attempted to answer are the questions of why interest is not annihilated by the accumulation of capital, why the marginal utilities of present and future consumption are not thereby equalized, and thus, why the return on capital is positive even after the stock of capital has reached its long-run equilibrium level.

To answer the enigma of interest, Böhm-Bawerk introduced three causes for positive interest. The first cause is the "difference between the relation of supply and demand as it exists at one point in time and that relation as it exists at another point in time" (Böhm-Bawerk 1959: 265–6). Thus, interest is due to relative physical scarcities. Relatively abundant future goods suffer a discount, or relatively scarce present goods earn a premium. As Irving Fisher points out, only Böhm-Bawerk's second reason – the "fact" that "we systematically undervalue our future wants and also the means which serve to satisfy them" (Böhm-Bawerk 1959: 268) – is able to account for a positive rate of return on capital in stationarity, putting a break upon the accumulation process before capital's value product, the third cause, vanishes (Fisher 1997a). Böhm-Bawerk believed that the first two causes "work cumulatively", independent of the third (ibid.: 283–4).

Also for Mises, determination and causation were analytically separate concepts. Even as late as 1949, ten years after the publication of John R. Hicks's *Value and Capital* (1939), he still attempted to find a single cause accounting for the positive sign of interest! Even though he employed general equilibrium reasoning in atemporal price theory, he refused to follow F. A. Hayek (1928, 1984a) or Irving Fisher (1997b), and to extend the marginal approach to intertemporal price relations. Mises's hostility towards utility analysis as developed at the London School of Economics (the Ordinalist Revolution; see part one) accounts for the fact that he introduced 'marginal utility' as psychic effects: It is because any quantity of a given good "can only produce a limited [psychic] effect, some things are considered scarce and treated as means" (Mises 2009: 120). Mises's ordinal measure of this "limited effect", his utility function, is negative. He refers to "felt uneasiness", which is to be minimized: "Utility means in this context simply this: causal relevance for the removal of felt uneasiness."(ibid.) The term 'causal' is meant to exclude simultaneous determination. Marginal utilities are disproportionally attached to "the various portions of a supply of *homogeneous* means (diminishing marginal utility). *Each portion is valued separately*" (ibid.: italics mine). Small quantities of any given stock satisfy the most important pleasures or reduce the most unbearable pain. High quantities can additionally satisfy less urgent ends or reduce more tolerable disutility.

They are introduced independent of relative physical scarcities and are as such given data to the economic system. Yet, not only is utility independently assigned to single units of a homogeneous stock, the respective utility assignments are independent of the availability of stocks with different physical characteristics and, for that matter, independent of the date at which these stocks are made available. Otherwise, marginal utilities would be simultaneously determined, and not 'caused' and, thus, of single origin.

The psychological root of Mises's utility analysis is often overlooked, since he vehemently distanced himself from the psychological approaches like the Weber-Fechner law (ibid.: 126), or Gossen's law of the saturation of wants:

> "In treating marginal utility we deal neither with sensuous enjoyment nor with saturation and satiety. We do not transcend the sphere of praxeological reasoning in establishing the following definition: We call that employment of a unit of a homogeneous supply which a man makes if his supply is n units, but would not make if, other things being equal, his supply were only $n-1$ units, the least urgent employment, or the marginal employment, and the utility derived from it marginal utility. In order to to attain this knowledge we do not need any physiological or psychological experience, knowledge, or reasoning. It follows *necessarily* from our assumption that people act (choose) and that in the first case acting man has n of a homogeneous supply and in the second case $n-1$ units. *Under these conditions no other result is thinkable.* Our statement is formal and aprioristic and does not depend on any experience." (ibid.: 124; my italics)

Statements like this have convinced friends and foes alike that Mises's utility analysis has no psychological underpinnings. Yet, Mises essentially argued that *economists* do not have to refer to experience of psychological effects, when they derive the law of diminishing utility. By contrast, marginal utility is introduced as a primitive concept such that the *agents* of his 'model' *do feel* uneasiness (and thus utility), as he himself pointed out. In his quest for the Holy Grail, that is, for the *single, apodictic* cause of a positive rate of interest, Mises refused to introduce utility as a linear transformation of preference relations. He therewith evaded concepts like the 'marginal rate of intertemporal substitution' (subjective opportunity costs), by which general equilibrium theorists like Fisher and Hayek introduced a tradeoff-based subjective relationship between dated commodities. He dismissed Fisher's approach by which he reconciled Böhm-Bawerk's three causes (Fisher 1997b):

The first cause, the preference for consumption smoothing, is given by the convexity of intertemporal indifference curves, which translates into a concave utility function. The second cause, the rate of pure time preference, is given by the slope of indifference curves at the bisecting line across individual choice sets (given the Euclidean textbook exposition for the two-period case). The third cause, the marginal productivity of investment, is introduced by individual transformation curves (objective opportunity costs). Diminishing returns account for increasing opportunity costs. All primitives *simultaneously* determine intertemporal equilibrium prices, i.e., interest rates (in the case of one commodity, or for steady-states, they determine a single rate of interest). As long as the marginal rate of intertemporal substitution at the bisecting line (indicating constant per-capita consumption) exceeds one, a positive rate of interest in stationary economies is no riddle at all: capital accumulates until subjective and objective opportunity costs are equal. Only if the slope at the bisecting line equals one, investments drive the net rate of interest down to zero.

Given the conflict between simultaneous determination and causal hierarchies, Mises's refused to employ Hayek's notion of intertemporal equilibrium (Hayek 1984a). He relied on the archaic concept of the "Evenly Rotating Economy" (ERE) instead, which is an infinite perpetuation of a timeless economy under certainty, in which "today does not differ from yesterday and tomorrow will not differ from today" (Mises, 2009: 248; see Cowen/Fink 1985 for a critical discussion). Note that ERE is so narrow a concept that it even cannot support balanced growth paths! The stationarity of primitives translates into stationary outcomes. Most importantly, constant individual consumption accounts for constant marginal utilities 'over time'. After all, the economy at date $t + 1$ is just a copy of the economy at t. Yet, if marginal utilities at t and $t + 1$ are equal, what then accounts for a positive interest? We have come full circle to Böhm-Bawerk's initial riddle.

So what then is Mises's single, apodictic cause of a positive rate of interest? He dismissed Böhm-Bawerk's first cause: not only does the rate of time preference depend on the shape of consumption profiles, the preference for consumption smoothing has no significance in stationary states. Even worse, decreasing consumption profiles may account for a negative rate of individual time preference (see also Mises 1940: 442–4). He also dismissed Böhm-Bawerk's third cause: in the ERE, all net income is computed to the non-produced or, equivalently, "original" factors of production. Original factors, with the usual emphasis on common labor, are called "future goods", earning according to their marginal product defined in terms of present consumption. Mises is explicit that capital is of zero value productivity in ERE, and that profit income is always and everywhere a disequilibrium phenomenon:

> "[. . .] it becomes evident that it is absurd to speak of a 'rate of profit' or a 'normal rate of profit' or an 'average rate of profit'. Profit is not related to or dependent on the amount of capital employed by the entrepreneur. Capital does not 'beget' profit. Profit and loss are entirely determined by the success and failure of the entrepreneur to adjust production to the demand of the consumers. There is nothing 'normal' in profits and there can never be an 'equilibrium' with regard to them." (Mises 2009: 295)

Somehow, the ERE-rate of interest is supposed to be positive despite the fact that the equilibrium return on investment is zero, that is, despite the fact that capital cannot appropriate - on a value basis - its net productive contribution to society.

Given that the first and third cause are dismissed, only Böhm-Bawerk's second cause remains as the single source of positive interest (here Mises sides with Frank Fetter 1902, 1907, 1927). The systematic undervaluation of future consumption, is introduced as a categorical aspect of human action, in the sense that "the very act of gratifying a desire implies the gratification at the present instant is preferred to that at a later instant" (Mises [2009: 481). In equilibrium and disequilibrium alike, the rate of pure time preference is positive simply because individuals consume (act); after all, they could have consumed later. Q.E.D., so Mises from his praxeological point of view. That's it! Even worse, Mises's built his real analysis entirely on this weak, 'one-footed' basis. A given positive rate of time preference *unidirectionally* determines the equilibrium rate of interest and, as will be shown in the subsequent section, also the choice of technique, the equilibrium level of income, saving, and investment, the equilibrium wage rate, ... just everything. It will be shown that also the hawkish prediction of his monetary theory ultimately depends on his time preference 'theory': the dominance of time preference is responsible for Mises's overemphasis on consumer sovereignty and the proclaimed necessity of mean-reversion.

Finally, note that since ERE-rates of profit are zero, the subjective discount of future needs does not play the role of a limit to the supply of capital that sustains a positive scarcity value and, thus, a source of interest. However, if all equilibrium income is computed to the non-produced factors, and yet the rate of interest positive, *interest must be a transfer payment*, an 'unearned' income (like rent in Adam Smith's model, see part one). This is possible because of the peculiarities of the Austrian capital theory, which introduces consumption goods as a part of the aggregate capital stock (free capital as the 'flexible' fraction of the overall time-fund which, in turn, is measured by the average period of production).[1] The income 'earned' by original factors first remains in the hands

[1]Note also that the average period of production does not determine the rate of interest. Mises's therefore sidesteps the well-established critique of Böhm-Bawerk's theory of interest.

of capitalists, who own firms and their assets. Most importantly, the capitalist class owns all output before sale. In this sense, ownership relations are decisive in establishing a strategic dominance of capitalists which, in turn, accounts for interest payments. Mises meets Marx!

In fact, interest is introduced as a tax on non-ownership: income to capital is a bribe that non-owners pay to owners for the maintenance of the given level of income. Since maintenance means 'food and shelter' for original factors during the average period of production, owners with positive time preference are categorically biased to consume the real savings necessary to sustain the even provision of services over time. Indeed, the entire initial subsistence fund is in danger. At zero interest, yet positive time preference, capitalists gradually eat up the maintenance fund, that is, as consumption goods mature until the entire fund has successively depleted. To induce them not to do so, non-owners earn discounted wages at a rate given by the rate of time preference. At zero time preference, all income would be distributed for free (Mises 2009: 481). Given the initial subsistence fund, only the capitalists' average time preference regulates the bliss level of consumption. In fact, the ERE is a golden-rule economy – at least golden for those with non-productive ownership. Of course, in contrast to Karl Marx, who introduced profit as a surplus, Mises argued that interest is well deserved. After all, the capitalist must be compensated for his sacrifice in terms of present consumption.

Chapter 13

Real Analysis as a Chain of Tautologies

13.1 The Evenly Rotating Economy

Mises's analysis builds on the following primitives: There are finitely many agents that are partitioned into two functional classes. As capitalists, agents own fixed and free capital, and know ways to transform them productively in time-consuming processes of varying duration (working capital is an intermediate notion). The second class is characterized by non-ownership. Membership is described by the supply of non-durable and non-reproducible services. To produce, owners must 'sustain' non-owners during the period of production by 'investing' free capital, that is, by supplying 'food and shelter'. In this sense, 'original' factors are introduced as 'present goods' that mature via the process of production to 'future goods'. For the sake of simplicity, it is assumed that only the services of 'common labor' are available. Total labor supply is given. Beside their initial stocks, all agents are described by the marginal utilities they assign to units of different consumption goods (free capital), and to the different units of a homogeneous stock (diminishing marginal utility) (Mises 2009: 480–99). Each agent is endowed with a positive rate of time preference.

To overcome the problem of how to aggregate over a heterogeneous stock, Mises followed the Austrian tradition, and isolated the time dimension involved in the process of capital deepening: physical capital is introduced as the embodiment of waiting-capacity, a homogeneous substance that is supposed to render aggregation admissible (Mises, [1934]1980: 399; Mises 2009: 483–4): physical capital is "labor, nature, and time stored up" (ibid.: 490). The aggregate waiting-capacity is measured (or indexed) by the average period of production. Note that Mises extended the classical concept of the subsistence fund to include fixed capital (ibid.). All kinds of capital goods are goods-in-process, transformed to intermediaries of 'lower' order over time (Mises 2009: 487-8).

However, even though Mises explicitly introduced fixed capital in form of machinery, the production processes are somehow still of a flow-input-point-output type!

Taking any production process as a reference point, technology is classified according to Böhm-Bawerkian standards: Processes that are more productive and less roundabout are strictly superior. Processes that are less productive and more roundabout, are strictly inferior. In both cases, no economic problem exist. Independent of relative prices (i.e., the interest rate), the former are always favored, and the latter always dismissed. A tradeoff exist for techniques that are less productive, but also less roundabout (capital consumption). Equivalently, a tradeoff exists for techniques that are more productive as well as more roundabout (capital accumulation). At the margins of choice, the interest rate is inversely related to profitability of increased roundaboutness. For any initial level of capital, accumulation runs into diminishing returns, since shorter and more productive investments are chosen first. This gives Mises's aggregate production function,

$$Y_{t+\tau} = f(\tau, L_t), \text{ with } \frac{\partial Y}{\partial \tau} > 0, \frac{\partial^2 Y}{\partial \tau^2} < 0,$$

where $Y_{t+\tau}$ denotes the aggregate point output initiated at date t, τ denotes the average period of production, and $L_t = \overline{L}$ is the total supply of labor, i.e., the total supply of 'present goods', invested at date t. The future is collapsed into single point in time, being 'later' only in respect to the average period of production, a technological variable. Mises's equilibrium outline sheds no light on the processes during the waiting period. Compounding is dispelled.

Let $\mathbf{x} = (x_1, ..., x_N)$ be the complete list of capital goods; $\tau(\mathbf{x})$ is a scalar (an index) on $\mathbf{x}$. Aggregate profits, P, in terms of consumption goods is then given by

$$P = Y - r\tau(\mathbf{x}) - w\overline{L},$$

where r denotes the interest rate, and w the wage rate. Profit is maximized and zero in ERE. The only choice variable of the owners is τ. Thus, the ERE is described by

$$\frac{\partial Y}{\partial \tau} = r.$$

Note that due to diminishing returns, $\partial\tau/\partial r < 0$. Since the ERE is a stationary equilibrium such that $Y_t \equiv C_t, \forall t \in \mathbb{N}$, we have

$$C^* = f(\tau^*, \overline{L}), \text{ with } \frac{\partial C^*}{\partial \tau^*} > 0, \frac{\partial^2 C^*}{\partial \tau^2} < 0,$$

where C denotes total consumption (including free capital advanced to labor), and where primes indicate ERE-values. It follows that

$$\frac{\partial C^*}{\partial r} < 0, \frac{\partial^2 C^*}{\partial r^2} > 0.$$

As Jack Hirshleifer has pointed out with regard to the Austrian capital theory, the rate of interest unidirectionally controls the ERE-level of consumption/output via the choice of technique:

> "To put the matter most forcefully, while Böhm-Bawerk's system (as formalized by Wicksell) purports to be a theory of interest it is actually consistent with any interest rate whatever! Given an arbitrary exogenous determined r, the rest of the system will adapt to conform to that r. Clearly, there is something missing from the model - and the missing element is the 'subjective' or time-preference factor." (Hirshleifer 1967: 195)

As discussed in the previous section, Mises introduced interest income as transfer payment. In ERE,

$$C^* \equiv R + W \equiv rC^* + w\overline{L},$$

such that R denotes interest income due to the privilege of ownership, and W is the wage bill. The interest rate is thus defined by an equivalence relation:

$$r \equiv \frac{C^* - w\overline{L}}{C^*} \equiv \frac{C^* - S}{C^*} \equiv \rho > 0,$$

with $S = w\overline{L}$ as gross saving, i.e., the free capital advanced to labor at each date to maintain the ERE-level of total consumption ad infinitum. Evidently, saving *is* investment! And *the interest rate is the expression of the average rate of pure time preference*, denoted by ρ, and defined as the fraction of total ERE-consumption that is necessary to bribe owners to sustain τ or, equivalently, to convince them not to disrupt the ERE by consuming 'future goods' today. Since they own consumption goods before sale, unproductive owners (in value terms) are free to pay themselves any amount they deem appropriate to compensate their sacrifice without any outside interference. Thus, $r \equiv \rho$, which is how Mises introduced time preference as the single cause of "*originary*" interest:

> "Originary interest is not a price determined on the market by the interplay of the demand for and the supply of capital or capital goods. Its height does not depend on the extent of this demand and supply. It is rather the rate of originary interest that determines both the demand for and the supply of capital or capital goods.
>
> [...] People do not save and accumulate capital because there is interest. Interest is neither the impetus to saving nor the reward or the compensation granted for abstaining from immediate compensation. It is the ratio in the mutual valuation of present goods as against future goods. " (Mises 2009: 523–4)

Evidently, Mises deliberately substituted tautological relations for a system of equations; the fiercest advocate for free markets saw no role for markets in the determination of the equilibrium rate of interest.

The reader may be confused by his claim that interest is no reward for present sacrifices. Yet, in fact, the capitalist class forgoes nothing: instead of paying the laboring class according to its marginal product, the average preference for 'present goods' relative to 'future goods' is enforced by cutting the wage rate. The average rate of time preference *gives* the originary rate of interest, which unidirectionally determines the average period of production and, thus, the marginal productivity of roundaboutness and the ERE-level of total consumption. Given the total supply of labor, also the *adjusted* wage rate is unidirectionally determined:

$$w \equiv (1 - r)\frac{C^*}{\overline{L}}.$$

The inverse relationship between r and w indicates class struggle. Since ρ 'rules the roost' via r, the property-based system favors the capitalist class.

In a nutshell, to arrive at categorically true statements, Mises *translates ex-post identities into ex-ante identities.* The rate of time preference is introduced as the dominant primitive. It is positive simply because 'humans act'. "No other result is thinkable" (see section 2). All the rest then just follows from a short chain of tautologies, and a minimum of production theory. With respect to Mises's business cycle theory, it is of particular importance that the rate of time preference alone controls the equilibrium level of total output and consumption via the choice of technique. Since the rate of time preference is assumed to be given, there are no other factors - monetary or real - that could ever affect the ERE-levels.

13.2 Traverse and the Rate of Time Preference

Mises built his disequilibrium analysis on the infertile sole introduced above. Not only is the ERE a proper mental framework to depict disequilibrium phenomena, so Mises, it is also superior to competing equilibrium notions:

> "It is [...] preposterous to maintain that the construction of the evenly rotating economy does not elucidate conditions within a changing universe and to require the economists to substitute the study of 'dynamics' for their alleged exclusive occupation with 'statics'. The so-called static method is precisely the proper mental tool for the examination of change." (Mises 2009: 248)

Not only business cycle theory, he insisted, but any analysis of economic change must be embedded into the ERE. Net saving becomes the single source of pure profits, per definition unexpected, due to exogenous increase in the average rate of time preference. Mises defined 'net profit', P_n, as the excess of overall windfall profits, $\widehat{P}$, over aggregate losses, L, being positive only if net savings are positive, indicating a change in the rate of time preference (ibid.: 292–3):

$$P_n \equiv \widehat{P} - L \equiv \frac{dS}{d\rho} \equiv \frac{S_n}{dr},$$

where S_n gives net saving as a disequilibrium phenomenon.

All uncertainty usually associated with windfalls, is relegated to the composition of consumption. The level of net profits is always and by necessity equal to the rate of change in the rate of originary interest. Over time, Mises is confident, factors combined in new processes can appropriate more and more of their contribution to long-run equilibrium, until stationarity is finally reached and net saving and thus net profit again disappear. The economy is stable:

> "What happens in the short-run is precisely the first stages of the chain of successive transformations which tend to bring about the long-run effects. The long-run effect is in our case the disappearance of entrepreneurial profits and losses. The short-run effects are the preliminary stages of this process of elimination which finally, if not interrupted by a further change in data, would result in the emergence of the evenly rotating economy." (Mises 2009: 294)

Not a "rational theory of saving" based on dynamic notions of equilibrium as developed by Wicksell (2002) and Hayek (citeyearhayek1936), but the law of supply and demand regulates Mises's traverse from one stationary state to another. Excess demands and price adjustments, of course, do not fit the use of ex-ante identities. Mises escaped into monetary analysis and introduces the market for loanable funds. Note however that the capital market is only operative in disequilibrium; in ERE, it is redundant. Money is a veil, yet weakening the link between time preference and the rest of his system in the case of changes in (circulating) money. But also in the absence of additions to money, net saving becomes an excess supply of capital. Net savings drive down the market rate of interest, so that market prices reflect the underlying change of the rate of pure time preference. The dominant capitalists thereby feed the money-price system with their preferences, signaling a change in the optimal technique.

A lower rate of interest induces an excess demand for labor in higher stages of production, the competition between entrepreneurs driving up the net income of higher-order labor. The excess demand for labor in roundabout process is matched by an excess supply of labor in stages of lower order. Reallocation is driven by the law of one price on the labor market. The rising profit of using labor in lower stages of production reduces the supply of consumption and interrupts processes with below average period of production. More and more labor moves upwards in the vertical chain of production, each time leaving interrupted processes that otherwise would have matured, yet at different points in time depending on their respective maturities. The decrease in the production of consumption goods is initiated by a decrease in the capitalists' consumption, the tax on wages falling with the rate of time preference, allowing labor to consume at the initial level over the longer period of production. The price level remains constant until the new processes ripen and total consumption is distributed to the original factors by a falling price level (minus the lower fraction consumed by the capitalist).

The investor's only job is to transform capital and to control the composition or structure of capital, not to control the growth of capital. They are entrepreneurs only with regard to the cost-minimizing combination of heterogeneous inputs, while savers determine the period of production (Mises 2009: 555). They rely on the interest rate to reflect such local information. Savers, in turn, lack any detailed technological knowledge related to their choice of technique and therefore depend on the investors entrepreneurial abilities.

When coming to money's nonneutrality, Mises subscribed to the classical version of forced saving, which emphasizes the impact of disproportional additions to the supply of money on the social rate of net time preference by initiating a redistribution of wealth and income among households with different saving propensities, that is, by Cantillon-effects (Mises, [1934]1980: 386, Mises 2009: 545–7; see also Hagemann and Trautwein 1998), with saving and investment however being always equal. This concept of forced saving must be distinguished from its post-classical version, especially prominent during the Interwar period and employed by Hayek (1935). This later version of forced saving rests on the existence of macroeconomic disequilibria, of excess aggregate expenditures, and which Keynes accordingly called 'investment over saving' (Keynes 1930; see also Hayek 1932: 133). Here, Say's Law is an equilibrium condition. Very much like the British monetary analysis he admires, Mises's credit market simply allows investing in other people's productive opportunities (Mises 2009: 543–4).

Mises asserted to be empirically evident what his business cycle theory attempts to prove analytically, namely, "that 'forced savings' [in his sense] can reduce the 'natural rate of interest' only fractionally, as compared with the reduction in the 'money rate of interest' which produces the 'forced savings"' (Mises 2006: 112) Thus, Mises derived the business cycle from a spread between the money rate of interest and a new, lower equilibrium level of the originary rate, "indirectly" reduced by a money-induced redistribution of wealth and income. Not a spread between the money rate of interest and the marginal productivity of capital is Mises's measure of macroeconomic disequilibria, but the spread between the net rate of time preference and the market rate of interest. If the money rate of interest is lowered below the reduced social rate of time preference, capital gains turn positive and investors aim at income above the warranted level. This gap between the money and the new rate of time preference induces what Mises names the 'direct effect' of credit expansion, which 'at first exerts a stronger influence than the displacement of the social distribution which occurs as a consequence of it' (Mises 1957: 391).

Chapter 14

Implications for Business Cycle Analysis

The ‘Austrian’ tradition in business cycle theory begins with Ludwig Mises’s *Theorie des Geldes und der Umlaufsmittel* in 1912. It arises from his attempt to integrate British monetary theory into Carl Menger’s marginal utility analysis. Always threatened to sink into oblivion, yet unfading, the Austrian Business Cycle Theory originates as an isolated chapter (chapter 19 of the English edition, Mises 1957). It is a corollary of Mises’s more comprehensive endeavour to overcome the classical dichotomy. In the first edition of *Theorie des Geldes*, yet only here, Mises is explicit: “A theory of business cycles is not the purpose of this study” (translated from Mises 1912: 433).

Mises introduced his business cycle theory as a refinement and extension of classical monetary orthodoxy. He adored the Currency School, especially David Ricardo and Samuel J. Loyd (Lord Overstone since 1850) for their analyses of monetary cycles. Triggered by the creation of inconvertible banknotes and checked by the specie-flow mechanism, the classical business cycle allegorizes money’s short-run nonneutrality. Correcting for what he considered to be the gravest analytical error of the Currency School – the exclusion of deposits from effective money supply (Mises 1912: ix) – Mises envisioned a world beyond the gold standard, where commercial and central banks can freely manipulate the money supply, unconstrained by the costs of obtaining gold. What mechanism, if any, imposes a check on monetary expansion in a closed or global economy, in the absence of international choice and the threat of external drain? What constrains the creation of credit in excess of savings in Wicksell’s Pure Credit Economy? Here, internal drains are excluded by definition and no rise in absolute prices and credit demand provides the incentive to increase interest rates (Mises 1957: 394). The supply of finance is perfectly elastic, “the credit-issuing banks are able to extend their issues indefinitely” (ibid.: 396). Aggregate expenditure increases in excess of real output, reflecting the excess supply of credit, and deteriorates money’s purchasing power.

It is only in *Theorie des Geldes* where Mises makes extensive use of Wicksell's relation between the money and the natural rate of interest. Wicksell states three conditions for macroeconomic equilibrium (neutrality): first, the equality between the loan rate of interest and the equilibrium rate on capital, second, the equality between saving and investment, and third, the stability of the price level (Wicksell 1935: 193, 207; Myrdal 1939: 387). Either of the first two conditions suggests a constant level of aggregate expenditures. Obviously, for all three conditions to prevail simultaneously, the economy must be stationary. Otherwise, Wicksell's first two conditions do not coincide with the third (see Hayek 1933a, and J. G. Koopmans 1933).

In contrast to Wicksell, however, Mises declined the study of price-level dynamics and applies Wicksell's interest rates gap to analyze the impact of money-induced income effects on relative prices. Insofar, Mises focused on the impact of money and credit expansion on intertemporal coordination. Given stationary intertemporal preferences, money-induced excess demand for consumption goods will sooner or later reverse and annihilate the accumulative impact of any prior excess in investment spending. This hawkish prediction of Mises's business cycle theory is placed on new ground in *Human Action*: mean-reversion becomes the outcome of the dominance Mises attributes to ownership relations. If capitalists do not voluntary save, that is, do not advance consumption to labor so as to keep wages constant over a longer period of production and instead consume, whereas investors choose longer investment periods due to the 'false price' of intertemporal exchange, the subsistence fund depletes before the new processes can mature. Even if total demand remains constant, somehow unaffected by increases in nominal spending, the excess demand for consumption goods turn positive, driving down the price of future goods (i.e. labor) in terms of present consumption. This implies a rise in the market rate of interest to the level predetermined by the rate of time preference. The traverse reverses to the initial level of consumption. To assert that additions to money and nominal demand can permanently increase output above its long-run equilibrium value – so the essence of Mises's cycle theory – is to assume that the increase can be maintained against the owner's will and resistance.

Thus, supply-driven credit expansion only temporarily disconnects the money rate of interest from the rate of time preference and imposes a signal-extracting problem. Mises's investors blindly follow the ups and downs of the loan rate, systematically fooled by the banking system that, in turn, is driven by a wrong mercantilist worldview, reinforced by the fact that the general public persistently tends to fall victim to inflationist ideologies. Even in the first German edition of *Theorie des Geldes*, where Mises still allowed for credit demand to exert some influence on the supply of credit, it is only the entrepreneurs' "call for cheap money" by which to circumvent the limitation imposed by thrift, and by their pressure on political processes that banks create excess credit. What Mises had in mind was the pressure exerted by the sovereign and rent-seekers on the banking system to overcome established "routines" and "conservatism", which is the pressure to move first (Mises 1912: 435). Mises claimed that "according to the Circulation Credit Theory [his label for the business cycle theory], it is clear that the direct stimulus which provokes the fluctuations is to be sought in the conduct of the banks." (Mises 2006: 120). Accordingly, the regularity of cycle is due to ideological ups and down, to the struggle of ideas, and to "the mania for lower interest" (ibid.: 121; Mises 2009: 569) as substitutes for the waves of optimism and panic of older analysis that Mises condemns as unscientific. The only way out comes by a general understanding of Mises's business cycle theory (Mises 2006: 123–4).

Further, it is unclear how malinvestments due to forced saving can possibly occur. As Hicks pointed out against Hayek's early version of business cycle theory, all dynamic analysis is built on time-lags, on frictions in market adjustments (Hicks 1967: 207–8). Mises refers to the money-induced distributional effects, which allow those who receive the money first to have a temporary gain at the cost of those, who receive money later. All prices, including wages, are perfectly flexible. Yet, if prices are flexible, and if all new processes begin with the employment of original factors alone, consumption demand increases in proportion to their higher nominal incomes. Investment spending initiating new processes is not associated with the purchase of capital, but with outlays to labor and land, to wages (and rents). Given the strict verticality of Mises's production processes, any excess supply on the money market is immediately reflected by an excess demand on the labor market and, according to the prevailing rate of time preference, by an inflationary excess demand for consumption goods. Thus, overinvestment – in nominal terms – implies nominal wage hikes.

In this light, Mises's business cycle theory depends on a price-lag, on nominal wages increasing while the prices of final outputs remain temporarily constant (the boom). Otherwise, forced saving could not drive the economy away from its trend path. But this cannot be. If, at full-employment, the circulating supply of money increases, flexible wages and prices increase together. Mises was explicit that there is no time-lag between the increase of capital goods prices and the increase of consumers' goods prices. "With the rise in wage rates the prices of consumers' goods rise too," and they increase at equal rate (Mises 2009: 550) It is indeed this initial rise of consumption-good prices that account for the boom.

> "The general upswing in prices spreads optimism. If only the prices of producers' goods had risen and those of consumer's goods had not been affected,

the entrepreneurs would have become embarrassed. They would have had doubts concerning the soundness of their plans, as the rise in costs of production would have upset their calculations. But they are reassured by the fact that the demand for consumers' goods is intensified and makes it possible to expand sales in spite of rising prices." (ibid.: 550–1; see also 554)

"At any rate, the immediate consequence of credit expansion is a rise in consumption on the part of those wage earners whose wages have risen on account of the intensified demand for labor by the expanding entrepreneurs." (ibid.: 553; italics mine)

Mises's claim that a more than proportional increase in the supply of money can prolong the boom is again inconsistent with his use of original factors of production (Mises 2009: 552). Even if money grows at an accelerating rate, the increase in nominal incomes increases in proportion to the excess funds at any point in time. From the onset, consumers are equipped with additional purchasing power to defend their chosen consumption level.

Wicksell did not apply his apparatus for price-level dynamics to the analysis of business cycles. He is well aware of the fact that increases in nominal wages reduce profit, if prices remain constant, that is, if aggregate demand is fixed. In such a case, investors switch technique, favoring more capitalistic processes. As soon as the level of expenditures shifts, and this is the starting point of Mises's business cycle theory, the economy faces real-wage rigidities. As he pointed out in his critical review of *Theorie des Geldes* (Wicksell 1914: 147–8), and what remains equally true for all of Mises's later formulations of business cycle theory, it remains to be proven that excess credit and the loan rate below its neutral rate induce a switch in technique towards longer processes. In this case, there is nothing left of Mises's business cycle theory.

Chapter 15

Concluding Remarks

A most astonishing fact about Mises is that the solution to the proper relationship between the three causes of interest is right in front of him, elaborated by the most prominent of his contemporaries. For instance, the excellent general equilibrium theorist Fisher stressed the simultaneous determination of interest, treating intertemporal preference and investment opportunities pari passu as primitives (Fisher 1997a, 1997b). Others were even close to or within the Austrian tradition like Wicksell and Hayek. In *Zur Zinstheorie – Böhm-Bawerk's dritter Grund*, written 1926 and published post mortem in 1928, Wicksell referred to Böhm-Bawerk's "cardinal error" (Bortkiewicz 1906). There, he correctly argued that during the accumulation process all three causes of interest collaborate. Very early, Wicksell insisted on a "rational theory of saving" (Wicksell 1954). Indeed, as Boianovsky (1998: 145) pointed out, "in his suggested rule for the determination of saving Wicksell all but wrote Ramsey's equation." Wicksell, like Ramsey, assumes that until stationary conditions prevail, it is the marginal productivity of capital that determines the rate of interest. Pure time preference, important to determine bliss levels, does not bear the burden of adjustment. Over time, the return on new capital approaches the exogenous rate of net time preference until the two rates become equal and Böhm-Bawerk's first cause evaporates (Wicksell 1934: 209).

Most importantly, Ramsey's bliss is not the starting point of analysis, but becomes the long-run goal of optimizing individuals who do not only care about the size of equilibrium income, but also about its time shape during the transition process. As Ramsey (1928: 548) pointed out, "the most remarkable feature of the [his] rule is that it is altogether independent of the production function [...], except in so far as this determines bliss, the maximum rate of utility obtainable." The dynamics of capital formation become separated from the aggregate production function which determines the long-run equilibrium level of income, given the rate of pure time preference.

Mises, however, treated all attempts to improve on the underlying equilibrium framework by reaching beyond the classical long-run concept as a waste of time. It is of course true that optimal growth theory and Ramsey's prominence are post WWII phenomena. However, Mises read extensively and surely knew the works of Wicksell, Fisher and Hayek. And yet, in subtlety and analytical depth, *Human Action* falls short of their contribution. Hicks's *Value and Capital* (1939), published ten years before *Human Action*, is neither mentioned, nor had it left any discernible impression on Mises's general outline. In siding with Frank Fetter once and continuing to do so ever after, Mises persistently declines all major contributions to our present understanding of capital theory since Böhm-Bawerk.

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